# Math 254 Assignment 3 Solutions



Section 6.1 2, 4, 16 Section 6.2 4, 16, 28, 6, 18, 30, 42
Section 6.3 4, 6, 18

6.1 2. The set of all $2\times 3$ matrices whose entries have the sum of 1: usual matrix operations

$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ both satisfy condition

but $A_1 + A_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ does not satisfy condition (entries sum to 1)

NOT closed under addition.

or - Does NOT contain zero $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

So NOT a vector space.

4. The set of all polynomials of degree less than or equal to 2 whose coefficients have sum of zero: the usual operations.

$P_2$ - set of all polynomials of degree less than or equal to 2 is a vector space. So it suffices to show this set is a subspace.

<u>closed under addition:</u>

If $p_1(x) = a_0 + a_1x + a_2x^2$ with $a_0 + a_1 + a_2 = 0$
$p_2(x) = b_0 + b_1x + b_2x^2$ with $b_0 + b_1 + b_2 = 0$

Then $p_1 + p_2 = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2$

$a_0 + b_0 + a_1 + b_1 + a_2 + b_2 = a_0 + a_1 + a_2 + b_0 + b_1 + b_2 = 0 + 0 = 0$ ✓

Closed under scalar multiplication ②

Let $p(x) = a_0 + a_1x + a_2x^2$ have $a_0 + a_1 + a_2 = 0$ & $c \in \mathbb{R}$.

$$cp(x) = c(a_0 + a_1x + a_2x^2) = ca_0 + ca_1x + ca_2x^2$$

$$ca_0 + ca_1 + ca_2 = c(a_0 + a_1 + a_2) = c(0) = 0 \quad \checkmark$$

$\therefore$ This is a vector space

16. The set of all 2x2 matrices of the form $\begin{bmatrix} a & b \\ b & a+b \end{bmatrix}$

Subspace of $M^{2,2}$?

<u>Closed under addition</u> Let $A_1 = \begin{bmatrix} a_1 & b_1 \\ b_1 & a_1+b_1 \end{bmatrix}$ $A_2 = \begin{bmatrix} a_2 & b_2 \\ b_2 & a_2+b_2 \end{bmatrix}$

$$A_1 + A_2 = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ b_1+b_2 & a_1+b_1+a_2+b_2 \end{bmatrix} = \begin{bmatrix} a_3 & b_3 \\ b_3 & a_3+b_3 \end{bmatrix} \text{ where } \begin{matrix} a_3 = a_1+a_2 \\ b_3 = b_1+b_2 \end{matrix}$$

So closed under addition.

<u>Scalar multiplication</u> Let $A = \begin{bmatrix} a & b \\ b & a+b \end{bmatrix}$ $c \in \mathbb{R}$

$$cA = \begin{bmatrix} ca & cb \\ cb & c(a+b) \end{bmatrix} = \begin{bmatrix} ca & cb \\ ca & ca+cb \end{bmatrix} = \begin{bmatrix} a' & b' \\ b' & a'+b' \end{bmatrix} \text{ where } \begin{matrix} a' = ca \\ b' = cb \end{matrix}$$

So closed under scalar multiplication.

$\therefore$ This is a subspace

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6.2 4. $\{x-1, x+1\}$ in $P_1$

LI? $c_1(x-1) + c_2(x+1) = 0 + 0x$

$-c_1 + c_2 + (c_1 + c_2)x = 0 + 0x \Rightarrow \begin{matrix} -c_1 + c_2 = 0 \\ c_1 + c_2 = 0 \end{matrix}$

$\Rightarrow c_1 = c_2 = 0$

Only trivial solution, vectors are LI.

