Midterm A

MATH 100:11, St. Francis Xavier University

February 14, 2012 **2:15pm-3:30pm**

Instructor: Tara Taylor

NAME (PRINT) SOLUTIONS

STUDENT NUMBER _____

SIGNATURE _____

You can use calculators and one index-card sized formula sheet. Please write answers on the question sheets, and use the back sides for scrap paper. The midterm consists of written answer questions. One of the problems has a choice of two, and if you do both then the best one will be taken for marks. The entire midterm is out of 30 marks. Good luck!

---

1. Use the correct order of operations to simplify [2]

$$\frac{-3^2 + 8 \times 4 - 7}{-(4-6)^2 + |1-13|}$$

$$= \frac{-9 + 32 - 7}{-(-2)^2 + |-12|} = \frac{23-7}{-4+12} = \frac{16}{8} = 2$$

2. Simplify to a single fraction in reduced form: [2]

$$\frac{20}{75} \div \left(\frac{2}{3} + \frac{2}{5}\right)$$

$$= \frac{20}{75} \div \left(\frac{2}{3}\times\frac{5}{5} + \frac{2}{5}\times\frac{3}{3}\right) = \frac{20}{75} \div \left(\frac{10+6}{15}\right) = \frac{20}{75} \div \frac{16}{15}$$

$$= \frac{\cancel{20}^{\,4}}{\cancel{75}_{\,15}} \times \frac{\cancel{15}^{\,1}}{\cancel{16}_{\,4}} = \frac{1}{4}$$

3. Convert 5/12 to a decimal and show your work. [2]

$$12 \overline{)\,5.0000\ldots} \quad 0.4166\ldots$$

$$48$$
$$20$$
$$12$$
$$80$$
$$72$$
$$80$$

$$\frac{5}{12} = 0.41666\ldots$$
$$= 0.41\overline{6}$$

4. (a) Use the definition of rationals to prove that the sum of two rationals is always a rational.

(b) Prove or give a counterexample: the sum of two irrationals is always irrational. [3]

a) let z_1, z_2 be rationals. Then $z_1 = \frac{p_1}{q_1}$, $z_2 = \frac{p_2}{q_2}$ where p_1, p_2, q_1, q_2 are integers, and $q_1, q_2 \neq 0$.

$$z_1 + z_2 = \frac{p_1}{q_1} + \frac{p_2}{q_2} = \frac{p_1 q_2 + p_2 q_1}{q_1 q_2} = \frac{p_3}{q_3}$$ where p_3, q_3 are integers & $q_3 \neq 0$.

b) Counterexample: $\sqrt{2} + (-\sqrt{2}) = 0$

5. Simplify to just one term (and don't use decimals): [2]

$$\sqrt{32} + \sqrt{50} - \sqrt{98}$$

$$\sqrt{2\times16} + \sqrt{2\times25} - \sqrt{2\times49} = 4\sqrt{2} + 5\sqrt{2} - 7\sqrt{2}$$

$$= 2\sqrt{2}$$

6. Explain in your own words what the density property of the rationals is trying to say. [1]

Between every 2 rationals there is another rational

7. Convert $0.618181818... = 0.6\overline{18}$ to a fraction in lowest terms. [3]

$$= 0.6 + .0\overline{18}$$

$$0.6 = \frac{6}{10} = \frac{3}{5}$$

$$.0\overline{18} = \frac{18}{99} = \frac{2\times9}{11\times9} = \frac{2}{11} \rightarrow .01\overline{8} = \frac{2}{110} = \frac{1}{55}$$

$$\frac{3}{5} + \frac{1}{55} = \frac{3}{5}\times\frac{11}{11} + \frac{1}{55} = \frac{33+1}{55} = \frac{34}{55}$$ in lowest terms because no common factors

