Intermediate Microeconomics II, ECON 301
General Equilibrium II: Production

Whereas previously we examined an exchange economy free of production, where agents are born with a fixed set of goods, and exchange or trade towards a mutually agreeable equilibrium, we will now include the production facet into our consideration, but within the context of general equilibrium still. When production is included the amount of goods within a market is no longer fixed, but varies dependent on the market prices for the goods.

1 The Robinson Crusoe Economy

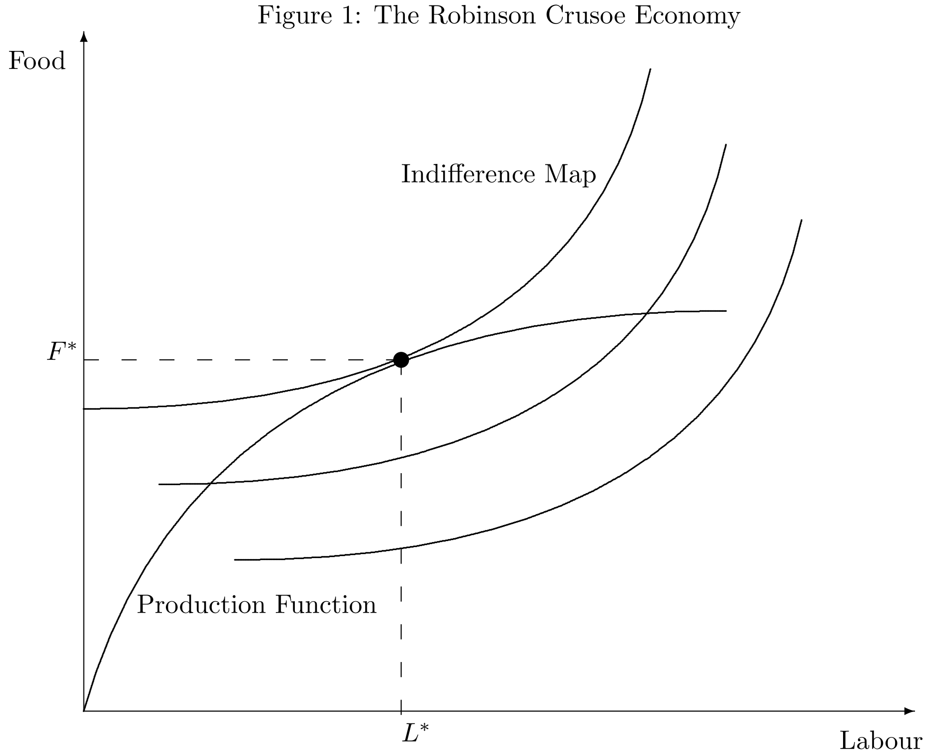
Just as in the previous discussion, we had stipulated the framework first, the minimum assumptions to examine general equilibrium with production are as follows;

1. There are two agents again, one of which is a consumer, and the other is a producer.
2. There are two goods produced within the economy.

Such an economy is typically called a **The Robinson Crusoe Economy**. Robinson Crusoe multi-tasks in the economy by being both the producer and consumer, where he can either choose to enjoy the scenery of the remote island on which he has found himself, or spend his time hunting and gathering. He needs to balance his utility increasing pursuit of day-dreaming or utility diminishing pursuit of production and relieving his hunger. We can depict such an economy in the same manner as before with some modifications.

Noting a few not so subtle differences with respect to your prior understanding of Intermediate Microeconomics, the previous classes, the indifference map now is such that the indifference curves are all upward sloping. This is because labour supply is now on the horizontal axis (and yields lower utility as you increase its supply. Note that in your typical labour-leisure choice, the horizontal axis has leisure as a good.). The production function is as we have always understood it, the greater the inputs (here labour by Robinson, yes, I'm on first name basis with him.) produced. And due to diminishing marginal product of labour, the shape of the production function is concave. The optimal choice of Robinson should then be where his production function just meets his indifference curve, i.e. the tangency point. Suppose it is not, and that equilibrium is where the indifference curve intersects the production function. However, at either of the intersections, Robinson can always raise his utility by increasing (if the intersection is the lower) labour supply, or decreasing it (if the intersection is at the higher point). This choice is given the technology available to Robinson, which is represented by the production function. Should technology change, the production function changes, and consequently

Figure 1: The Robinson Crusoe Economy



the equilibrium choice would be altered. This means that we can represent the equilibrum as;

$$MP_L = MRS_{L,F}$$

2 Robinson decides to Incorporate his Company

The first extension of our exchange model yields results that is expected. However, it is hardly a full depiction of how a real life economy might work. To do so we can separate the goods and labour market. Going with the same story, suppose Robinson chooses to alternate his roles where some days he behaves as a worker, and other days he is the sole shareholder of a firm so that there are now two markets, one for the good and one for labour. As the shareholder and manager of Crusoe Inc (he unimaginatively names himself in his firm, and is the sole stock on the island's borse). The firm first chooses its output based on the labour and good prices that prevail in each market, and determine the optimal mix of input and output, that is how much labour to supply and how much to produce to maximize his profits. As a worker, Robinson would receive an income for his efforts from himself, while as a worker he will collect the profits. In his role as a consumer, he would choose how much to consume given the market prices.

