December Exam

MATH 100:11, St. Francis Xavier University

December 9, 2011 — **2pm-4pm**
Instructor: Tara Taylor — **Location: NH315**

NAME (PRINT) SOLUTIONS

STUDENT NUMBER ______________________

SIGNATURE ______________________

- You can use a calculator, one index-card sized formula sheet (both sides), and your square for the symmetries of the square.
- Please write answers on the question sheets, and use the back sides for scrap paper.
- There are two sections to this exam. The first section consists of 10 mandatory questions each worth 2 marks, for a total of 20 marks. The second section consists of 6 long answer questions, each worth 4 with the best 5 taken, for a total of 20 marks.
- The entire exam is out of 40 marks.

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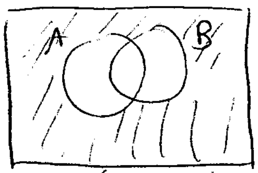
1 Mandatory Questions

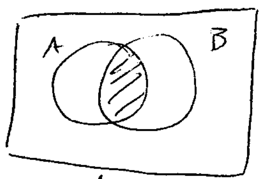
Each question is worth 2 marks, for a total of 20 marks. These questions are all mandatory.

1. Given two arbitrary sets A and B, give a Venn Diagram to illustrate the set

$$(A \cup B)' \cup (A \cap B)$$

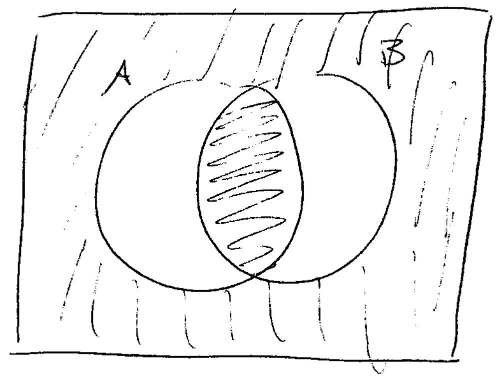


$(A \cup B)'$



$A \cap B$

So



2. The 4^{th} row of Pascal's triangle is 1 4 6 4 1. Use **this information** to determine how many subsets with 3 elements the set A has if $n(A) = 5$.

5^{th} row: 1 5 10 10 5 1
0 1 2 3 4 5

If $n(A) = 5$, then A has 10 subsets with 3 elements.

3. Find the sum $100 + 101 + 102 + \cdots 199 + 200$. (show your work)

$= (1 + 2 + \ldots 200) - (1 + \ldots + 99)$

$= \frac{200(201)}{2} - \frac{(99)(100)}{2} = 20100 - 4950$

$= 15150$

4. Find $6542_{seven} - 2456_{seven}$. (leave everything base 7)

$$\begin{array}{r} {}^{4}\,{}^{\cancel{3}\,10}\,{}^{9} \\ 6\cancel{5}\cancel{4}\cancel{2}_{seven} \\ -\ 2456_{seven} \\ \hline 4053_{seven} \end{array}$$

5. Use the fact that the set of prime numbers is a subset of the naturals $\{1, 2, 3, \cdots\}$ to find the cardinal number of the set of primes.

There are infinitely many primes.
The cardinal # of naturals is $\aleph_0$ so
cardinal number of primes must also be $\aleph_0$

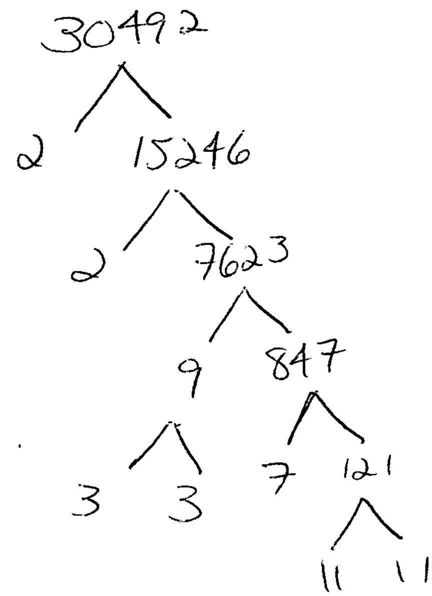
6. Give an example of an object which possesses the golden ratio and explain how the golden ratio shows up.

ex CN tower - ratio of total height to height to look off deck

7. Write out the negation to the statement "Some students do live in residence or do not have a car."

Every student does not live in residence and has a car.

