

Midterm B

MATH 100:11, St. Francis Xavier University

February 14, 2012

2:15pm-3:30pm

Instructor: Tara Taylor

NAME (PRINT) _____

SOLUTIONS

STUDENT NUMBER _____

SIGNATURE _____

You can use calculators and one index-card sized formula sheet. Please write answers on the question sheets, and use the back sides for scrap paper. The midterm consists of written answer questions. One of the problems has a choice of two, and if you do both then the best one will be taken for marks. The entire midterm is out of 30 marks. Good luck!

1. Use the correct order of operations to simplify

[2]

$$\frac{-4^2 + 7 \times 3 + 5}{-(1-4)^2 + |1-15|}$$
$$= \frac{-16 + 21 + 5}{-(-3)^2 + |-14|} = \frac{5+5}{-9+14} = \frac{10}{5} = 2$$

2. Simplify to a single fraction in reduced form:

[2]

$$\frac{75}{20} \div \left(\frac{3}{4} + \frac{4}{5} \right)$$
$$= \frac{75}{20} \div \left(\frac{3 \times 5}{4 \times 5} + \frac{4 \times 4}{5 \times 4} \right) = \frac{75}{20} \div \left(\frac{15+16}{20} \right) = \frac{75}{20} \div \left(\frac{31}{20} \right)$$
$$= \frac{75}{20} \times \frac{20}{31} = \frac{75}{31} \text{ in reduced form because no common factors}$$

3. Convert 7/12 to a decimal and show your work.

[2]

$$12 \overline{) 7.00000000} = 0.583333... = .58\overline{3}$$

Handwritten long division steps:

$$\begin{array}{r} 6.0 \\ \hline 100 \\ 96 \\ \hline 40 \\ 36 \\ \hline 40 \\ 36 \\ \hline \dots \end{array}$$

4. (a) Use the definition of rationals to prove that the sum of two rationals is always a rational.
 (b) Prove or give a counterexample: the sum of two irrationals is always irrational. [3]

see midterm A

5. Simplify to just one term (and don't use decimals): [2]

$$\begin{aligned} & \sqrt{18} + \sqrt{72} - \sqrt{50} \\ & \sqrt{9 \times 2} + \sqrt{36 \times 2} - \sqrt{25 \times 2} \\ & = 3\sqrt{2} + 6\sqrt{2} - 5\sqrt{2} \\ & = 4\sqrt{2} \end{aligned}$$

6. Explain in your own words what the density property of the rationals is trying to say. [1]

between every rational is another rational

7. Convert $0.9363636\dots = 0.9\overline{36}$ to a fraction in lowest terms. [3]

$$= 0.9 + 0.0\overline{36}$$

$$0.9 = \frac{9}{10}$$

$$0.\overline{36} = \frac{36}{99} = \frac{4}{11} \quad \text{so } 0.0\overline{36} = \frac{4}{110} = \frac{2}{55}$$

$$\frac{4}{11} + \frac{4}{110} = \frac{4 \times 10}{11 \times 10} + \frac{4}{110} = \frac{40 + 4}{110} = \frac{44}{110} = \frac{4 \times 11}{10 \times 11} = \frac{4}{10} \quad \text{oops!}$$

$$\frac{9}{10} + \frac{4}{110} = \frac{9 \times 11}{10 \times 11} + \frac{4}{110} = \frac{99 + 4}{110} = \frac{103}{110} \quad \text{no common factors}$$

8. Solve the following equation for x and be sure to check your answer:

[2]

$$\frac{1}{4}x + \frac{5}{6}x - 6 = \frac{2}{3}x - 1$$

① multiply by LCD to get rid of fractions LCD = 12

$$12\left(\frac{1}{4}x\right) + 12\left(\frac{5}{6}x\right) - 12(6) = 12\left(\frac{2}{3}x\right) - 12(1)$$