Robinson decides to develop a currency on which everything else is traded. He bases the value of all other goods, labour particularly, in terms of the good, and he arbitrarily values the good at $1, and you should now recall that when we do this, we are making the good produced by Crusoe Inc. the numeraire good. Let the wages Robinson gets as a worker be w

Let's examine first Crusoe Inc. Whenever Robinson decides on holding a board meeting with himself, he wants to decide how much of his worker self he wants to hire, L, and how much good he will produce as a worker, C. Suppose we wants to achieve π profits in any period, then his problem is just;

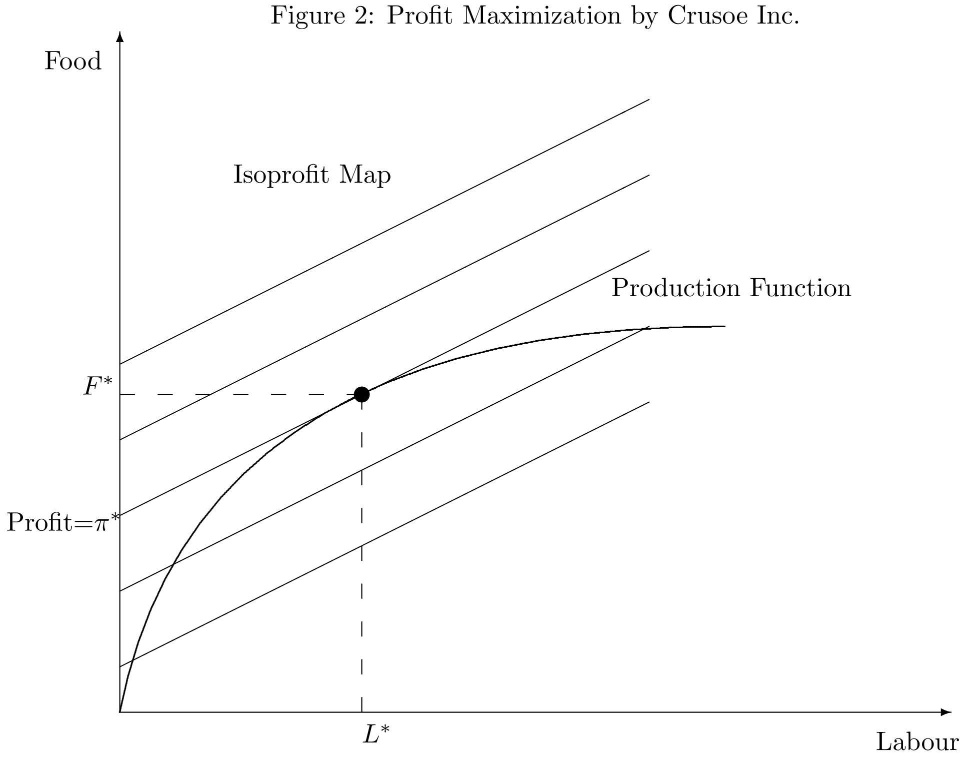
$$\pi = C - wL$$

which means that the amount of goods Robinson will produce is;

$$C = \pi + wL$$

which is just the equation of the isoprofit line. The analysis is depicted below;

Figure 2: Profit Maximization by Crusoe Inc.

