

INDUSTRIAL ORGANIZATION  
OLIGOPOLISTIC COMPETITION

Both the monopoly and the perfectly competitive market structure has in common is that neither has to concern itself with the strategic choices of its competition. In the former, this is trivially true since there isn't any competition. While the latter is so insignificant that the single firm has no effect. In an oligopoly where there is more than one firm, and yet because the number of firms are small, they each have to consider what the other does. Consider the product launch decision, and pricing decision of Apple in relation to the IPOD models. If the features of the models it has in the line up is similar to Creative Technology's, it would have to be concerned with the pricing decision, and the timing of its announcement in relation to that of the other firm. We will now begin the exposition of Oligopolistic Competition.

## 1 Bertrand Model

Firms can compete on several variables, and levels, for example, they can compete based on their choices of prices, quantity, and quality. The most basic and fundamental competition pertains to pricing choices. The Bertrand Model is examines the interdependence between rivals' decisions in terms of pricing decisions.

The assumptions of the model are:

1. 2 firms in the market,  $i \in \{1, 2\}$ .
2. Goods produced are homogenous,  $\Rightarrow$  products are perfect substitutes.
3. Firms set prices simultaneously.
4. Each firm has the same constant marginal cost of  $c$ .

What is the equilibrium, or best strategy of each firm? The answer is that both firms will set the same prices,  $p_1 = p_2 = p$ , and that it will be equal to the marginal

cost, in other words, the perfectly competitive outcome. This is a very powerful model in that it says that price competition is so intense that all you need is two firms to achieve the perfect competitive outcome. We will show this through logical arguments and contradictions, as well as through the use of a diagram.

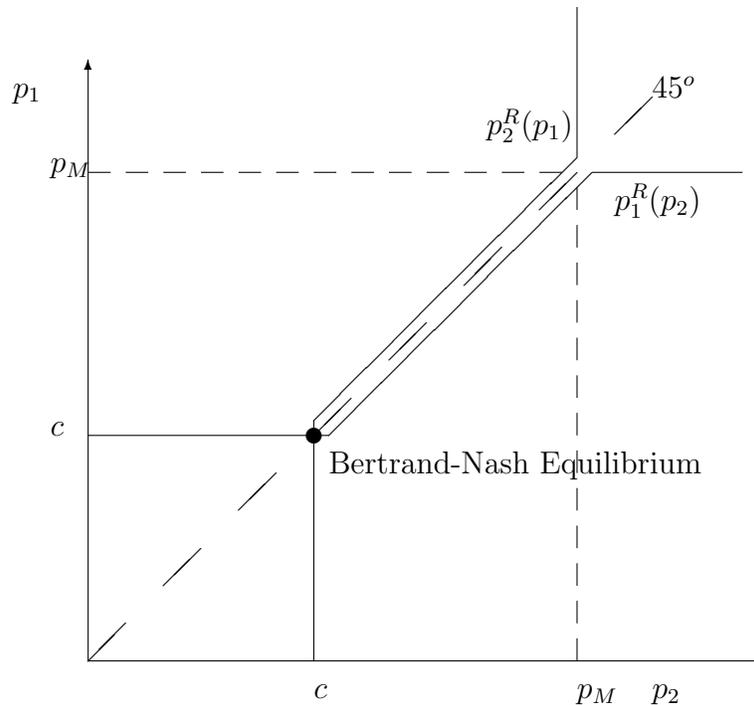
Using logical arguments:

1. **Firm's will never price above the monopoly's price:** Suppose not. And suppose firm 1 believes that firm 2 would choose a price  $p_2$  above the monopoly's price, then the **best response** of firm 1 is to price at the monopoly's price since at that point, its profit is maximized. And firm 2 would be driven out of the market. Therefore no firm would ever price above the monopoly's price.
2. **In equilibrium, all firm's prices are the same:** Suppose firm 2 chooses to price at the monopoly's price, what is the best response of firm 1? Firm 1 would realize that by pricing at a slightly lower price, it would be able to capture the entire market since the goods are perfectly substitutable, that is  $p_1 = p_M + \epsilon$ , where  $p_M$  is the monopoly's price, and  $\epsilon > 0$ . Then only one firm is left. Therefore the equilibrium where firms charge a different prices cannot be an equilibrium,  $p_1 = p_2 = p$ .
3. **In equilibrium, prices must be at the marginal cost:** Suppose not, then  $p_1 = p_2 = p > c$ . However, either firm would always find it is in their best interest or their best response to undercut its competition and obtain the entire market for itself, by reducing its prices a little bit more, say  $\epsilon > 0$ . By induction, it is in fact not possible then to have an equilibrium above the marginal cost, since it is only at the marginal cost that firms have no incentives to deviate from the equilibrium prices.

