Math 254 Assignment 5 Winter 2015

Section 7.1 6, 12, 16, 22, 24 Section 7.2 4, 6, 16, 21, 26

Section 7.1

6. $T: P_3 \to P_3$ given by

$$T(a_0+a_1x+a_2x^2+a_3x^3) = (a_0+a_2) - (a_1+2a_3)x^2$$

Let $p, q \in P_3$. So $p = a_0+a_1x+a_2x^2+a_3x^3$

$q = b_0+b_1x+b_2x^2+b_3x^3$

T1: $T(p+q) = T\left(a_0+b_0+(a_1+b_1)x+(a_2+b_2)x^2+(a_3+b_3)x^3\right)$

$= (a_0+b_0+a_2+b_2) - (a_1+b_1+2(a_3+b_3))x^3$

$= (a_0+a_2) - (a_1+2a_3)x^3 + (b_0+b_2) - (b_1+2b_3)x^3$

$= T(p) + T(q)$ preserves addition

T2: Let $c \in \mathbb{R}$, so $cp = ca_0 + ca_1x + ca_2x^2 + ca_3x^3$

$T(cp) = (ca_0+ca_2) - (ca_1+2ca_3)x^3$

$= c\left[(a_0+a_2) - (a_1+2a_3)x^3\right] = c\,T(p)$

preserves scalar multiplication

$\therefore$ T is LT

12. $K(x,y) = (x, \sin y, 2x+y)$ $K: \mathbb{R}^2 \to \mathbb{R}^3$

Does not preserve addition since $\sin(y_1+y_2) \neq \sin y_1 + \sin y_2$

16. $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $T: \mathbb{R}^3 \to \mathbb{R}^3$ $T(\underline{x}) = A\underline{x}$

$T(x_0) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ x \\ z \end{bmatrix}$

So $T(x,y,z) = (y,x,z)$

22. $T_{-\pi/2}$ rotation by $-\pi/2$

$$A_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

so $$A_{-\pi/2} = \begin{bmatrix} \cos -\pi/2 & -\sin -\pi/2 \\ \sin -\pi/2 & \cos -\pi/2 \end{bmatrix} = \begin{bmatrix} 0 & -(-1) \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$T_{-\pi/2}(x,y) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = (y, -x)$$

24. T^0 reflection in x axis

$$A = \begin{bmatrix} \cos 0 & \sin 0 \\ \sin 0 & -\cos 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

so $$T^0(x,y) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = (x, -y)$$

(makes sense)

Section 7.2

4. $K+S$

$K(x,y,z) = (x,\ x+y,\ x+y+z)$ so $K: \mathbb{R}^3 \to \mathbb{R}^3$

$S(x,y,z) = (z, y, x)$ $\quad S: \mathbb{R}^3 \to \mathbb{R}^3$

$K+S: \mathbb{R}^3 \to \mathbb{R}^3$ is defined

$$(K+S)(x,y,z) = (x+z,\ x+2y,\ 2x+y+z)$$

6. GS $\quad G: \mathbb{R}^3 \to M^{2,2}$ $\quad G(x,y,z) = \begin{bmatrix} y & z \\ -x+y & -x \end{bmatrix}$

$GS: \mathbb{R}^3 \to M^{2,2}$ $\quad GS(x,y,z) = G(z,y,x) = \begin{bmatrix} y & x \\ -z+y & -z \end{bmatrix}$

(switch x and z)

16 $T(x,y) = (2x+y,\ x+y,\ x-y,\ x-2y)$

$e_1 = (1,0) \quad e_2 = (0,1)$

$T: \mathbb{R}^2 \to \mathbb{R}^4$ so

$$\text{matrix} = \begin{bmatrix} T(e_1) & T(e_2) \\ & \\ & \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & -2 \end{bmatrix}$$

21. Composition of reflection T^θ followed by T^ψ is rotation by angle $2(\psi-\theta)$

reflection matrices

$$\begin{bmatrix} \cos 2\psi & \sin 2\psi \\ \sin 2\psi & -\cos 2\psi \end{bmatrix} \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\psi \cos 2\theta + \sin 2\psi \sin 2\theta & \cos 2\psi \sin 2\theta - \sin 2\psi \cos 2\theta \\ \sin 2\psi \cos 2\theta - \cos 2\theta \sin 2\theta & \sin 2\psi \sin 2\theta + \cos 2\psi \cos 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(2\psi - 2\theta) & -\sin(2\psi - 2\theta) \\ \sin(2\psi - 2\theta) & \cos(2\psi - 2\theta) \end{bmatrix} = \begin{bmatrix} \cos 2(\psi-\theta) & -\sin 2(\psi-\theta) \\ \sin 2(\psi-\theta) & \cos 2(\psi-\theta) \end{bmatrix}$$

rotation by $2(\psi-\theta)$

(see matrices on p 324 & 326)

2e. Composition of contraction/dilation with another contraction/dilation is a ~~contraction~~ dilation

$T_c(x,y) = (cx, cy)$ for some $c \neq 0$

$T_d(x,y) = (dx, dy)$ for some $d \neq 0$

$T_d T_c(x,y) = (cdx, cdy)$ $cd \neq 0$

contraction if $cd < 1$

dilation if $cd > 1$