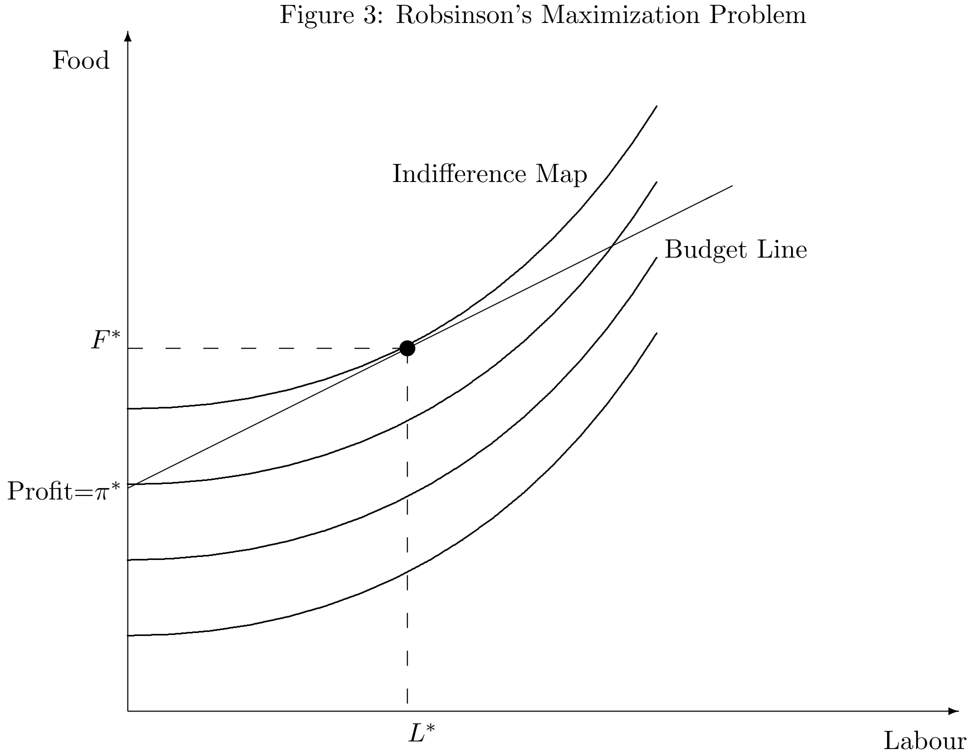


We know that the higher the isoprofit line the greater is the profit derived from an operation. All point greater than the production function is not attainable, while all points to the southeast of the hull of the production function yields sub optimal production levels. Consequently, the best place to produce at is where the isoprofit is just tangent to the production function. The profit at equilibrium, is just the intercept of the isoprofit line that it tangent to the production function. Note that in this economy, the postal service is provided free and yet efficient since after profits are declared, Robinson immediately gets his dividends in his coconut mailbox. Boy, this is corny! Note that this choice, given wages, w, determines also the optimal level of C, the optimal level of production and L, the amount of labour to hire.

We have not completed our story as yet, since we have not consider Robinson the consumer and worker! I wonder if this story would make onto some best seller list! As a consumer, with the profits π^* in hand, he has to determine how much to consume, and how much to supply in labour.

At the baseline, Robinson could just simply spend what he has in his mailbox, his dividends which you can think of as his endowment. To describe his problem, we could use the tools from Intermediate Microeconomics I which is depicted below. Notice that in this instance the budget constraint is upward sloping besides the indifference curve. This is because labour that he supplies is not a "good".

Figure 3: Robsinson's Maximization Problem



You can easily reconfigure this in the usual manner so that on the horizontal axis we have leisure instead of labour. **Perform this variation of the diagram yourself**. The analysis remains unchanged. Essentially, to the south east is the budget set where Robinson could afford. However, the area to the north west is where he desires to be, loads of money and sunshine, without a care in the world. Using the usual rationale for consumer equilibrium, you should realize that equilibrium occurs when Robinson's indifference curve is just tangent to his budget line, and that choice is optimal. Notice also that the endowment is where the intercept of the budget line is since at labour supply of zero is when he just sits under the coconut tree. The slope of the budget constraint is just the wage rate, w, since the price of the good he produces is based at $1. The equilibrium point be stated as,

$$MPS = w = MRT$$

We can now bring all the market equilibrium together now to show what constitutes a general equilibrium in the product, and labour market, and you would have noted that it is exactly the same as the very first equilibrium, where we have the indifference curve being tangent to the production function. To state the equilibrium again, together with the wage rate this time,

