Complexity of Universality and Related Decision Problems for Unary Two-Dimensional Automata

MCU 2026

Taylor J. Smith

Department of Computer Science
St. Francis Xavier University
Antigonish, Nova Scotia, Canada

July 27, 2026

Table of Contents

Table of Contents

Two-Dimensional Automata

- A **two-dimensional** (2D) **automaton** is a generalization of a one-dimensional automaton.
- Two major differences:
  1. Different input word
  2. Different transition function

Two-Dimensional Automata

- A **two-dimensional** (2D) **automaton** is a generalization of a one-dimensional automaton.
- Two major differences:
  1. **Different input word**
  2. Different transition function

$$
\begin{array}{cccccc}
\# & \# & \# & \cdots & \# & \# \\
\# & a_{1,1} & a_{1,2} & \cdots & a_{1,n} & \# \\
\# & a_{2,1} & a_{2,2} & \cdots & a_{2,n} & \# \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\# & a_{m,1} & a_{m,2} & \cdots & a_{m,n} & \# \\
\# & \# & \# & \cdots & \# & \#
\end{array}
$$

Two-Dimensional Automata

- A **two-dimensional** (2D) **automaton** is a generalization of a one-dimensional automaton.
- Two major differences:
  1. Different input word
  2. **Different transition function**

$$\delta : (Q \setminus q_{\text{accept}}) \times (\Sigma \cup \{\#\}) \to Q \times \{U, D, L, R\}$$

Deterministic four-way (2DFA-4W)

$$\delta : (Q \setminus q_{\text{accept}}) \times (\Sigma \cup \{\#\}) \to 2^{Q \times \{U,D,L,R\}}$$

Nondeterministic four-way (2NFA-4W)

Restricted and Unary 2D Automata

- 2D automata do not have to be **four-way** automata.
- Restrict the transition function to get:
  - **Three-way** (3W) automata: $\{D, L, R\}$
  - **Two-way** (2W) automata: $\{D, R\}$
- Three-way automata cannot return to a row after moving downward, but they can read symbols multiple times in a row.
- Two-way automata are "read-once".
  - Similar to a one-way one-dimensional automaton.

Restricted and Unary 2D Automata

- 2D automata do not have to be **four-way** automata.
- Restrict the transition function to get:
  - **Three-way** (3W) automata: $\{D, L, R\}$
  - **Two-way** (2W) automata: $\{D, R\}$
- Three-way automata cannot return to a row after moving downward, but they can read symbols multiple times in a row.
- Two-way automata are "read-once".
  - Similar to a one-way one-dimensional automaton.
- If the alphabet Σ of a 2D automaton consists of only one letter, we say the 2D automaton is **unary**.
- We denote this by the shorthand -1Σ.

Decision Problems

- A 2D automaton $\mathcal{A}$ recognizes a language $L(\mathcal{A})$.
- Some common decision problems we can ask about two languages $L(\mathcal{A})$ and $L(\mathcal{B})$:
  - **Membership:** $w \in L(\mathcal{A})$ for some 2D word w
  - **Emptiness:** $L(\mathcal{A}) = \emptyset$
  - **Universality:** $L(\mathcal{A}) = \Sigma^{**}$ (the set of all 2D words)
  - **Equivalence:** $L(\mathcal{A}) = L(\mathcal{B})$
  - **Inclusion:** $L(\mathcal{A}) \subseteq L(\mathcal{B})$
  - **Disjointness:** $L(\mathcal{A}) \cap L(\mathcal{B}) = \emptyset$

