

Midterm A

MATH 100:12, St. Francis Xavier University

October 17, 2014

10:15am-11:05am

Instructor: Tara Taylor

NAME (PRINT) SOLUTIONS

STUDENT NUMBER \_\_\_\_\_

SIGNATURE \_\_\_\_\_

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You can use calculators and one index-card sized formula sheet (both sides). Please write answers on the question sheets, and use the back sides for scrap paper. There are two sections to this midterm. The first section consists of 8 True/False questions, each worth 2 marks, for a total of 16 marks. The second section consists written answer questions for a total of 24 marks. The entire midterm is out of 40 marks. Good luck!

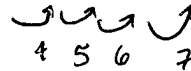
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# 1 True/False Questions

Each question is worth 2 marks, for a total of 16 marks. No explanation is required, just fill in T for true or F for false in the blank before the statement.

1. T Deductive reasoning is taking a general rule and applying it to a specific case.

2. T The next probable number in the sequence 2, 6, 11, 17, ... is 24.



3. T  $3 + 6 + 9 + \dots + 3n = \frac{3n(n+1)}{2}$

4. F For  $A = \{x \mid x \text{ is a letter in the word "mathematics"}\}$ ,  $n(A) = 11$ . ~~7~~

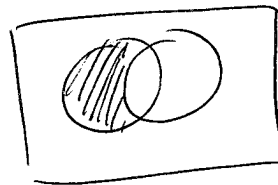
5. F For any sets  $A$  and  $B$ , we have  $(A \cup B)' = A' \cup B'$ .

6. T If  $n(A) = 2$  and  $n(B) = 3$ , then  $A \times B$  has 64 subsets.  $n(A \times B) = 6$   $2^6 = 64$

7. T If  $A = \{2, 4, 6, \dots\}$  and  $B = \{10, 20, 30, \dots\}$  then  $A$  and  $B$  are equivalent.

both equiv to  $\mathbb{N}$

8. T For any sets  $A$  and  $B$ ,  $A - B = A \cap B'$ .



in  $A$  and not in  $B$

## 2 Long Answer Questions

This section has a total of 24 marks.

1. The following three equations are all true:

[5]

$$\left. \begin{array}{l} \text{triangular} \\ \text{numbers} \end{array} \right\} \begin{array}{l} 3^2 - 1^2 = 2^3 \\ 6^2 - 3^2 = 3^3 \\ 10^2 - 6^2 = 4^3 \end{array} \left. \vphantom{\begin{array}{l} \text{triangular} \\ \text{numbers} \end{array}} \right\} \begin{array}{l} \text{cubes going} \\ \text{up by 1} \end{array}$$

(a) Based on the pattern of the equations, what would the next two equations be? Explain your reasoning and check if the equations are true.

(b) What would the  $n^{\text{th}}$  equation be? Hint: consider the triangular numbers  $T_n$ .

Triangular  
numbers

1, 3, 6, 10, 15, 21, 28, ...

next 2 equations

$$15^2 - 10^2 = 5^3$$

$$21^2 - 15^2 = 6^3$$

$$\begin{array}{l} \text{check: } 225 - 100 = 125 = 5^3 \checkmark \\ 441 - 225 = 216 = 6^3 \checkmark \end{array}$$

$$b) T_1 = 1 \quad T_2 = 3 \quad \dots \quad T_n = \frac{n(n+1)}{2}$$

$$n=1: T_2^2 - T_1^2 = 2^3$$

$$n: T_{n+1}^2 - T_n^2 = (n+1)^3$$

$$\text{or } \left[ \frac{(n+1)(n+2)}{2} \right]^2 - \left[ \frac{n(n+1)}{2} \right]^2 = (n+1)^3$$

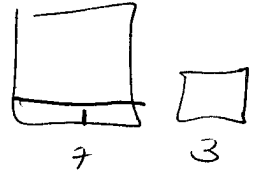
Do any 1 of the following 2 questions (and explain your answers). If you do both, they will both be marked and the best one will be taken. [5]

- (a) Max, Sam and Brett were playing basketball. One of them broke a window, and the other two saw him break it. Max said, "I am innocent." Sam said, "Max and I are both innocent." Brett said, "Max and Sam are both innocent." If only one of them is telling the truth, who broke the window?
- (b) You have brought two unmarked buckets to a stream. The buckets hold 7 gallons and 3 gallons of water, respectively. How can you obtain exactly 5 gallons of water to take home?

a) IF Max is true, others false: Max is innocent, Sam is not  
 IF Sam is true, others false: Max is not innocent - Sam contradicts himself - not possible

IF Brett is true - this contradicts Max.  
 Thus Max is true & Max is innocent. Sam broke window

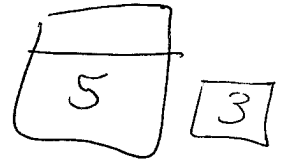
b) Fill 7 up. Transfer to 3 twice & empty 3



Transfer 1 to 3, fill up 7



Transfer 2 to 3 to fill, 7 left with 5



2. Let  $A = \{4, 7, 10, 13, \dots\}$ . Show that  $A$  is infinite either by using the definition of infinite set or by showing it is equivalent to  $\mathbb{N} = \{1, 2, 3, \dots\}$ . [3]

① def of infinite: equivalent to proper subset

let  $B = \{7, 10, 13, \dots\} \subset A$ .