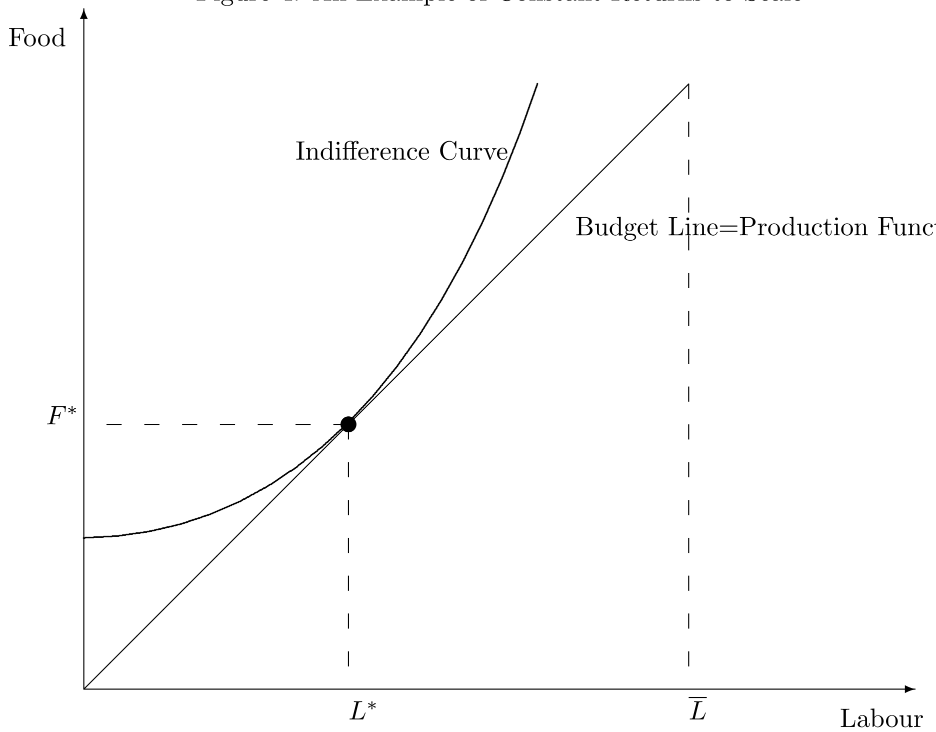
$$MP_L = w = MRT = MRS$$

Although the manner in which we arrive at this insight seems bizarre, if you consider the larger picture where each market here is constituted by large numbers of firms, and consumers, the equilibrium we have just found is very powerful. All firms need essentially is to use the signals provided by the price of their products to decide how much to produce, since it is just not feasible to ask everyone how much they would like to have. The market prices reflects the marginal values of the goods that the firms use as inputs and outputs. Further, if the firms use the market prices to value their profits, then their decisions will reflect the marginal values that consumers place on their goods.

We have considered only one technology used above, that of diminishing returns to scale, or diminishing marginal product of labour. However, we know that that is not the sole description of technology available to Robinson, let alone what we might observe in reality. How would the equilibrium look like in a constant or increasing returns to scale economy.

If the technology available to Robby, now we're getting really friendly with Robinson, the production function would simply be a straight upward sloping line such as that depicted below,

Figure 4: An Example of Constant Returns to Scale



Notice first that the production function is the same as the budget line, and since the intercept of the budget line is determined by his profit, this means that Crusoe Inc. is actually earning zero profits. The rationale is as follows; If the firm could earn zero profits, what is there to stop him from earning profit as large as he imagines, i.e. indefinitely (That is there is no maximum profit). While if profits were less than zero, the firm would do best no to produce at all. Thus we are left with the **competitive** firm earns zero profits. In that case, the amount he can consume of the good, food, is dependent on the total amount of labour Robinson can supply, $\overline{L}$. Further since what he earns is what he can spend, the production function corresponds with Robinson's budget line. Another way to think about it is as follows; We know that the firm is not making profits, and that we have denominated the price of food at \$1, i.e. it is the numeraire good. This means that the equation of the budget line is;

$$w \times L = p \times F = F$$

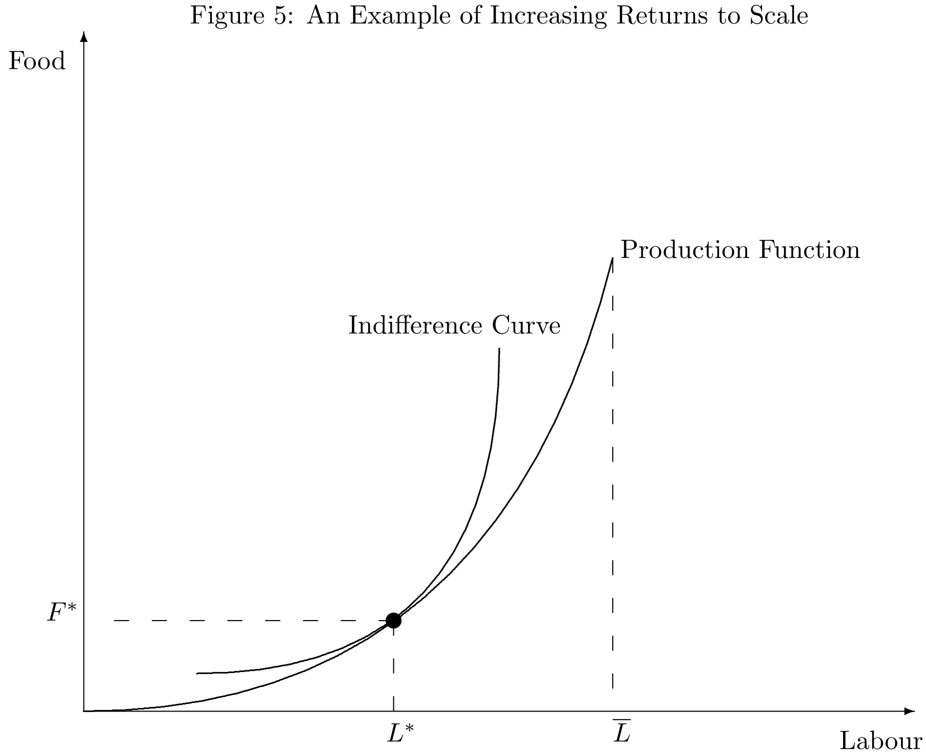
while the production function is,

$$F = MP_L \times L = w \times L$$

It is clear then that the former corresponds with the latter.