Decision Problems

- A 2D automaton $\mathcal{A}$ recognizes a language $L(\mathcal{A})$.
- Some common decision problems we can ask about two languages $L(\mathcal{A})$ and $L(\mathcal{B})$:
  - **Membership:** $w \in L(\mathcal{A})$ for some 2D word w
  - **Emptiness:** $L(\mathcal{A}) = \emptyset$
  - **Universality:** $L(\mathcal{A}) = \Sigma^{**}$ (the set of all 2D words)
  - **Equivalence:** $L(\mathcal{A}) = L(\mathcal{B})$
  - **Inclusion:** $L(\mathcal{A}) \subseteq L(\mathcal{B})$
  - **Disjointness:** $L(\mathcal{A}) \cap L(\mathcal{B}) = \emptyset$
- Note that, for unary 2D automata, many questions about languages reduce to simple dimension-testing!
  - E.g., membership = does $\mathcal{A}$ accept a word of dimension $m \times n$?
  - E.g., universality = does $\mathcal{A}$ accept words of *every* dimension?
- We can represent unary languages as **accepted dimension sets**:

$$D(\mathcal{A}) = \{(m, n) \mid \mathtt{a}^{m \times n} \in L(\mathcal{A})\}.$$

Decidability: Four-Way

- The decidability landscape isn't very interesting.

	2DFA-4W	2NFA-4W
membership	✓	✓
emptiness	✗	✗
universality	✗	✗
equivalence	✗	✗
inclusion	✗	✗
disjointness	✗	✗

Decidability: Unary Four-Way

- The decidability landscape *still* isn't very interesting.

	2DFA-4W-1Σ	2NFA-4W-1Σ
membership	✓	✓
emptiness	✗	✗
universality	✗	✗
equivalence	✗	✗
inclusion	✗	✗
disjointness	✗	✗

Decidability: Three-Way and Two-Way

- We start to recover some positive results.

	2DFA-3W	2NFA-3W
membership	✓	✓
emptiness	✓	✓
universality	✓	✗
equivalence	?	✗
inclusion	✗	✗
disjointness	✗	✗

	2DFA-2W	2NFA-2W
membership	✓	✓
emptiness	✓	✓
universality	✓	✗
equivalence	✓	✗
inclusion	✓	✗
disjointness	✓	?

- Yet more positive results, but not as well-studied.

	2DFA-3W-1Σ	2NFA-3W-1Σ
membership	✓	✓
emptiness	✓	✓
universality	✓	?
equivalence	✓	?
inclusion	✓	?
disjointness	✓	?

	2DFA-2W-1Σ	2NFA-2W-1Σ
membership	✓	✓
emptiness	✓	✓
universality	✓	?
equivalence	✓	?
inclusion	✓	?
disjointness	✓	?

Decidability: Unary Three-Way and Two-Way

- Yet more positive results, but not as well-studied. *Until now!*

	2DFA-3W-1Σ	2NFA-3W-1Σ
membership	✓	✓
emptiness	✓	✓
universality	✓ coNP-hard	∗ coNP coNP-hard
equivalence	✓ coNP-hard	?
inclusion	✓ coNP-hard	?
disjointness	✓	?

	2DFA-2W-1Σ	2NFA-2W-1Σ
membership	✓	✓
emptiness	✓	✓
universality	✓	?
equivalence	✓	?
inclusion	✓	?
disjointness	✓	?

Decidability: Unary Three-Way and Two-Way

- Yet more positive results, but not as well-studied. *Until now!*

	2DFA-3W-1Σ	2NFA-3W-1Σ
membership	✓	✓
emptiness	✓	✓
universality	✓ coNP-hard	* coNP coNP-hard
equivalence	✓ coNP-hard	?
inclusion	✓ coNP-hard	?
disjointness	✓	?

	2DFA-2W-1Σ	2NFA-2W-1Σ
membership	✓	✓
emptiness	✓	✓
universality	✓ P	✓ ELEMENTARY coNP-hard
equivalence	✓ P	✓ ELEMENTARY coNP-hard
inclusion	✓ P	✓ ELEMENTARY coNP-hard
disjointness	✓ P	✓ ELEMENTARY NL-hard

Table of Contents

3W Deterministic: Known Results

- For general alphabet 2DFA-3W:
  - **Membership**: L-complete (Lindgren et al.)
  - **Emptiness and universality**: PSPACE-hard (reduction from DFA-2W universality)
- For unary alphabet 2DFA-3W-1Σ:
  - **Emptiness**: upper bound of NP (follows from 2NFA-3W-1Σ result)
  - What about other complexity results?

