THE GEOMETRY OF ORIGAMI

By Aggie & Gen

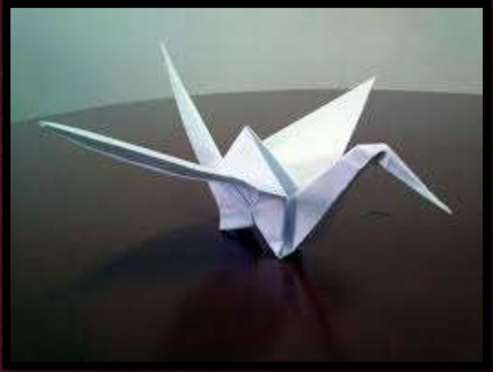
ORIGAMI

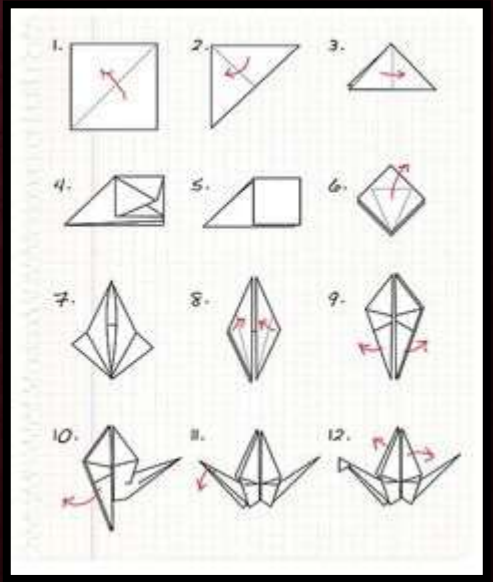
- **Origami** is the art of paper folding which is often associated with Japanese culture.
  - *ori* meaning "folding", and *kami* meaning "paper“




THE CRANE

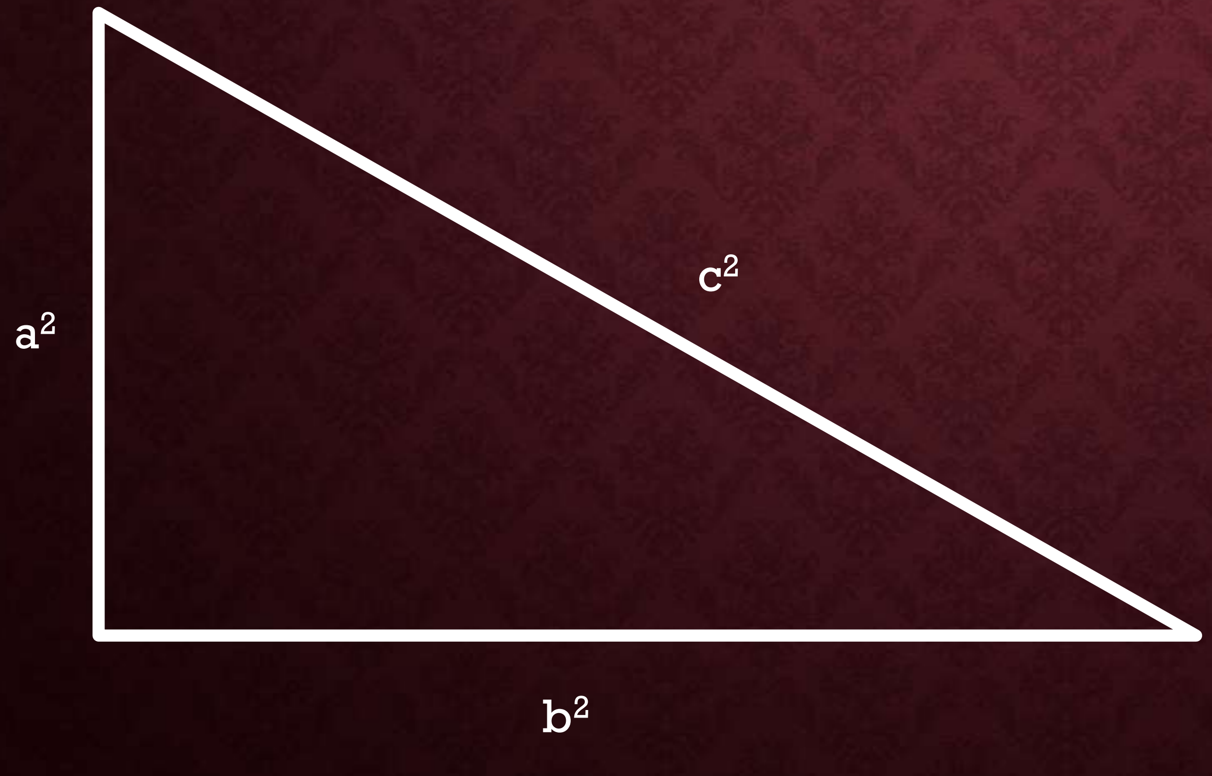
- Most famous design in Japanese origami
- Symbol of peace
- "Anyone who folds one thousand paper cranes will have their heart's desire come true."
- Basis of mathematical theories:
  - Kawasaki
  - Maekawa