Define matching  $A \leftrightarrow B$  by  $n \leftrightarrow n+3$ . This is a one to one correspondence, so  $A \sim B$ .  $A$  must be infinite

②  $\mathbb{N} = \{1, 2, 3, \dots\}$   
 $A = \{4, 7, 10, \dots\}$

matching  $n \leftrightarrow 3n+1$   
 gives one to one correspondence.  
 $\mathbb{N}$  is infinite, so  $A$  must be as well.

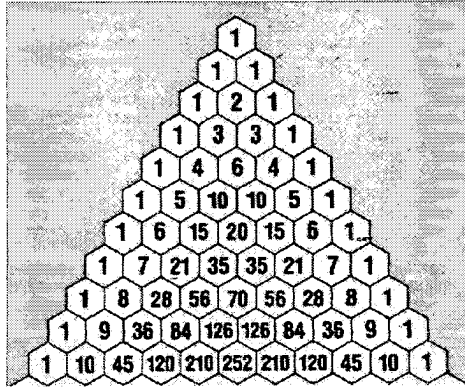


Figure 1: Pascal's Triangle

3. (a) Using the triangle, how many ways are there to order a pizza with 6 toppings if there are 9 toppings to choose from? (assuming there is only one size and you can't double up on toppings) [2]
- (b) Looking at the fifth row (starting 1 5...), find the sum of the numbers in the row. How does this row and sum of the numbers connect with the number of subsets of a set  $A$  if  $n(A) = 5$ ? Explain. [3]

a)  $1 \quad 9 \quad 36 \quad 84 \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1$   
 $0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$

84 ways to order pizza with 6 toppings

b)  $1 + 5 + 10 + 10 + 5 + 1 = 32$

if  $n(A) = 5$

1	subset with	0	elements
5	"	1	"
10	"	2	"
10	"	3	"
5	"	4	"
1	"	5	"

∴ we know there are  $2^5 = 32$  total subsets

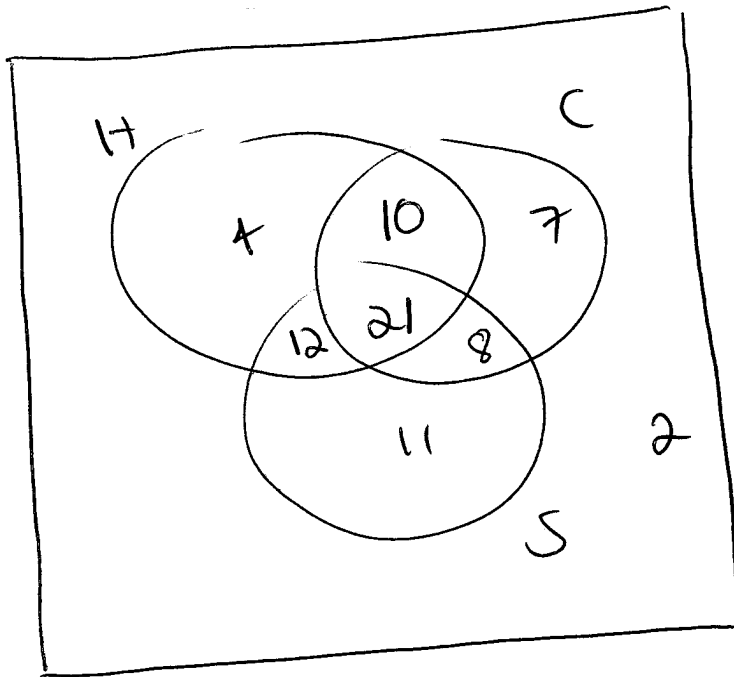
4. Jessica Fisher conducted a survey among 75 patients admitted to the cardiac unit of a Halifax hospital during a two month period. She investigated three risk factors: high blood pressure, high cholesterol and smoking. [6]

- ⑤ • 47 patients had high blood pressure  $47 = 10 + 21 + 12 + \underline{4}$
- ⑥ • 46 patients had high cholesterol  $46 = 10 + 21 + 8 + \underline{7}$
- ⑦ • 52 patients were smokers  $52 = 12 + 21 + 8 + \underline{11}$
- 33 patients had high blood pressure and were smokers ②  $33 = 21 + 12$
- 31 patients had high blood pressure and high cholesterol ③  $31 = 21 + 10$
- 21 patients had all three risk factors ①
- 51 patients had at least two of the risk factors ④  $51 = 12 + 21 + 10 + \underline{8}$

- (a) Draw a corresponding Venn diagram (and be sure to first define your sets).
- (b) Find the number of patients who had no risk factors.
- (c) Find the number of patients who were not smokers.
- (d) Find the number of patients who were smokers but did not have high blood pressure or high cholesterol.

Let  $H$  = set of people with high blood pressure  
 $C$  = " " " " cholesterol  
 $S$  = " " " " smoke

total  
 $75 = 4 + 10 + 7 + 12 + 21 + 8 + 11 + \underline{2}$



- b) 2 people had no risk factors
- c)  $75 - 52 = 23$  people were not smokers
- d) 11 were smokers but did not have high blood pressure or high cholesterol