$\therefore$  in equilibrium,  $p_1 = p_2 = p = c$ . Notice that in making the arguments we have always stated the firm's choice as a function of the other firm's choice,  $p_i^*(p_j)$ , where  $i \neq j$ , and  $i, j \in \{1, 2\}$ . This is known as a reaction function. Depicting our argument on a diagram with prices on both the axes. It is obvious that equilibrium is achieved only at the point where the reaction functions meet, since it is only at the intersection that each firm's best response corresponds with the other's. Any other point cannot be

an equilibrium since the actions that one believes the other would do would never be realized. Only at  $c$  does their expectations match, and the equilibrium is sound since both firms are the same, symmetric.

Figure 1: The Bertrand Model and Equilibrium



## 2 Pricing with Capacity Constraints

However, you might be thinking that the equilibrium is highly unrealistic, since most firms do earn positive profits in even in markets with more firms, and you would be correct. What is then missing in the model? Consider the following considerations;

1. **Product Differentiation:** In reality, most firms produce products that at the least, their consumers perceive as different from a rival's. This then implies that when products are differentiated, either real or otherwise, a competitor's choice to undercut the other will not necessarily raise profits substantially, consequently

price competition does not have the power to drive prices down to marginal cost. We will consider this again when we examine product differentiation.

2. **Dynamic Competition:** The Bertrand Model assumes that the pricing game is a one shot game, which is hardly what really occurs since the lifespan of a firm is typically more than one period. Recall our multi period simultaneous game where we found that agents could achieve mutually beneficial outcomes that would otherwise not have been possible. Extending the idea, when we consider the pricing competition game over a finitely large number of periods, and that the game is repeated in each, it is possible for the firms to achieve an equilibrium where prices are greater than marginal cost.
3. **Capacity Constraints:** The implicit assumption of the last model is that the firm in deviating and undercutting its competitor obtains the entire market, it is able to meet the full demand of the market. However, this need not be true all the time, that is the firm has an endogenous constraint in the sense that it is not possible for it to meet all of the demand of the market should it undercut its competition.

Maintaining all of the previous assumptions, we augment them with the additional assumption that each firm has a capacity constraint of  $k_i$ ,  $i \in 1, 2$ , such that even if the demand for their homogenous product is greater than they can produce, they are not able to meet it. Further, without loss of generality, suppose  $k_1 \leq k_2$ , and for simplicity, assume here that  $c = 0$

The equilibrium under this scenario is that of  $p_1 = p_2 = P(k_1 + k_2)$ , that is both firms price at the point where there is no unused capacity. Suppose for simplicity, and clarity that the marginal cost is zero. This assumption allows us to focus on the pricing decision on hand without concern with the marginal gain in profit. (You should think about this in more detail after the proof to convince yourself that had we allowed marginal cost to be greater than zero, the arguments would have far more conditions distracting from the root of the strategic concern.) And finally suppose total industry capacity is sufficient small in relation to market demand.

Let us examine what the optimal price would be should the firms act in concert as one, i.e. a monopoly. Given that marginal cost is zero, they would maximize their profit when they utilize their capacity to the maximum, i.e. a corner solution, since the marginal revenue would never be zero given positive prices. We will now adopt similar arguments to prove our above conjecture.

1. **There is no incentive for firm 2 to deviate:** Suppose  $p_1 = P(k_1 + k_2)$ , can firm 2 do better?
  - (a) Suppose firm 2 chooses to price above this price, but that would mean that the profit could always be increased by raising their output. Consequently, they would never do this.
  - (b) Suppose firm 2 chooses to price below the price of  $P(k_1 + k_2)$ . But since the price set by firm 1 is at the point where both firms would be producing at capacity. By firm 2 choosing to deviate and pricing below  $P(k_1 + k_2)$ , it actually would not be able to capture any additional market since it does not have the capacity to meet the increased demand.

Therefore firm 2 would never deviate, and would set the price equal to  $P(k_1 + k_2)$ .