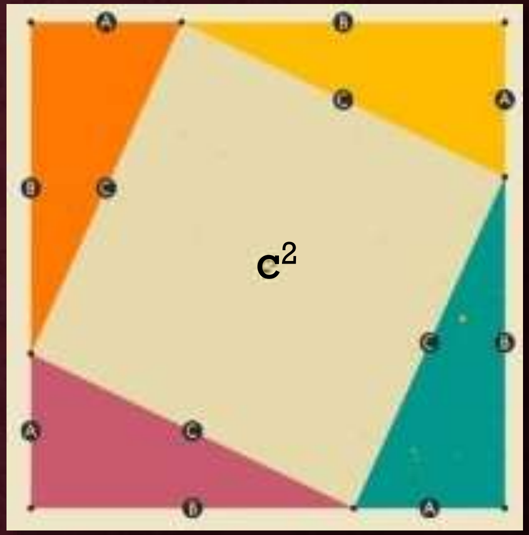


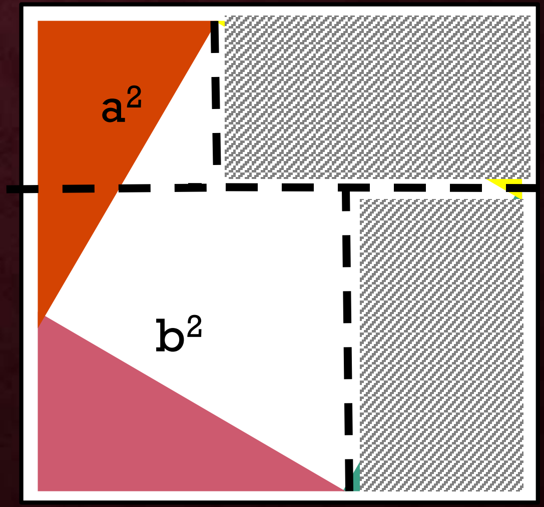


APPLICATIONS IN GEOMETRY

- Pythagoras' Theorem
  - $c^2 = a^2 + b^2$



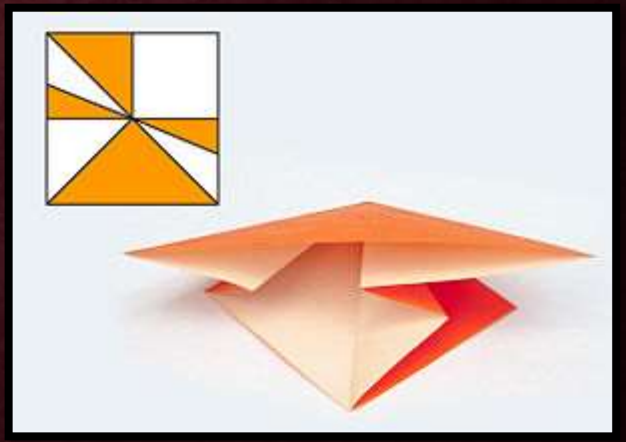




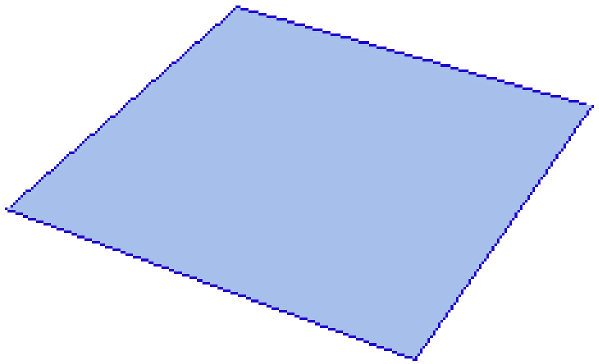
KAWASAKI’S THEOREM

- Theorem: A crease pattern is considered a flat origami construction if and only if the sum of the alternating angles surrounding each vertex is 180°.