	M	E	U	Eq	In	Dis
2DFA-3W	✓	✓	✓	?	✗	✗
2DFA-3W-1Σ	✓	✓	✓	✓	✓	✓

K. Lindgren, C. Moore, and M. Nordahl. Complexity of two-dimensional patterns. *J. Statist. Phys.* 91(5/6):909–951, 1998.

Theorem

The 2DFA-3W-1Σ universality problem is coNP*-hard.*

- It is well known that 3-CNF-SAT is NP-complete.
- From this, we know that 3-CNF-UNSAT is coNP-complete.
- Consider what happens when we encode an instance of 3-CNF-UNSAT into a unary 2D language recognized by a three-way 2D automaton.

R. Karp. Reducibility among combinatorial problems. In *Complexity of Computer Computations*, pages 85–103, 1972.

Theorem

The 2DFA-3W-1Σ universality problem is coNP*-hard.*

Proof Idea

- Let $\phi = C_1 \wedge \cdots \wedge C_s$ be a Boolean CNF formula with at most 3 literals per clause.
- Suppose ϕ contains variables $x_1, \ldots, x_t$.
- Enumerate the first t primes $p_1, \ldots, p_t$ and associate to variable x_i the prime number p_i.
- Given an $m \times n$ 2D word, interpret the number of columns n as an encoding of a truth value assignment:

$$x_i = \begin{cases} \text{false}, & \text{if } n \equiv 0 \bmod p_i; \\ \text{true}, & \text{if } n \equiv 1 \bmod p_i. \end{cases}$$

Theorem

The 2DFA-3W-1Σ universality problem is coNP*-hard.*

Proof Idea (cont'd)

- Construct a unary 3W deterministic 2D automaton $\mathcal{A}_\phi$ that does the following:
  - Compute the residue for the current literal to determine the encoded truth value.
  - Use the sign of the current literal (stored in the finite-state control) to determine whether the current literal is true under the encoded assignment.
  - Update a flag as needed to indicate whether any literal seen in the current clause has evaluated to true.
- If no literal in a clause is true, then $\mathcal{A}_\phi$ accepts.
- If all clauses are satisfied, then $\mathcal{A}_\phi$ does not accept.

3W-1Σ Deterministic Equivalence/Inclusion

- Applying the same logic as our universality result, we get the following.

Corollary

The 2DFA-3W-1Σ equivalence and inclusion problems are coNP*-hard.*

3W-1Σ Deterministic Equivalence/Inclusion

- Applying the same logic as our universality result, we get the following.

Corollary

The 2DFA-3W-1Σ equivalence and inclusion problems are coNP*-hard.*

- We cannot use the same argument to show 2DFA-3W-1Σ emptiness/disjointness is coNP-hard.
  - This model is closed under complement
  - But there is no known poly-time construction to produce a poly-size unary three-way deterministic 2D automaton recognizing the complement language
- We also cannot establish membership in coNP itself.
  - We would need to show nonuniversality is in NP
  - But there is no known way to ensure poly-size witnesses
  - We may not be able to efficiently verify dimensions

Table of Contents

3W Nondeterministic: Known Results

- For general alphabet 2NFA-3W:
  - **Membership**: NL-complete (Lindgren et al.)
  - **Emptiness**: PSPACE-hard (follows from 2DFA-3W result)
- For unary alphabet 2NFA-3W-1Σ:
  - **Emptiness**: NP-complete (Smith and Salomaa)
  - What about other decidability/complexity results?

	M	E	U	Eq	In	Dis
2NFA-3W	✓	✓	✗	✗	✗	✗
2NFA-3W-1Σ	✓	✓	?	?	?	?

Lindgren, Moore, and Nordahl. *J. Statist. Phys.*, 1998.