The case of increasing returns to scale is depicted below.

Figure 5: An Example of Increasing Returns to Scale



As observable, diagrammatically, we can show the optimal choice of labour and consumption. However, conceptually the equilibrium cannot be supported by a competitive market. This is because if the firm were to face the prices given by the marginal rate of substitution, the firm would like to produce more than Robinson could and would like to consume. To see this, first note that if at the optimal choice, the firm exhibits increasing returns to scale, it means that the average cost of production is greater than the marginal cost of production. Since we are considering a competitive firm, or a firm that behave like a competitive, where $p = MC$, this means that the firm would be making negative profits. The objective of profit maximization would encourage the firm to produce more, that is greater than the demand, and incongruent with the labour supply. Therefore there is no price that can support the equilibrium, or no price that would cause demand to equate with supply. This is an example of **Non-convexity**, or in other words, the common tangent will not separate the preferred points (area to the northwest of the indifference curve) from the feasible points (reflected by the area to the south east of the production function). The significance of non-convexity is that prices are no longer able to convey the necessary information to bring about an efficient allocation.

3 First & Second Welfare Theorem

The First Welfare Theorem continues to hold even when we include production. There are however some key qualifications that need to be noted.

1. Recall that the First Welfare Theorem says nothing about distribution, that is profit maximization ensures efficiency is achieved, but says nothing about whether the allocation is equitable or otherwise. However, as noted before, the definition of equitability needs to be well defined as well. (Recall that the **First Welfare Theorem** says that a Competitive Equilibrium is **Pareto Efficient**).

2. A Competitive Equilibrium needs to exist in the first place. This means that incidences of Increasing Returns to scale would rule out a Competitive Equilibrium, and consequently the theorm.

3. The theorem assumes that there are no **Production Externalities**, that is the choices made by firms do not affect other firms. This is a rather strong assumption, and is unlikely fulfilled.

4. The theorem assumes that there are no **Consumption Externalities**, that is the decision made by the firms do not affect the consumers' choices, which is also a strong assumption.

Similarly, the **Second Welfare Theorem** holds in the extension of the **Pure Exchange Economy** as well with the following qualifications. (Recall that the **Second Welfare Theorem** says that any **Pareto Efficient Allocation** can be supported as a Competitive Equilibrium).

1. Both consumers' preferences and producers' production set be convex, consequently ruling out incidences of increasing returns to scale.

2. For us to achieve the Pareto Efficient Allocation, we need to redistribute the endowments of the players, and let the competitive market use prices as signals to bring about a Competitive Equilibrium. Here with production, the redistribution includes both income from labour, and ownership shares.

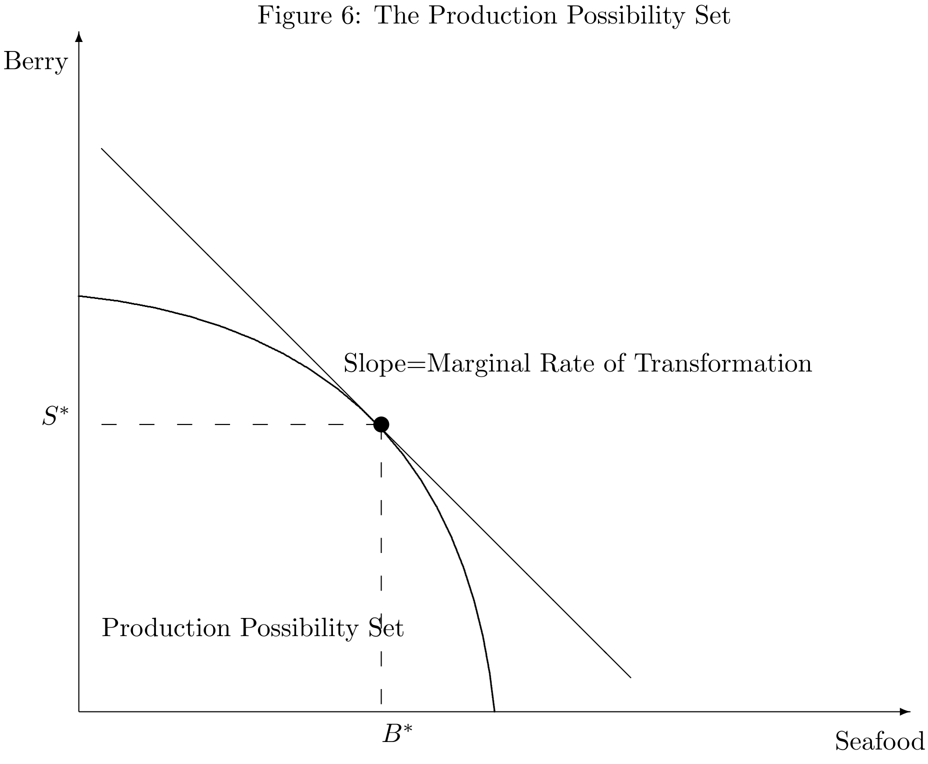
Regarding the **Second Welfare Theorem** recall that the difficulties associated with redistribution of endowment. Those difficulties remain true here.