- A few definitions:
  - Flat origami construction: configuration that flattens to a plane without additional creasing
  - Vertex: point of intersection formed by creases



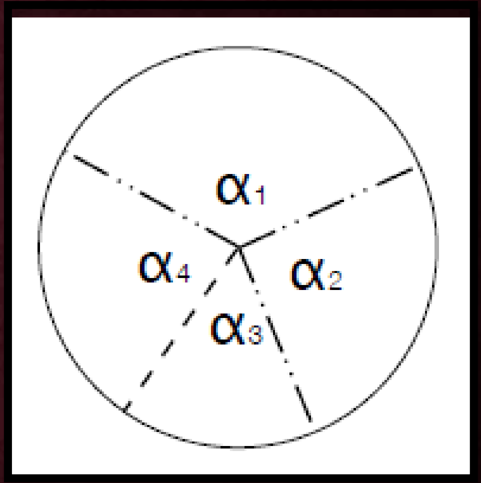
THE FLAT-FOLDABLE CRANE



PROOF OF KAWASAKI'S THEOREM

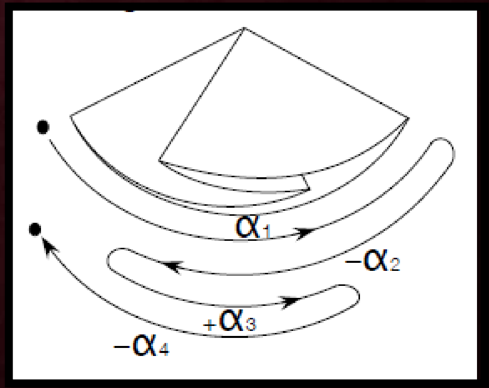
General Proof (with 2n angles):

- Step 1: $\alpha_1-\alpha_2+\alpha_3-\alpha_4+\ldots-\alpha_{2n}=0$
- Step 2: $\alpha_1+\alpha_2+\alpha_3+\alpha_4+\ldots+\alpha_{2n}+(\alpha_1-\alpha_2+\alpha_3-\alpha_4\ldots-\alpha_{2n})=360°$
- Simplify: $2\alpha_1+2\alpha_3+\ldots+2\alpha_{2n-1}=360°$
  $\alpha_1+\alpha_3+\ldots+\alpha_{2n-1}=180°$