T. J. Smith and K. Salomaa. Decision problems and projection languages for restricted variants of two-dimensional automata. *Theoret. Comput. Sci.* 870:153–164, 2021.

3W-1Σ Nondeterministic Universality

- Universality is undecidable over general alphabets.
- Over a unary alphabet, the problem still seems pretty tough.
  - We won't even bother then...

- Universality is undecidable over general alphabets.
- Over a unary alphabet, the problem still seems pretty tough.
  - We won't even bother then...
- Let's consider a **'bounded size'** variant of universality instead.
- Consider finite sets $R, C \in \mathbb{N}_{>0}$.
- Denote by

$$D_{R,C} = (R \times \mathbb{N}_{>0}) \cup (\mathbb{N}_{>0} \times C)$$

  the domain of all 2D words having
  - dimension $r \times \mathbb{N}_{>0}$ for $r \in R$, or
  - dimension $\mathbb{N}_{>0} \times c$ for $c \in C$.

- We may split this $\boldsymbol{D_{R,C}}$**-universality** problem into two cases.

Fixed-Column Universality

Fix $c \geq 1$. Given a unary 3W nondeterministic 2D automaton $\mathcal{A}$, is $\mathtt{a}^{m \times c} \in L(\mathcal{A})$ for all $m \geq 1$?

3W-1Σ Nondeterministic Universality

- We may split this $\boldsymbol{D_{R,C}}$**-universality** problem into two cases.

Fixed-Column Universality

Fix $c \geq 1$. Given a unary 3W nondeterministic 2D automaton $\mathcal{A}$, is $\mathrm{a}^{m \times c} \in L(\mathcal{A})$ for all $m \geq 1$?

Lemma

For every unary 3W nondeterministic 2D automaton $\mathcal{A}$ and for all $c \geq 1$, we can construct a unary NFA $\mathcal{B}_{\mathcal{A},c}$ such that, for all $m \geq 1$,

$$b^m \in L(\mathcal{B}_{\mathcal{A},c}) \Leftrightarrow a^{m \times c} \in L(\mathcal{A}).$$

Proof Idea

Construct a row-entry configuration graph and use standard reachability techniques.

- We may split this $\boldsymbol{D_{R,C}}$**-universality** problem into two cases.

Fixed-Row Universality

Fix $r \geq 1$. Given a unary 3W nondeterministic 2D automaton $\mathcal{A}$, is $\mathtt{a}^{r \times n} \in L(\mathcal{A})$ for all $n \geq 1$?

- We may split this $\boldsymbol{D_{R,C}}$**-universality** problem into two cases.

Fixed-Row Universality

Fix $r \geq 1$. Given a unary 3W nondeterministic 2D automaton $\mathcal{A}$, is $\mathrm{a}^{r\times n} \in L(\mathcal{A})$ for all $n \geq 1$?

Lemma

For every unary 3W nondeterministic 2D automaton $\mathcal{A}$ and for all $r \geq 1$, we can construct a unary NFA-2W $\mathcal{C}_{\mathcal{A},r}$ such that, for all $n \geq 1$,

$$c^n \in L(\mathcal{C}_{\mathcal{A},r}) \;\Leftrightarrow\; a^{r\times n} \in L(\mathcal{A}).$$

Proof Idea

Simulate reading one row of the 2D word and maintain a row counter value separately.

3W-1Σ Nondeterministic Universality

- It is known that unary NFA universality is coNP-complete (Stockmeyer and Meyer).
  - Applies to fixed-column universality!
- It is also known that unary NFA-2W universality is coNP-complete (Galil).
  - Applies to fixed-row universality!

L. Stockmeyer and A. Meyer. Word problems requiring exponential time. In *Proc. of STOC 1973*, pages 1–9, 1973.

Z. Galil. Hierarchies of complete problems. *Acta Inform.* 6:77–88, 1976.

3W-1Σ Nondeterministic Universality

- It is known that unary NFA universality is coNP-complete (Stockmeyer and Meyer).
  - Applies to fixed-column universality!
- It is also known that unary NFA-2W universality is coNP-complete (Galil).
  - Applies to fixed-row universality!