8. Find the prime factorization of 30492.



So $30492 = 2^2 3^2 7 \, 11^2$

9. Use truth tables to show that conditional statement $p \to q$ is equivalent to a statement involving "or". (so you first need to give the "or" statement)

$p \to q \equiv \sim p \vee q$

p	q	$\sim p$	$p \to q$	$\sim p \vee q$
T	T	F	T	
T	F	F	F	
F	T	T	T	
F	F	T	T	

10. When you stack your cds in stacks of 6, there is 1 left over. When you stack them in stacks of 10 there are 5 left over. Use modular systems to determine the smallest number of cds you could have.

Let x = # of cds

So $x \equiv 1 \bmod 6$

$x = 5 \bmod 10$

numbers 1 mod 6: 1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67

numbers 5 mod 10: 5, 15, 25, 35 ...

Smallest number is 25.

2 Long Answer Questions

6

This section has a total of 20 marks. There are ~~7~~ questions, each worth 4. The best 5 will be taken.

1. Recall that the Fibonacci sequence is given by $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. The following four equations are true:

$$\begin{aligned} 2^2 - 1^2 &= 3 & F_3^2 - F_1^2 &= F_4 \\ 3^2 - 1^2 &= 8 & F_4^2 - F_2^2 &= F_6 \\ 5^2 - 2^2 &= 21 & F_5^2 - F_3^2 &= F_8 \\ 8^2 - 3^2 &= 55 & F_6^2 - F_4^2 &= F_{10} \end{aligned}$$

   (a) Use inductive reasoning to make a conjecture as to what the next equation would be, and verify it.

   (b) Use your reasoning to come up with a general equation in terms of the F_n's.

$F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, $F_7 = 13$, $F_8 = 21$, $F_9 = 34$, $F_{10} = 55$, $F_{11} = 89$, $F_{12} = 144$

a) next: $13^2 - 5^2 = 144$ ✓ $169 - 25 = 144$ true

b) general: $F_n^2 - F_{(n-2)}^2 = F_{2n-2}$

2. Many electronic devices use the ASCII coding system. This system uses 7 digit binary codes to represent characters. The binary equivalents of the decimal numbers 65 through 90 represent the capital letters A through Z. So A is represented by 1000001 and so on. Strings of binary digits can be put together to make words, with each block of 7 digits representing a certain letter. Using this information, translate the following binary string into a word:

1001000|1001111|1010000|1000101

64, 32, 16, 8, 4, 2, 1

A	= 65	N	78
B	= 66	O	79
C	67	P	80
D	68	Q	81
E	69	R	82
F	70	S	83
G	71	T	84
H	72	U	85
I	73	V	86
J	74	W	87
K	75	X	88
L	76	Y	89
M	77	Z	90

1001000 $= 64 + 8 = 72 =$ H

1001111 $= 64 + 8 + 4 + 2 + 1 = 79$ = O

1010000 $= 64 + 16 = 80$ = P

1000101 $= 64 + 4 + 1 = 69$ = E

Word is HOPE

3. Recall that in the symmetries of the square we have the following eight elements: M is rotation clockwise by 90, N is rotation by 180, P is rotation by 270, Q is rotation by 360, R is a horizontal flip, S is a vertical flip, T is a flip about the diagonal through the top left corner and bottom right corner, V is a flip about the diagonal through the top right corner and bottom left corner.