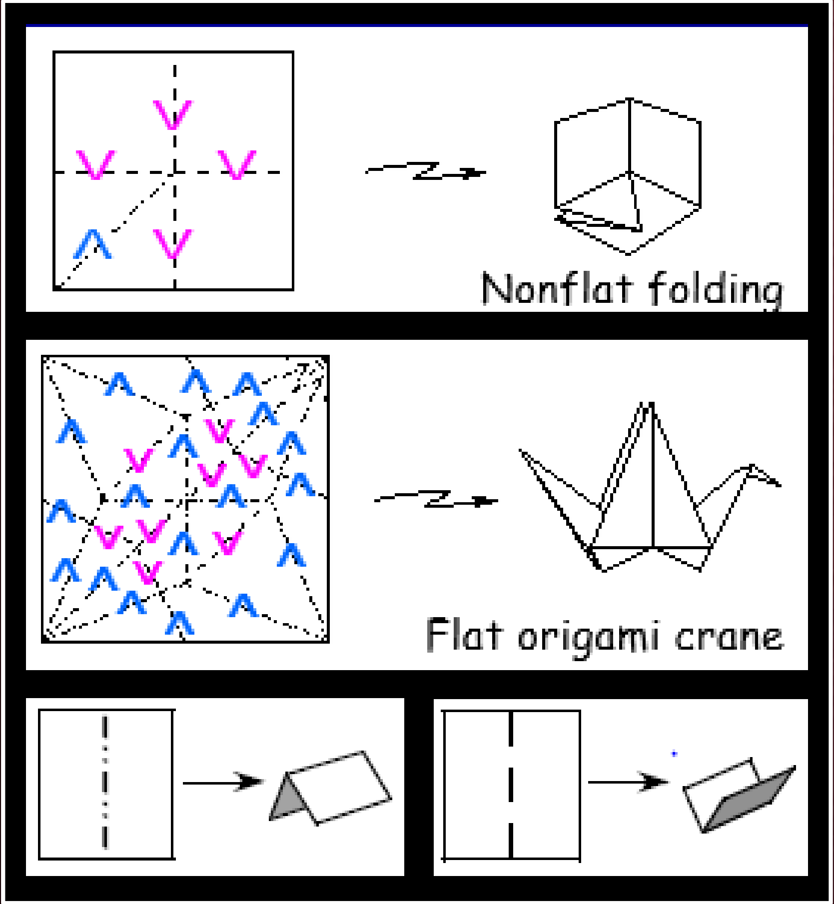


Proof (n=2):

- Step 1: $\alpha_1-\alpha_2+\alpha_3-\alpha_4=0$
- Step 2: $\alpha_1+\alpha_2+\alpha_3+\alpha_4+(\alpha_1 -\alpha_2+\alpha_3 -\alpha_4)=360°$
- Simplify: $2\alpha_1+2\alpha_3=360°$
  $\alpha_1+\alpha_3=180°$



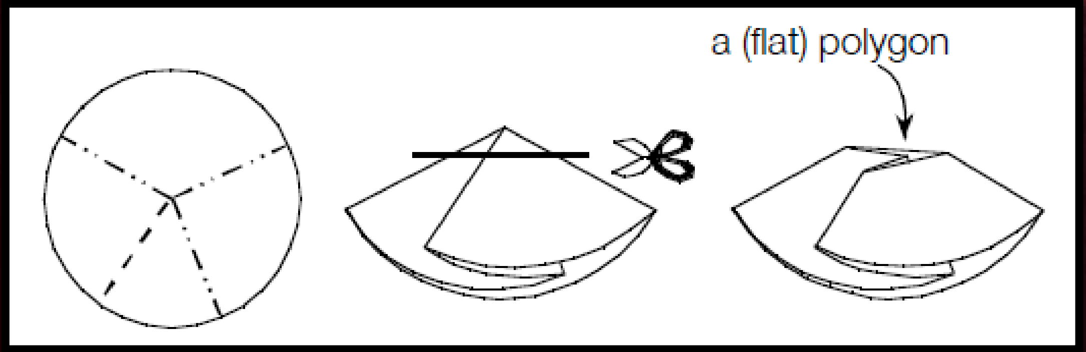
MAEKAWA'S THEOREM

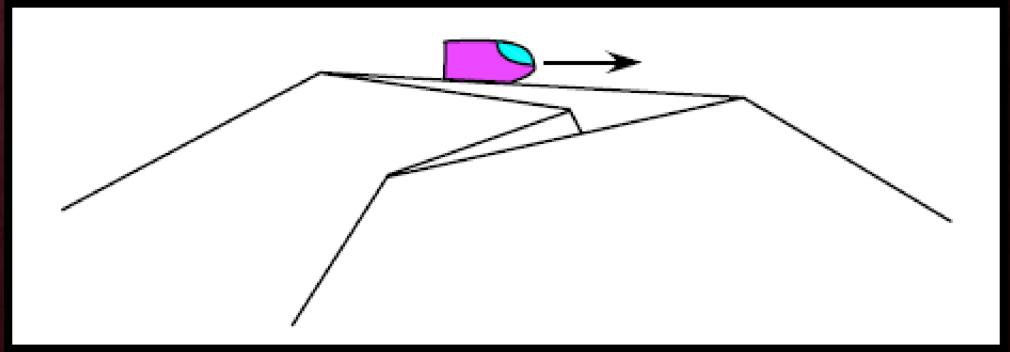


- Theorem: The difference between the number of mountain and valley creases in a flat-foldable vertex is always two.
  - $|M - V| = 2$
- A few definitions:
  - Mountain: crease that elevates the paper when unfolded
  - Valley: crease that forms a concave shape when unfolded

PROOF OF MAEKAWA'S THEOREM

- 180° rotation at each mountain crease
- -180° rotation at each valley crease
- $180 \cdot M - 180 \cdot V = 360$
  - Where M is the number of mountain creases and V is the number of valley creases
  - The sum of the exterior angles is 360°
- After simplification: $M - V = 2$





EXAM QUESTION

- Is it possible to have an odd number of creases surrounding a vertex for a flat-foldable construction? Hint: Use both theorems presented in the slideshow to explain your answer.

REFERENCES

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