Theorem

The 2NFA-3W-1Σ $D_{R,C}$-universality problem is coNP*-complete for every fixed finite $R, C \subseteq \mathbb{N}_{>0}$, or when finite sets $R, C \subseteq \mathbb{N}_{>0}$ are given in unary as part of the input.*

L. Stockmeyer and A. Meyer. Word problems requiring exponential time. In *Proc. of STOC 1973*, pages 1–9, 1973.

Z. Galil. Hierarchies of complete problems. *Acta Inform.* 6:77–88, 1976.

Table of Contents

2W Deterministic: Known Results

- For general alphabet 2DFA-2W:
  - **Membership**: L-complete (Lindgren et al.)
  - **Emptiness**: P (Smith and Salomaa)
- For unary alphabet 2DFA-2W-1Σ:
  - **Emptiness**: P (Smith and Salomaa)
  - What about other complexity results?

	M	E	U	Eq	In	Dis
2DFA-2W	✓	✓	✓	✓	✓	✓
2DFA-2W-1Σ	✓	✓	✓	✓	✓	✓

Lindgren, Moore, and Nordahl. *J. Statist. Phys.*, 1998.
Smith and Salomaa. *Theoret. Comput. Sci.*, 2021.

IBR-Accepting 2D Automata

- When working with two-way 2D automata, the following variant of the model often makes our lives easier.

Definition

An **IBR-accepting** two-way 2D automaton is one where, when the input head reads a boundary marker #, either the automaton enters its accepting state on the next transition, or the transition is undefined.

- Every two-way 2D automaton can be converted to an equivalent IBR-accepting 2D automaton of the same size!
- Going forward, we will assume all two-way 2D automata are IBR-accepting.

2W-1Σ Deterministic Decision Problems

Theorem

The 2DFA-2W-1Σ universality, equivalence, inclusion, and disjointness problems are in P*.*

- Since every input word is over a unary alphabet, consider the sequence of states visited by repeatedly reading the symbol a.
- We read the symbol a until:
  - we enter the accepting state
  - we enter a nonaccepting state with no outgoing transition
  - we enter a nonaccepting state we saw before
- **Two-way**: the sequence of input head movements is monotone.
- **Deterministic**: the sequence of states is unique.
  - These two facts make our job much easier!

Theorem

The 2DFA-2W-1Σ universality, equivalence, inclusion, and disjointness problems are in P.

Proof Idea

- For each transition, keep track of the number of downward and rightward moves made so far in the computation before that transition.
- Classify the sequence of states visited into one of three cases:
  - Enters the accepting state
  - Enters a nonaccepting state with no outgoing transition
  - Enters a repeated nonaccepting state
- Also place the dimension (m, n) of the input word into an **accepting set** $A_{\mathcal{A}}$ or a **rejecting set** $R_{\mathcal{A}}$ as appropriate.

Theorem

The 2DFA-2W-1Σ universality, equivalence, inclusion, and disjointness problems are in P*.*

Proof Idea (cont'd)

- Let $D(\mathcal{A})$ denote the **accepted dimension set** (i.e., the union of all dimensions in $A_{\mathcal{A}}$).
- Let $X(\mathcal{A})$ denote the **rejected dimension set** (i.e., the union of all dimensions in $R_{\mathcal{A}}$).
- Now observe:
  - **Universality**: is $X(\mathcal{A}) = \emptyset$?
  - **Equivalence**: is $(D(\mathcal{A}) \cap X(\mathcal{B})) \cup (D(\mathcal{B}) \cap X(\mathcal{A})) = \emptyset$?
  - **Inclusion**: is $D(\mathcal{A}) \cap X(\mathcal{B}) = \emptyset$?
  - **Disjointness**: is $D(\mathcal{A}) \cap D(\mathcal{B}) = \emptyset$?