4 The More General Case

The final extension from the **Pure Exchange Economy** is that where we examine the case of a multiple input and output economy. To be precise, we will only be extending to only a two good case, but an extension beyond this is quite natural.

Suppose Robbie now chooses between two goods to produce, Berries, and Seafood (such as clams and fishes). In that case, the most natural way to depict a production set for him would be that in figure 6 below, where movements towards the north east requires an increase in technological capabilities so that Robbie could produce more. The area within the concave curve is the **Production Possibility Set** (which you have learned in your first year), while the boundary or hull created is known as the **Production Possibilities Frontier**. Note that while previously we had depicted a production function where is depicts the relationship between the input (labour) and output (food), here the production possibility set depicts all the production that is possible, or **feasible**. Also note that the slope of the production possibility frontier measures the **marginal rate of transformation** given a particular production technology.

Figure 6: The Production Possibility Set



In the depiction of the production possibilities frontier, it has been implicitly assumed that the technology available to Robbie exhibits decreasing returns to scale. If instead we have constant returns to scale, the production possibilities frontier would be a downward sloping straight line (Why?).

Consider the following; suppose that Robbie can produce b berries for every hour of work, and s seafood for every hour of work. This then means that for a labour supply of R_b towards berry production, and R_s for seafodd production, and letting the total production of berry and seafood be denoted by B and S, then he can produce,

$$B = R_b \times b$$

$$S = R_s \times s$$

Given that Robbie works a total of R (which is just a constant) hours a day, his total time allocation,

$$R_b + R_s = R$$
$$\Rightarrow \frac{B}{b} + \frac{S}{s} = R$$
$$\Rightarrow B = R - S\frac{b}{s}$$

which is nothing but the equation of a straight downward sloping line with a slope or marginal rate of transformation of $\frac{b}{s}$.

The depiction of the above production possibilities frontier reveals only one technology available to production. The technology available would change if we suppose that another worker exists, and here comes Friday.

Suppose Friday is a better fisherman then Robbie, while Robbie's talents lay in working around the island collecting berries. His nose is just made for it. Suppose the skills available to Friday are such that;