8. Solve the following equation for x and be sure to check your answer: [2]

$$\frac{1}{6}x + \frac{3}{4}x - 5 = \frac{2}{3}x - 2$$

① get rid of fractions - LCD is 12

$$12 \times \frac{1}{6}x + 12 \times \frac{3}{4}x - 12(5) = 12\left(\frac{2}{3}x\right) - 12(2)$$

$$2x + 9x - 60 = 8x - 24$$

$$11x - 60 = 8x - 24$$

$$11x - 8x = -24 + 60$$

$$3x = 36 \rightarrow x = 36/3 = 12$$

check $\frac{1}{6}(12) + \frac{3}{4}(12) - 5 = \frac{2}{3}(12) - 2$

$$2 + 9 - 5 = 8 - 2$$

$$6 = 6 \checkmark$$

9. Find an equation of the line that is perpendicular to the line $y = -2x + 3$ and goes through the point $(4, 1)$. Draw a graph of each line (on the same graph). [3]

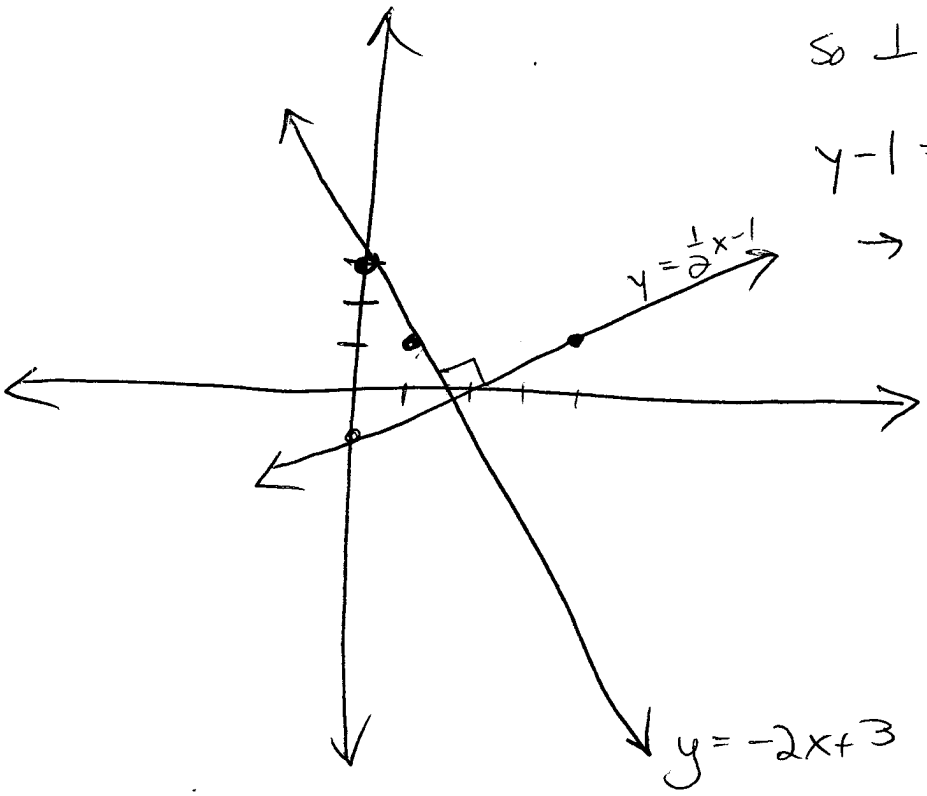
m for $y = -2x + 3 = -2$

So $\perp$ line has $m = \frac{-1}{-2} = \frac{1}{2}$

$$y - 1 = \frac{1}{2}(x - 4)$$

$$\rightarrow y - 1 = \frac{1}{2}x - 2$$

$$y = \frac{1}{2}x - 1$$



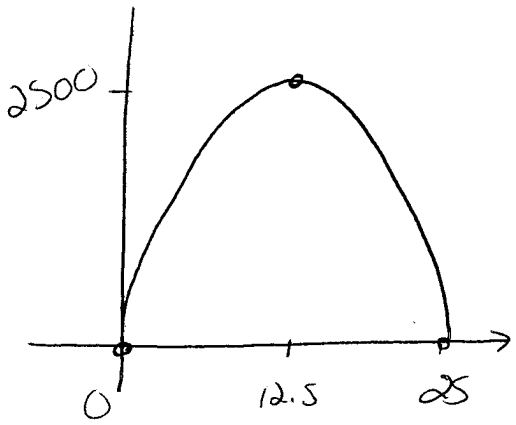
10. An object is fired into the air so that its distance in feet above the ground t seconds after firing is given by

$$s(t) = -16t^2 + 400t$$

Find the maximum height it reaches and the number of seconds it takes to reach that height. (Hint: consider the graph) [2]

$s(t) = -16t^2 + 400t = -16t(t-25)$ $t=0$ or $t=25$

Vertex is halfway between so $\frac{25}{2} = 12.5$



$s(12.5) = -16\left(\frac{25}{2}\right)^2 + 400\left(\frac{25}{2}\right)$

$= \frac{-16}{4}(25)^2 + 200(25)$

$= -4(625) + 5000$

$= -2500 + 5000$

$= 2500$

Max height is 2500 ft at 12.5 seconds

11. Solve the equation either by factoring or by using the quadratic formula: [2]

$$6x^2 - 6x - 72 = 0$$

$6x^2 - 6x - 72 = 6(x^2 - x - 12) = 6(x-4)(x+3) \Rightarrow x = 4, -3$

