Math 254 Winter 2015 Assignment 1

Section 5.1 12, 18, 26, 30

12. $\{(2,0,1),(1,-2,0),(4,-4,1),(1,1,1)\}$

4 vectors in $\mathbb{R}^3$ $4>3$ so must be linearly dependent

$$\begin{bmatrix} 2 & 1 & 4 & 1 \\ 0 & -2 & -4 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad R_3 \to 2R_3 - R_1 \quad \begin{bmatrix} 2 & 1 & 4 & 1 \\ 0 & -2 & -4 & 1 \\ 0 & -1 & -2 & 1 \end{bmatrix}$$ 2

$$R_3 \to 2R_3 - R_2 \quad \begin{bmatrix} 2 & 1 & 4 & 1 \\ 0 & -2 & -4 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{array}{l} R_2 \to R_2 - R_3 \\ R_1 \to R_1 - R_3 \end{array} \quad \begin{bmatrix} 2 & 1 & 4 & 0 \\ 0 & -2 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to -\tfrac{1}{2}R_2 \quad \begin{bmatrix} 2 & 1 & 4 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad R_1 \to R_1 - R_2 \quad \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to \tfrac{1}{2}R_1 \quad \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$v_3 = v_1 + 2v_2$ 1

check $(2,0,1) + (2,-4,0) = (4,-4,1)$ ✓

18. $\{v_3, v_4, v_5\}$

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$ 2

Linearly independent - could row reduce 3 columns to get 3 leading ones

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to -R_2 \quad \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_1 \to R_1 - 3R_2 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

26. $\{\underbrace{(1,0,-1,0)}_{v_1}, \underbrace{(3,6,1,2)}_{v_2}, \underbrace{(-2,0,2,0)}_{v_3}\}$ LD?

can't tell from # of vecters so row reduce to see if you can get 3 leading ones

2

OR - notice that $v_3 = -v_1$ so linearly dependent

30. Let $\{v_1, v_2, v_3, v_4\}$ be LI in $\mathbb{R}^m$. Show that $\{v_1, v_2, v_3\}$ is also LI

LI $\Leftrightarrow$ only solution to $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 = 0$ is all $c_i = 0$.

2

Suppose $\{v_1, v_2, v_3\}$ is LD. Then $\exists\ c_1, c_2, c_3$ not all 0 with $c_1v_1 + c_2v_2 + c_3v_3 = 0$

But then $c_1v_1 + c_2v_2 + c_3v_3 + 0v_4 = 0 + 0 = 0$ has non-trivial solution. This is a contradiction. Thus $\{v_1, v_2, v_3\}$ is LI.

Section 5.2 6, 10, 18, 24

6. The set of all vectors (x, y, z) such that $x+y+z=0$

i. Let v_1, v_2 be in S.
So $v_1 = (x_1, y_1, z_1)$ with $x_1+y_1+z_1=0$
$v_2 = (x_2, y_2, z_2)$ with $x_2+y_2+z_2=0$

$v_1+v_2 = (x_1+x_2,\ y_1+y_2,\ z_1+z_2)$

$x_1+x_2+y_1+y_2+z_1+z_2 = x_1+y_1+z_1 + x_2+y_2+z_2$
$= 0+0 = 0$

3

So $v_1+v_2 \in S$.

ii Let $v \in S$, $c \in \mathbb{R}$. So $v = (x, y, z)$ with $x+y+z=0$.
$cv = (cx, cy, cz)$.
$cx+cy+cz = c(x+y+z) = c(0) = 0$
So $cv \in S$.

S is closed under addition & scalar multiplication so S is a subspace.

10. Set of all 4-vectors except $(1,1,1,1)$ of $\mathbb{R}^4$

So $v = (2,2,2,2)$ in S

2

$c = \frac{1}{2} \in \mathbb{R}$
$\frac{1}{2}v = (1,1,1,1) \notin S$ NOT closed under scalar multiplication

So NOT a subspace

18. $\{\cancel{\{}(1,1,0),(1,0,1)\}$ spans
subspace of $\mathbb{R}^3$ consisting of all vectors
(x,y,z) such that $x=y+z$

system $x=y+z$

let $y=s$, $z=t$ 2 parameters

Solution $(x,y,z)=(s+t,s,t)$

$=s(1,1,0)+t(1,0,1)$ 2

always LC of $(1,1,0)$ & $(1,0,1)$

so $\{(1,1,0),(1,0,1)\}$ spans.

24. for #6

(x,y,z) where $x+y+z=0$
so $x=-y-z$
let $y=s$, $z=t$, $x=-s-t$ 2
solutions $(-s-t,s,t)$
$=s(-1,1,0)+t(-1,0,1)$

spanned by $\{(-1,1,0),(-1,0,1)\}$

(more than one right answer)

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