$\{x-1, x+1\}$

16. Spans $P_1$? dim $P_1 = 2$ & vectors are LI
So they must span

or - See if $a_0 + a_1 x = c_1(x-1) + c_2(x+1)$ always has solution
$= (-c_1 + c_2) + (c_1 + c_2)x$

$$-c_1 + c_2 = a_0$$
$$c_1 + c_2 = a_1$$
$$2c_2 = a_0 + a_1 \Rightarrow c_2 = \frac{a_0 + a_1}{2}$$

$$c_1 = a_1 - c_2 = a_1 - \left(\frac{a_0 + a_1}{2}\right) = \frac{a_1 - a_0}{2}$$

So vectors span

28 $\{x-1, x+1\}$ basis for $P_1$? Yes - since LI & spans

6. $\left\{\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\right\}$ in $M^{2,2}$

LI? $c_1 \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$c_1 + c_3 = 0 \Rightarrow c_3 = -c_1$ — $c_1$ can be anything

$2c_1 - c_2 = 0 \Rightarrow c_2 = 2c_1$

$-c_1 + c_2 + c_3 = 0 \Rightarrow -c_1 + 2c_1 - c_1 = 0$ ✓ — Vectors are NOT LI

$c_2 + 2c_3 = 0 \Rightarrow 2c_1 + 2(-c_1) = 0$ ✓

18. $\dim(M^{2,2}) = 4$ so No way 3 vectors could span

30. Not a basis since not LI & doesn't span
( & not right # of vectors)

42. Subspace of $P_3$ consisting of all polynomials

$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

with $a_2 = 0$

So $W = \{ a_0 + a_1 x + a_3 x^3 \mid a_0, a_1, a_3 \in \mathbb{R} \}$

has basis $\{1, x, x^3\}$ dim $= 3$.

(LI & spans)

Section 6.3

4. $v = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$ $\quad \mathcal{B} = \left\{ \overset{v_1}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}, \overset{v_2}{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}} \right\}$

coordinate vector $\quad \underline{v} = -2v_1 + 3v_2$

so $\underline{x} = (-2, 3)$

6. $S_1 = \{ x^2 + x - 1,\ x^2 - 2x + 3,\ x^2 + 4x - 5 \}$ LI?

use standard basis $\{1, x, x^2\}$

coordinate vectors are $\{(-1,1,1), (3,-2,1), (-5,4,1)\}$

$\begin{bmatrix} -1 & 3 & -5 \\ 1 & -2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$ $\begin{matrix} R_2 \to R_2 + R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$ $\begin{bmatrix} -1 & 3 & -5 \\ 0 & 1 & -1 \\ 0 & 4 & -4 \end{bmatrix}$ $\begin{matrix} R_1 \to -R_1 \\ R_3 \to \frac{1}{4}R_3 - R_1 \end{matrix}$ $\begin{bmatrix} 1 & -3 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$R_1 \to R_1 + 3R_2$ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ so LD and $v_3 = 2v_1 - v_2$

So $x^2 + 4x - 5 = 2(x^2 + x - 1) - (x^2 - 2x + 3)$

check: $2x^2 + 2x - 2 - x^2 + 2x - 3$ ✓

18. $\{x^2 - x + 1,\ x^2 + 2x + 1,\ x - 2,\ x + 2,\ x^2 - 1,\ x^2 + 1\}$

reduce to set with same span

Use coordinate vectors with standard basis $\{1, x, x^2\}$

$\{(1,-1,1), (1,2,1), (-2,1,0), (2,1,0), (-1,0,1), (1,0,1)\}$

$$\begin{bmatrix} 1 & 1 & -2 & 2 & -1 & 1 \\ -1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - R_1 \end{matrix} \quad \begin{bmatrix} 1 & 1 & -2 & 2 & -1 & 1 \\ 0 & 3 & -1 & 3 & -1 & 1 \\ 0 & 0 & 2 & -2 & 2 & 0 \end{bmatrix}$$

$$\begin{matrix} R_3 \to \frac{1}{2} R_3 \\ R_2 \to \frac{1}{3} R_2 \end{matrix} \quad \begin{bmatrix} 1 & 1 & -2 & 2 & -1 & 1 \\ 0 & 1 & -1/3 & 1 & -1/3 & 1/3 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix}$$

3 leading ones

corresponding vectors $\{x^2 - x + 1,\ x^2 + 2x + 1,\ x - 2\}$