Theorem

The 2DFA-2W-1Σ universality, equivalence, inclusion, and disjointness problems are in P.

Proof Idea (cont'd)

- Both $A_{\mathcal{A}}$ and $R_{\mathcal{A}}$ are constructed according to a case-based system of linear inequalities (omitted here).
- Each linear inequality is in a fixed number of integer variables.
- The number of variables is bounded by a constant independent of $\mathcal{A}$.

Theorem

The 2DFA-2W-1Σ universality, equivalence, inclusion, and disjointness problems are in P*.*

Proof Idea (cont'd)

- Both $A_{\mathcal{A}}$ and $R_{\mathcal{A}}$ are constructed according to a case-based system of linear inequalities (omitted here).
- Each linear inequality is in a fixed number of integer variables.
- The number of variables is bounded by a constant independent of $\mathcal{A}$.
- This is an integer linear program!
  - We can solve these in poly-time.

H. Lenstra, Jr. Integer programming with a fixed number of variables. *Math. Oper. Res.* 8(4):538–548, 1983.

Table of Contents

2W Nondeterministic: Known Results

- For general alphabet 2NFA-2W:
  - Membership: NL-complete (Lindgren et al.)
  - Emptiness: P (Smith and Salomaa)
- For unary alphabet 2NFA-2W-1Σ:
  - Emptiness: P (Smith and Salomaa)
  - What about other decidability/complexity results?

	M	E	U	Eq	In	Dis
2NFA-2W	✓	✓	✗	✗	✗	?
2NFA-2W-1Σ	✓	✓	?	?	?	?

Lindgren, Moore, and Nordahl. *J. Statist. Phys.*, 1998.
Smith and Salomaa. *Theoret. Comput. Sci.*, 2021.

- Since our 2D automaton is nondeterministic, we no longer have the guarantee that a sequence of states is unique.
- But not all is lost!
- Consider the sequence of moves over $\{D, R\}^*$ made by a 2D automaton during its computation.

- Since our 2D automaton is nondeterministic, we no longer have the guarantee that a sequence of states is unique.
- But not all is lost!
- Consider the sequence of moves over $\{D, R\}^*$ made by a 2D automaton during its computation.

Proposition

A set is semilinear if and only if the set is definable in Presburger arithmetic.

S. Ginsburg and E. Spanier. Semigroups, Presburger formulas, and languages. *Pacific J. Math.* 16(2):285–296, 1966.

- Since our 2D automaton is nondeterministic, we no longer have the guarantee that a sequence of states is unique.
- But not all is lost!
- Consider the sequence of moves over $\{D, R\}^*$ made by a 2D automaton during its computation.

Proposition

A set is semilinear if and only if the set is definable in Presburger arithmetic.

- Turn the sequence of moves $v \in \{D, R\}^*$ into a Parikh map $\Psi(v) = (|v|_D, |v|_R)$.

S. Ginsburg and E. Spanier. Semigroups, Presburger formulas, and languages. *Pacific J. Math.* 16(2):285–296, 1966.

Lemma

Given a unary 2W nondeterministic 2D automaton $\mathcal{A}$, we can obtain a Presburger arithmetic formula $\phi_{\mathcal{A}}(m,n)$ such that, for all $m,n \geq 1$,

$$\phi_{\mathcal{A}}(m,n) = true \ \Leftrightarrow \ a^{m\times n} \in L(\mathcal{A}).$$

Lemma

Given a unary 2W nondeterministic 2D automaton $\mathcal{A}$, we can obtain a Presburger arithmetic formula $\phi_{\mathcal{A}}(m, n)$ such that, for all $m, n \geq 1$,

$$\phi_{\mathcal{A}}(m, n) = true \Leftrightarrow a^{m \times n} \in L(\mathcal{A}).$$

- It is known that Presburger arithmetic is decidable.

M. Presburger. Über die Vollständigkeit eines gewissen Systems der Arithmetik ganzer Zahlen, in welchem die Addition als einzige Operation hervortritt. In *Comptes Rendus du I Congrès des Mathématiciens des Pays Slaves*, pages 92–101, 1930.