2. **There is no incentive for firm 1 to deviate:** Now instead let  $p_2 = P(k_1 + k_2)$ , that is firm 2 sticks to its strategy, would firm 1 find deviation from the price of  $P(k_1 + k_2)$ ? By similar argument as before, no.

Consequently the following statement holds.

**For a sufficiently small industry capacity in relation to the market demand, then equilibrium prices are greater than marginal cost.**

### 3 Cournot Competition

Cournot competition is one where firms simultaneously choose their optimal quantity produced instead of prices. The manner in which we derive a solution is through examining what the **best strategy each has given their believes in what their competition would do.**

Before we begin, as usual we have to stipulate the assumptions:

1. There are two firms (though the problem can be generalized to the multiple firm case),  $i \in 1, 2$ .
2. Firms produce a homogenous product.
3. Firms choose optimal quantity produced simultaneously.
4. Marginal Cost of production are the same for both firms,  $c$ .

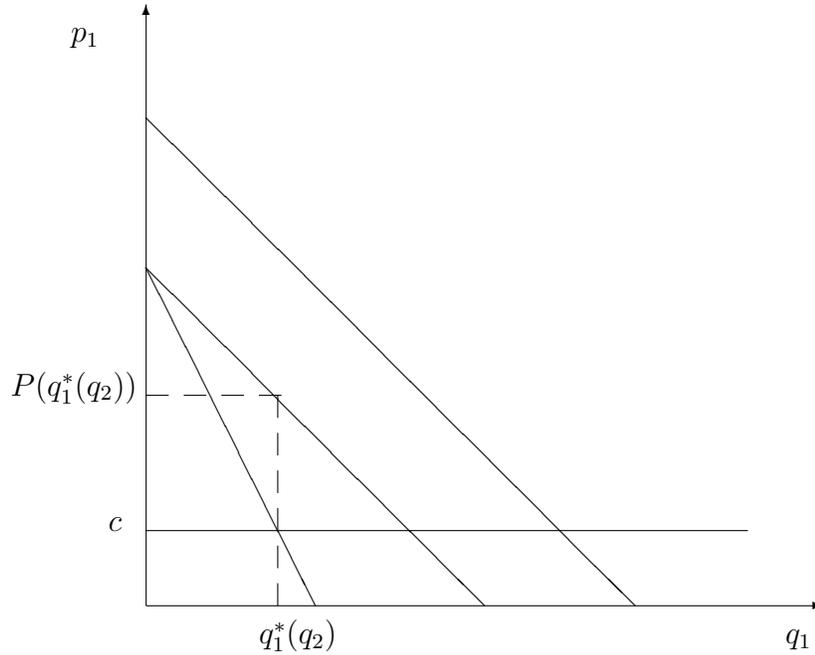
### 3.1 A Description of the Process

Let the output of each firm be  $q_i$ . The price that is sold is ultimately dependent on the joint choices of both firms, i.e.  $P \equiv P(q_1 + q_2)$ . That is given what firm  $j$  chooses, firm  $i$ 's choice will ultimately affect the prices of the market. If we were to plot this, what we will derive is the residual demand of the firm in question. Essentially, given this residual demand, each firm will then make their choices as if they were a monopoly in order to maximize their profit, i.e. by setting marginal revenue equal to marginal cost.

Considering some extreme considerations; suppose firm 2 chooses to produce nothing, then the best that firm 1 can and would do is to produce the monopoly quantity. On the other hand, if firm 2 chooses to produce at the competitive level, in which case, the best that firm 1 can do is to produce nothing. This illustrates how each firm's choices are tied to each other. We call, just as in the case of Bertrand competition,  $q_i(q_j)$  a reaction function of  $i$ , where  $i \neq j$ ,  $i, j \in 1, 2$ . The relationship, as you may discern is decreasing in the choice of the other firm, since the more the other firm chooses, the **Residual Demand** would be smaller, i.e. limiting the choices of the firm in question.

If we were to plot the choices of each firm given the other's choices, we would get a reaction function, as in the Bertrand case. Whereas in the latter, the reaction function is upward sloping, the case for Cournot competition is downward sloping since as noted before, the greater the choice of the competition, the smaller the residual demand.

