Math 254 Winter 2015 Assignment 2

Section 5.3: 4, 8, 20, 22, 28   5.4: 6, 12, 18

4. $\{(1,1,0),(1,1,1)\}$ for subspace of $\mathbb{R}^3$ of all (x,y,z) such that $y = x+z$

2

Consider $(1,1,1)$. $x=1$, $y=1$, $z=1$. $x+z = 2 \neq y$,
So this vector isn't even in the subspace.
$\therefore$ This is NOT a basis. (It is LI though)

8. $\{(2,0,2),(1,1,1)\}$ for the subspace of $\mathbb{R}^3$ generated by $(1,0,1)$ & $(0,1,0)$

vectors in subspace are of the form
$$a(1,0,1) + b(0,1,0) = (a,b,a)$$

So can we express any (a,b,a) as LC of $(2,0,2)$ & $(1,1,1)$?

Try to solve $(a,b,a) = c(2,0,2) + d(1,1,1)$
$$= (2c+d,\ d,\ 2c+d)$$

2

$$\Rightarrow a = 2c+d, \quad b = d$$
$$\Rightarrow d = b, \quad 2c = a-d = a-b \Rightarrow c = \frac{a-b}{2}$$

$$(a,b,a) = \frac{a-b}{2}(2,0,2) + b(1,1,1)$$

So $\{(2,0,2),(1,1,1)\}$ is a spanning set
The set is LI since neither vector is a multiple of the other.
$\therefore$ The vectors form a basis.
could show 2 LI vectors in subspace]

20. $\{(0,1,-1,0), (0,-1,2,0)\}$ is LI, so it spans subspace of $\mathbb{R}^4$ of all vectors of the form $(0,a,b,0)$

$$(0,a,b,0) = a(0,1,0,0) + b(0,0,1,0)$$

Subspace has basis $\{(0,1,0,0), (0,0,1,0)\}$
so dim $= 2$. 2

2 LI vectors from the subspace form a basis.
The 2 vectors are LI + in the subspace, so they must also span.

22. Solution space
$$\begin{bmatrix} 1 & -3 & 1 & 0 & -1 \\ 1 & -2 & 1 & -1 & 0 \\ 1 & -1 & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$
$$\begin{bmatrix} 1 & -3 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 2 & 0 & -2 & 2 \end{bmatrix}$$

$R_3 \to \frac{1}{2}R_3 - R_2$
$$\begin{bmatrix} 1 & -3 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

let $x_3 = r$, $x_4 = s$, $x_5 = t$

then $x_2 - x_4 + x_5 = 0 \to x_2 = x_4 - x_5 = s - t$
$x_1 - 3x_2 + x_3 - x_5 = 0 \to x_1 = 3x_2 - x_3 + x_5$
$= 3(s-t) - r + t$
$= -r + 3s - 2t$

vectors in Solution space are of the form
$(-r+3s-2t, s-t, r, s, t)$

$= r(-1,0,1,0,0) + s(3,1,0,1,0) + t(-2,-1,0,0,1)$

basis $\{(-1,0,1,0,0),(3,1,0,1,0),(-2,-1,0,0,1)\}$

spans & is LI , dim = 3 3

28. Subspace of $\mathbb{R}^3$ generated by

$\{(1,-1,0),(0,1,1),(2,-1,1),(1,0,1)\}$

Find subset that is LI, so row reduce

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ -1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \quad R_2 \to R_2 + R_1 \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

3

$$R_3 \to R_3 - R_2 \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2 leading ones

so $\{(1,-1,0),(0,1,1)\}$ dim = 2
LI & spans.

Section 5.4

6, 12 & 18

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$R_3 \to 2R_3 - R_2$ $\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 \end{bmatrix}$ $\quad$ $R_2 \to \frac{1}{2}R_2$, $R_3 \to -R_3$ $\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

3 leading ones so rank = 3

basis for $CS(A) = \{(1,1,1), (-1,1,0), (1,0,0)\}$
so col rank = 3

basis for $RS(A) = \{(1,-1,1,0), (1,1,0,0), (1,0,0,1)\}$
row rank = 3

verified : rank = col rank = row rank $\quad$ 4

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