Vector Spaces

A *vector space* is a non-empty set V of objects called vectors that can be added and multiplied by scalars in special ways. For any vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ and any scalars c and d,

1. (Closure under addition) $\mathbf{u}+\mathbf{v} \in V$.
2. (Closure under scalar multiplication) $c\mathbf{v} \in V$.
3. (Commutativity of addition) $\mathbf{u}+\mathbf{v} = \mathbf{v}+\mathbf{u}$.
4. (Associativity of addition) $(\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$.
5. (Zero) There is an element denoted $\mathbf{0}$ in V and called the *zero vector* with the property that $\mathbf{0}+\mathbf{v} = \mathbf{v}$ for any $\mathbf{v} \in V$.
6. (Negatives) There exists a vector called the *negative* of $\mathbf{v}$, denoted by $-\mathbf{v}$, with the property that $\mathbf{v}+(-\mathbf{v}) = \mathbf{0}$.
7. (Scalar associativity) $c(d\mathbf{v}) = (cd)\mathbf{v}$.
8. (Distributivity) $c(\mathbf{v}+\mathbf{w}) = c\mathbf{v}+c\mathbf{w}$ and $(c+d)\mathbf{v} = c\mathbf{v}+d\mathbf{v}$
9. (One) $1\mathbf{v} = \mathbf{v}$.