Figure 2: Quantity Choices



### 3.2 A Simple Algebraic Model

Given the intuition and insights we can now examine a algebraic example. Let the demand of the market be  $P(Q) = a - bQ$ , where  $Q = q_1 + q_2$ . Each firm would choose to maximize their profit which given constant marginal cost is,

$$\begin{aligned}\pi_i &= Pq_i - cq_i \\ \Rightarrow \pi &= aq_i - bq_i^2 - bq_iq_j - cq_i\end{aligned}$$

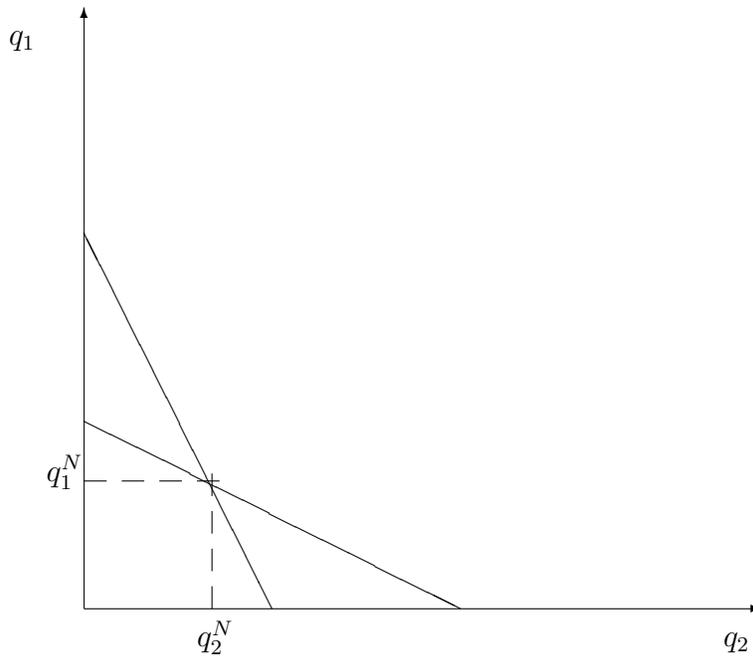
Their first order condition would be,

$$\begin{aligned}a - 2bq_i - bq_j - c &= 0 \\ \Rightarrow R_i(q_j) = q_i &= \frac{a - bq_j - c}{2b}\end{aligned}$$

where  $i \in 1, 2$ . In equilibrium, since all firms are **symmetric**,  $q_i = q_j$ , which means that

$$\Rightarrow R_i(q_j) = q_i = \frac{a - c}{2b} - \frac{q_j}{2}$$

Figure 3: The Cournot Equilibrium



$$\Rightarrow q_i^* = \frac{a - c}{2b} - \frac{q_i^*}{2}$$

$$\Rightarrow q_i^* = \frac{a - c}{3b}$$

And the equilibrium price is,

$$P^* = a - \frac{2(a - c)}{3}$$

$$\Rightarrow P^* = \frac{a + 2c}{3}$$

which is greater than the marginal cost of  $c$ . Note further that this duopoly's output is greater than the monopoly's but less than it would have been under perfect competition. Consequently, duopoly's prices are greater than perfect competition, but less than monopoly's. **Can you show this is true? Read page 112 of your text. How does the equilibrium quantity and prices change as the number of firms increase. What if the marginal cost of the firms are not the same, that is  $c_1 \neq c_2$**

## 4 Bertrand versus Cournot

Which model we ultimately choose to understand reality is ultimately dependent on the ease in which firm in reality adjust prices or quantity. If firms find it easier to adjust quantity, then Bertrand models are a better description of reality about the strategic choices, since the quantity decision is immediately consequent to the pricing choices. However, if instead firms find adjusting prices far easier, then the Cournot model would be a better model to use. Examples of the former include the software industry, while examples of the latter include automobile industries, where capacity is a strong constraint on choices.

Of course firms may choose the choice variables sequentially, in which case the models may be merged. For example, it is possible that capacity and pricing decisions are separate, while the latter presents a more binding constraint, then in modelling such a situation, we should have a two stage game, where firms first choose to make the long run decision, i.e. capacity, before the short run decision on prices. Of course when solving, you use backward induction, that is you find the choice of the firm pertaining to the pricing decision given the capacity choice, substitute this choice function in the first stage, and find the capacity equilibrium. The second stage equilibrium would then also be resolved.

**Read section 7.5 of your text. Your examinations will have similar content.**