Lemma

Given a unary 2W nondeterministic 2D automaton $\mathcal{A}$, we can obtain a Presburger arithmetic formula $\phi_{\mathcal{A}}(m, n)$ such that, for all $m, n \geq 1$,

$$\phi_{\mathcal{A}}(m, n) = true \iff a^{m \times n} \in L(\mathcal{A}).$$

- It is known that Presburger arithmetic is decidable.
- It is also known that Presburger arithmetic has a decision procedure in ELEMENTARY.

D. Oppen. A $2^{2^{2^{pn}}}$ upper bound on the complexity of Presburger arithmetic. *J. Comput. System Sci.* 16(3):323–332, 1978.

Theorem

The 2NFA-2W-1Σ universality, equivalence, inclusion, and disjointness problems are in ELEMENTARY*.*

Proof Idea

Use the following Presburger arithmetic formulas:

- **Universality**:
  $\forall m \, \forall n \, ((m \geq 1 \wedge n \geq 1) \Rightarrow \phi_{\mathcal{A}}(m, n))$
- **Equivalence**:
  $\forall m \, \forall n \, ((m \geq 1 \wedge n \geq 1) \Rightarrow (\phi_{\mathcal{A}}(m, n) \Leftrightarrow \phi_{\mathcal{B}}(m, n)))$
- **Inclusion**:
  $\forall m \, \forall n \, ((m \geq 1 \wedge n \geq 1) \Rightarrow (\phi_{\mathcal{A}}(m, n) \Rightarrow \phi_{\mathcal{B}}(m, n)))$
- **Disjointness**:
  $\forall m \, \forall n \, ((m \geq 1 \wedge n \geq 1) \Rightarrow \neg(\phi_{\mathcal{A}}(m, n) \wedge \phi_{\mathcal{B}}(m, n)))$

- We can also obtain complexity lower bounds, though the gap is quite large.

2W-1Σ Nondeterministic Decision Problems

- We can also obtain complexity lower bounds, though the gap is quite large.

Theorem

The 2NFA-2W-1Σ universality, equivalence, and inclusion problems are coNP*-hard.*

Proof Idea

Reduce from unary NFA universality.

2W-1Σ Nondeterministic Decision Problems

- We can also obtain complexity lower bounds, though the gap is quite large.

Theorem

The 2NFA-2W-1Σ universality, equivalence, and inclusion problems are coNP*-hard.*

Proof Idea

Reduce from unary NFA universality.

Theorem

The 2NFA-2W-1Σ disjointness problem is NL*-hard.*

Proof Idea

Reduce from ST-NONCONNECTIVITY.

Table of Contents

Conclusions

- We studied various decision problems for unary 2D automata.
- In some cases, we obtained all-new decidability properties.
- In other cases, we proved complexity bounds for known-decidable problems.

	2DFA-3W-1Σ	2NFA-3W-1Σ
universality	✓ coNP-hard	* coNP coNP-hard
equivalence	✓ coNP-hard	?
inclusion	✓ coNP-hard	?
disjointness	✓	?

	2DFA-2W-1Σ	2NFA-2W-1Σ
universality	✓ P	✓ ELEMENTARY coNP-hard
equivalence	✓ P	✓ ELEMENTARY coNP-hard
inclusion	✓ P	✓ ELEMENTARY coNP-hard
disjointness	✓ P	✓ ELEMENTARY NL-hard

Future Work

- What is an upper bound on the complexity of 2DFA-3W-1Σ universality, equivalence, and inclusion?
- What is the status of decidability (and complexity) for other 2NFA-3W-1Σ decision problems?
- Can we come up with a decision procedure for 2NFA-2W-1Σ universality, equivalence, inclusion, and disjointness that doesn't rely on Presburger arithmetic?

Thank you!

taylorjsmith.xyz

taylorjsmith.xyz/flarelab/