   (a) Find RM and MR (show your work). 2

   (b) Find the inverse of T (explain). 1

   (c) Is this system commutative? Why or why not? 1

$RM = T$

a) $\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix} \xrightarrow{R} \begin{bmatrix}3' & 2'\\ 4' & 1'\end{bmatrix} \xrightarrow{M} \begin{bmatrix}4' & 3'\\ 1' & 2'\end{bmatrix}$

$\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix} \xrightarrow{M} \begin{bmatrix}3 & 4\\ 2 & 1\end{bmatrix} \xrightarrow{R} \begin{bmatrix}2' & 1'\\ 3' & 4'\end{bmatrix}$ $MR = V$

b) Inverse of T: $\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix} \xrightarrow{T} \begin{bmatrix}4' & 3'\\ 1' & 2'\end{bmatrix} \xrightarrow{?} \begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$ Need to do T again

inverse is T

c) System is not commutative because order matters

ex $RM \neq MR$

4. Consider the following argument:

   If it is the weekend and it is snowing, Aidan will go skiing. Aidan does not go skiing. Therefore, it is not snowing.

   (a) Express the argument in symbols.

   (b) Determine if the argument valid or not. Be sure to explain why or why not.

p: It is the weekend

q: It is snowing

r: Aidan will go skiing

$$\begin{array}{l} (p \wedge q) \to r \\ \sim r \\ \hline \therefore \sim q \end{array}$$

$$\begin{array}{l} (p \wedge q) \to r \\ \sim r \\ \hline \therefore \sim (p \wedge q) \end{array}$$

is valid by Modus Tollens

$\sim(p \wedge q) \equiv \sim p \vee \sim q$ just need one of $\sim p$ or $\sim q$ to be true

Can't assume $\sim q$ has to be true

$\therefore$ NOT valid

5. The Dean of Student Services surveyed a group of students about which support services they were using to help them improve their academic performance and found the following results:

- 1 • 5 were using office hours, tutoring and online study groups
- 2 • 16 were using office hours and tutoring $= 5 + 11$
- 6 • 28 were using tutoring $= 11 + 5 + 9 + 3$
- 3 • 14 were using tutoring and online study groups $= 5 + 9$
- 4 • 8 were using office hours and online study groups but not tutoring
- 5 • 23 were using office hours but not tutoring $= 8 + 15$
- 7 • 18 were using only online study groups
- 8 • 37 were using none of the services

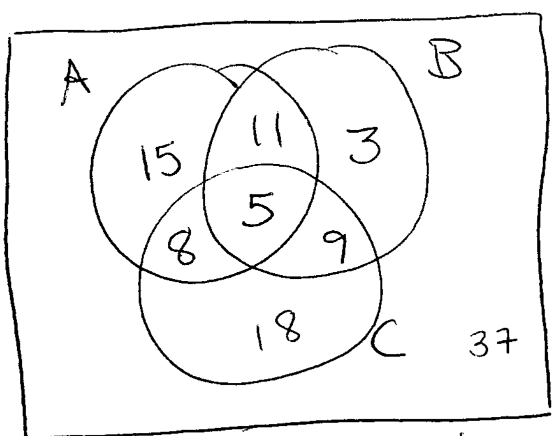
(a) Use a Venn Diagram to illustrate the survey results.

(b) How many students were using only ~~office hours?~~ one service

(c) How many students were using at ~~most~~ least 2 services?

Let A = set of students using office hours
B = " tutoring
C = " online study groups

a)



b) Only ~~office hours~~ one service - just 15 students $+ 3 + 18 = 36$

c) At least 2 services

$8 + 5 + 9 + 11 = 33$

33 students using at least 2

6. Consider the following statement:

If the sum of the digits of a number n add up to a multiple of 9, then the number n is a multiple of 3.

(a) Determine the truth value of the statement (explain). 1.5

(b) Write out in words the inverse of the statement. 1 1.5

(c) Determine the truth value of the inverse. If true, explain why. If false, give a counter-example.

a) If the sum of the digits of n add up to a multiple of 9, then n is a multiple of 9 (divisibility test). Any multiple of 9 is a multiple of 3 since 9 = 3x3, So this is true.

b) Inverse: If the sum of the digits of a number n do not add up to a multiple of 9, then the number n is not a multiple of 3.

c) ex. n = 12 1+2 = 3 not a multiple of 9 but 12 = 4(3) is a multiple of 3

So this is a counterexample.

The inverse is false.

**** HAVE A GREAT HOLIDAY! ****