$$B = F_b \times \beta$$

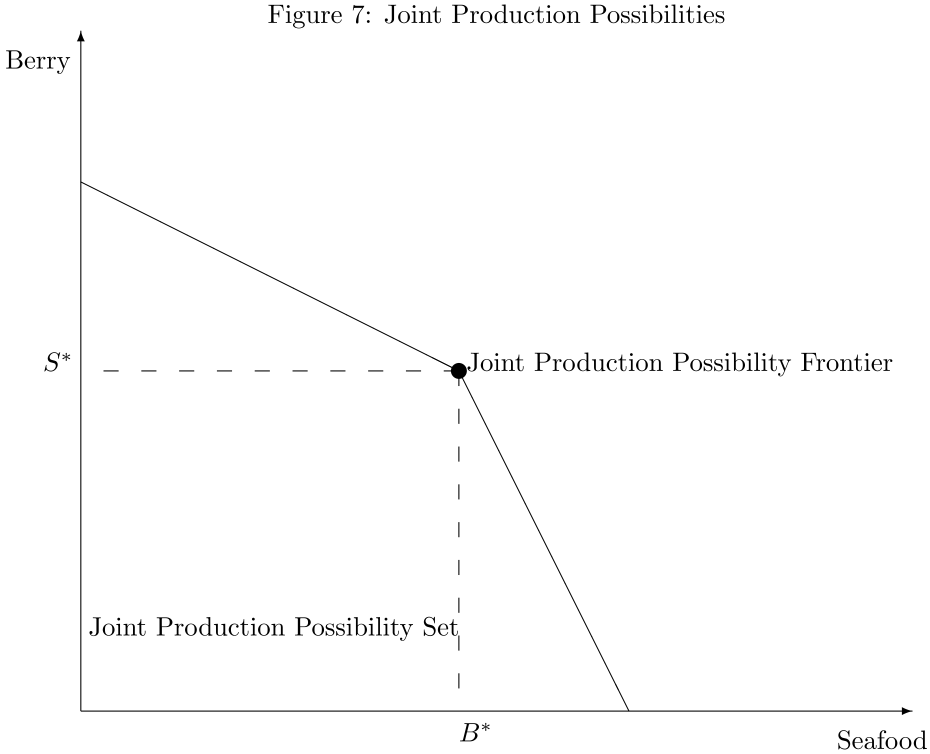
$$S = F_s \times \sigma$$

where $\beta < b$ while $\sigma > s$. Then, Friday's production function is,

$$F_b + F_s = F$$
$$\Rightarrow \frac{B}{\beta} + \frac{S}{\sigma} = F$$
$$\Rightarrow B = F - S\frac{\beta}{\sigma}$$

where F is the total labour supply of Friday, which as it was for Robbie, is a linear function with constant returns to scale. Further note that since $\frac{\beta}{\sigma} < \frac{b}{s}$, it thus implies that Friday has a **comparative advantage** in the production process in Fish, while Robbie's is in Berry production. This then means that if they both were to focus on the *forte*, they could increase they total consumption through a better production technology. Their combined production function as you should have seen in your first economics class is as follows;

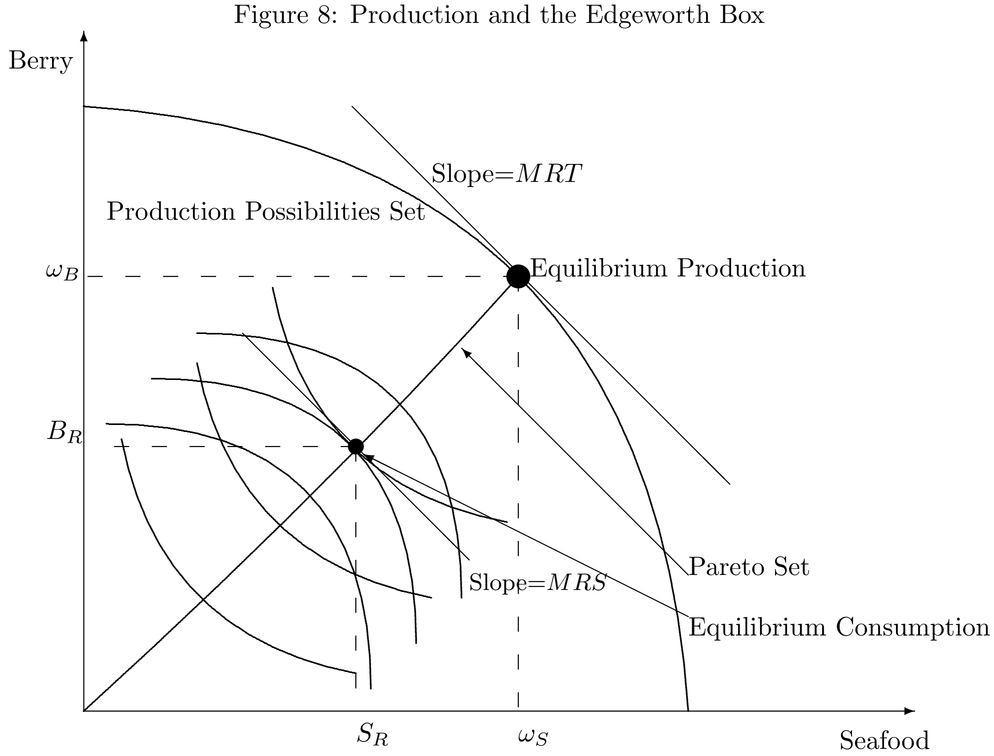
Figure 7: Joint Production Possibilities



We will now examine how to include consumption choices just as in the desert island scenario when Robinson was the lone inhabitant out there producing and consuming. The story now has both Robinson and Friday both producing and consuming (that is there are two workers, and consumers). We will now revert back to the more general case where the production possibility frontier is downward sloping and concave as in a typical economic analysis. Retaining the same notation for individual consumption of berry and seafood as B_i and S_i respectively, where the subscript $i \in \{R, F\}$ where R is for Robinson Crusoe, and F is for Friday. We will add in addition the total of both the goods available for production and denote it as ω_B and ω_S for total berries and seafood available for consumption to both Robinson and Friday.