$$3x + 10x - 72 = 8x - 12$$

$$13x - 72 = 8x - 12$$

$$13x - 8x = -12 + 72$$

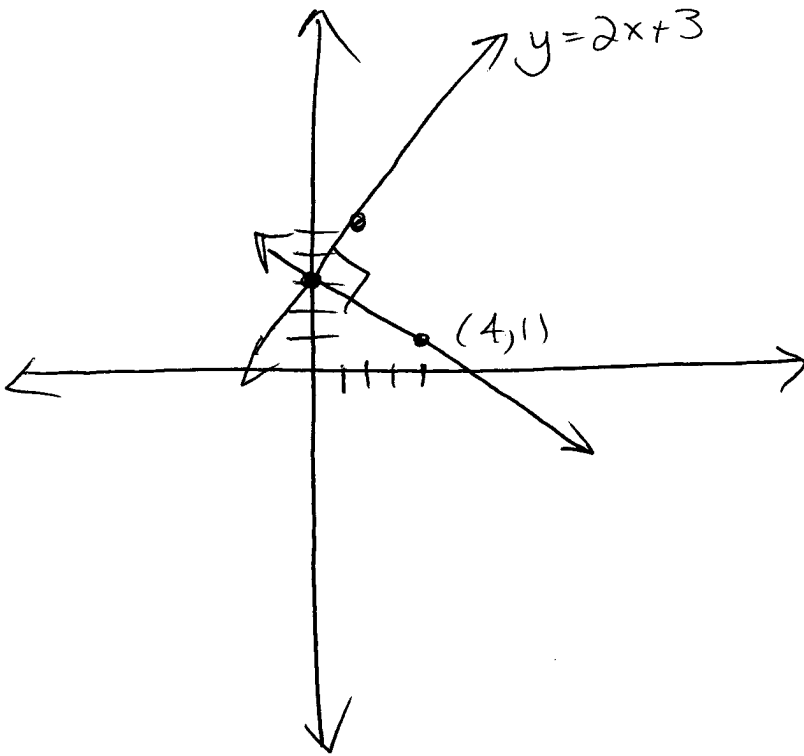
$$5x = 60 \rightarrow x = \frac{60}{5} = \boxed{12}$$

check: $\frac{1}{4}(12) + \frac{5}{6}(12) - 6 = \frac{2}{3}(12) - 1$

$$3 + 10 - 6 = 8 - 1$$

$$7 = 7 \quad \checkmark$$

9. Find an equation of the line that is perpendicular to the line $y = 2x + 3$ and goes through the point $(4, 1)$. Draw a graph of each line (on the same graph). [3]



$m = 2$ so \perp line has
Slope $m = -\frac{1}{2}$

$$y - 1 = -\frac{1}{2}(x - 4)$$

$$y - 1 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 3$$

10. An object is fired into the air so that its distance in feet above the ground t seconds after firing is given by

$$s(t) = -16t^2 + 400t$$

Find the maximum height it reaches and the number of seconds it takes to reach that height. (Hint: consider the graph) [2]

See Midterm A

11. Solve the equation either by factoring or by using the quadratic formula: [2]

$$6x^2 - 6x - 72 = 0$$

See midterm A

12. Convert the following from words into an equation (but don't solve): the quotient of a number and five is seven more than three times the number. [2]

let x be the number

$$\frac{x}{5} = 3x + 7$$

13. Solve ONE of the following by setting up a linear model and solving a linear equation. Check your answer. [5]

- (a) A chemist needs to mix 40 liters of 20% acid solution with some 70% acid solution to get a mixture that is 50% acid. How many liters of the 70% solution should be used?
- (b) Two cars leave Antigonish at the same time, going in the same direction. One travels at a speed of 100 km/h and the other travels at a speed of 110 km/h. How long does it take for them to be 15 km apart?

a) Let $x =$ amt of 70% in liters

strength	amt liquid	amt chemical
20%	40	$.2(40) = 8$
70%	x	$.7x$
50%	$40+x$	$.5(40+x)$

$$.5(40+x) = 8 + .7x$$

$$20 + .5x = 8 + .7x$$

$$20 - 8 = .7x - .5x$$

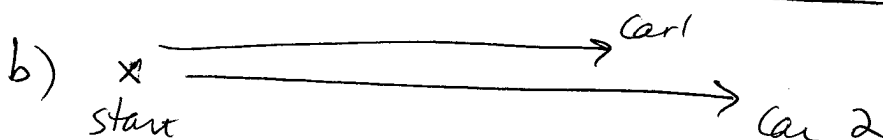
$$12 = .2x \rightarrow x = \frac{12}{.2} = 60L$$

Should add 60L of 70%

check: total liquid = $40 + 60 = 100L$

total acid = $.2(40) + .7(60) = 8 + 42 = 50L$

= 50% of total ✓



let $t =$ time to be 15 km apart in hours

then $110t - 100t = 15$

$$10t = 15 \rightarrow t = \frac{15}{10} = 1.5 \text{ hours}$$

check slow car travels $(100)(1.5) = 150 \text{ km}$ ✓

fast car travels $(110)(1.5) = 165 \text{ km} - 15 \text{ km more}$

Takes 1.5 hours to be 15 km apart