or -

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(6)(-72)}}{2(6)}$

$= \frac{6 \pm \sqrt{36 + 1728}}{12} = \frac{6 \pm 42}{12} = \frac{6+42}{12}, \frac{6-42}{12}$

$= \frac{48}{12}, \frac{-36}{12}$

$x = 4, -3$

12. Convert the following from words into an equation (but don't solve): the quotient of a number and seven is five more than three times the number. [2]

Let x be the number

$\frac{x}{7} = 3x + 5$

13. Solve ONE of the following by setting up a linear model and solving a linear equation. Check your answer. [5]

(a) A chemist needs to mix 20 liters of 40% acid solution with some 70% acid solution to get a mixture that is 50% acid. How many liters of the 70% solution should be used?

(b) Two cars leave Antigonish at the same time, going in the same direction. One travels at a speed of 95 km/h and the other travels at a speed of 105 km/h. How long does it take for them to be 15 km apart?

a) Let x = # liters of 70% solution

strength	amt of liquid	amt of acid
40%	20L	$.4(20) = 8$
70%	x	$.7x$
50%	$20+x$	$.5(20+x)$

$.5(20+x) = 8+.7x$

$10 + .5x = 8 + .7x \Rightarrow 10-8 = .7x - .5x$

$2 = .2x$

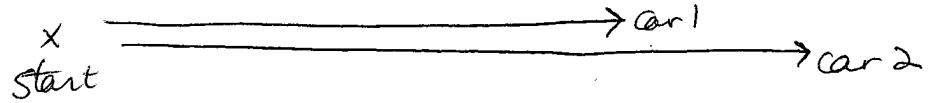
$\frac{2}{.2} = x \rightarrow x = 10$

Should add 10 l of 70%

check: total liquid $= 20+10 = 30$

total acid $= .4(20) + .7(10) = 8+7 = 15 = 50\%$ of liquid ✓

b)



let t = time it takes for cars to be 15 km apart (in hours)

distance of slow car $= 95t$

distance of fast car $= 105t$

want $105t - 95t = 15 \Rightarrow 10t = 15 \Rightarrow t = \frac{15}{10} = 1.5$ hrs

They will be 15km apart after 1.5 hours

check slow car $- 95(1.5) = 142.5$

fast car $105(1.5) = 157.5$ — difference is 15km ✓