We this setup, we can complete the examination in the more general case where we have two consumers with two producers. Once the two producers have decided on the total production in the economy, we would obtain ω_B and ω_S which must be on the production possibility frontier. This essentially is nothing but the endowment of the economy from the perspective of the consumers. That is given this total production, and using the prices as signals, both Robbie, and Friday could then trade toward a optimal level for their personal consumption such that their individual utility function is maximized. Noting also that this final consumer market equilibrium must be on the **Pareto Set**. The manner in which we can depict these concepts and ideas in one diagram is revealed below:

Figure 8: Production and the Edgeworth Box



The production equilibrium would determine the available goods that can be traded in the consumer market. We have found that in equilibrium, consumer utility will be tangent to each other, that is at equilibrium the **marginal rate of substitution** for all the consumers will be the same, which essentially means that the equilibrium will be **Pareto Efficient**.

Whereas in the exchange economy, where consumers can only trade one good for another, this instance here where we also have the production market, the exchange of goods can be achieved through another means, through changing the relative choice of products to be produced. The significance of this is that whereas in the exchange economy, all equilibrium must be along the Pareto Set, here, the equilibrium need not be so, principally the production sector could always alter the production combinations. This discussion is leading us to understanding why in equilibrium we need to have and will have the **Marginal Rate of Substitution** equating with the **Marginal Rate of Transformation**.

Consider the possibility that the Marginal Rate of Transformation (MRS) is not equal to the Marginal Rate of Substitution (MRT), for example an incidence where $MRS < MRT$ which means using our example $\frac{\Delta B_R}{\Delta S_R} < \frac{\Delta \omega_B}{\Delta \omega_S}$. Would such a situation be Pareto Efficient for the consumers? When the MRT is greater, it means that Robinson and Friday could decrease the production of seafood, and increase that of Berry at a rate faster than what would keep Robinson (and Friday as well since in the trading equilibrium, they both have the same MRS) on the same utility. Put another way, the production could be alter so that consumer could be made better off since the change in production could bring the consumers onto a higher indifference curve since more is always better.

Another way to think about it is as follows. Recall that in a competitive economy, prices relieves the need to collect information since all relevant information is captured in the prices. If both producer and consumer markets face the same prices, and we know that there cannot be excess demand (i.e. aggregate excess demand must be zero), necessarily the MRS must be equal to MRT. This idea is illustrated in the above diagram.

$$MRS = MRT$$

This must be attained for the equilibrium to be Pareto Efficient.

But Oops! Wait! We have not talked about the labour market. Realize that in this setup, there is Robinson and Friday the workers, besides being shareholders and consumers! We will deal with this now.

As before in the schizo Robinson case, we will first set up the firm problem where both Robinson and Friday are both the shareholders maximizing their profits. They are hiring themselves as labour, where the wage rate that they each get is w_R and w_F for Robinson and Friday respectively, and the price of the goods are p_B and p_S are the prices of berries and seafood respectively. Then the profit function is;

$$\max_{\omega_S,\omega_B,L_R,L_F} p_B\omega_B + p_S\omega_S - w_R L_R - w_F L_F$$

where they are subject to the production technology captured within the function of the quantities of the products they can produce. Suppose the problem gives rise to the following equilibrium labour demand, L_R^* and L_F^*, and let the total wage bill be $w_R L_R - w_F L_F = W^*$, then the company's profit would be;

$$\Pi = p_B\omega_B + p_S\omega_S - W^*$$
$$\Rightarrow \omega_B = \frac{\Pi + W^*}{p_B} - \omega_S \frac{p_S}{p_B}$$

which is nothing but the isoprofit line, noting as before that given the cost, Pi determined the intercept of the isoprofit line on the diagram we have above, and that it has a slope of $MRT = -\frac{p_S}{p_B}$. Note that this fact will also hold in the many firm case, that is the marginal rate of transformation must always equate with the price ratio of the products that the firms face since the prices tells us the relative value of the technical trade offs between the production of the two goods or goods on hand. Put it in another way, the profit maximizing choice of firms must be such that the Marginal Rate of Transformation equates with the negative value of the price ratio.

Now we can move on to the consumers, Robinson and his buddy Friday. There now have the dividends from the company, plus a pay check for their hard work. Since in a competitive market, or more precisely in our characterization, no money is added nor taken away from the system, both Friday and Robinson will have enough money to make all their purchases, i.e. everything they have produced. We know from the first course in Intermediate Microeconomics, that given the price of the goods, the individual will always make their best choice by choosing to consume the bundle which satisfies the following condition;

$$MRS = -\frac{p_B}{p_S}$$
$$\Rightarrow MRS = MRT$$

And consequently the Pareto Efficiency criterion is met in this economy, and the prices has done its job to signal the relative scarcity of the products, both production scarcity (given technology), and consumption scarcity (given their preferences).