Acknowledgements

The Department of Education gratefully acknowledges the contributions of the following individuals to the preparation of the Mental Math booklets:

Sharon Boudreau—Cape Breton-Victoria Regional School Board
Anne Boyd—Strait Regional School Board
Estella Clayton—Halifax Regional School Board (Retired)
Jane Chisholm—Tri-County Regional School Board
Paul Dennis—Chignecto-Central Regional School Board
Robin Harris—Halifax Regional School Board
Keith Jordan—Strait Regional School Board
Donna Karsten—Nova Scotia Department of Education
Ken MacInnis—Halifax Regional School Board (Retired)
Ron MacLean—Cape Breton-Victoria Regional School Board
Sharon McCready—Nova Scotia Department of Education
David McKillop—Chignecto-Central Regional School Board
Mary Osborne—Halifax Regional School Board (Retired)
Sherene Sharpe—South Shore Regional School Board
Martha Stewart—Annapolis Valley Regional School Board
Susan Wilkie—Halifax Regional School Board
Contents

Introduction .................................................................................................................. 1
Definitions ............................................................................................................... 1
Rationale .............................................................................................................. 1

The Implementation of Mental Computational Strategies ........................................... 3
General Approach ................................................................................................. 3
Introducing a Strategy ........................................................................................... 3
Reinforcement ....................................................................................................... 3
Assessment .......................................................................................................... 3
Response Time .................................................................................................... 4

A. Addition — Fact Learning .................................................................................. 5
Facts and the Fact Learning Strategies ............................................................... 5
Addition Facts Extended to 10s and 100s ......................................................... 6

B. Addition — Mental Calculations .......................................................................... 11
Using Addition Facts for 10s and 100s ........................................................... 11
Quick Addition — No Regrouping .................................................................... 11
Front End Addition ............................................................................................ 12
Finding Compatibles ......................................................................................... 13
Break Up and Bridge ......................................................................................... 13
Compensation .................................................................................................... 14

C. Subtraction — Fact Learning ............................................................................. 15
Facts and the Fact Learning Strategies ............................................................ 15
Subtraction Facts Extended to 10s and 100s ................................................... 15

D. Subtraction — Mental Calculations ................................................................. 18
Using “Think Addition” in Subtraction ............................................................ 18
Back Through 10 ............................................................................................ 19
Up Through 10 ................................................................................................. 19

E. Addition and Subtraction — Computational Estimation .................................... 20
Front End .......................................................................................................... 20
Adjusted Front End .......................................................................................... 21
Rounding .......................................................................................................... 22
Introduction

Definitions
It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

While there are many aspects to mental math, this booklet, *Mental Computation*, deals with fact learning, mental calculations, and computational estimation — mental math found in General Curriculum Outcome (GCO) B. Therefore, teachers must also remember to incorporate mental math strategies from the six other GCOs into their yearly plans for Mental Math, for example, measurement estimation, quantity estimation, patterns and spatial sense. For more information on these and other strategies see *Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one’s head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one’s head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale
In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people’s calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.
It is important to establish a rational for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost $1.90, can I buy them if I have $5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost $3.00?)
The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students’ abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student’s thinking, especially in situations where pencil-and-paper responses are weak.
Assessments, regardless of their form, should shed light on students’ abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time
Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.
A. Addition — Fact Learning

Facts and the Fact Learning Strategies
In grade two students were expected to recall addition facts to 18, with a three-second response. At the beginning of grade 3, it is important to ensure that students review the addition facts to 18 and the fact learning strategies addressed in previous grades. Students will use these facts and strategies when doing mental math addition with numbers in the 10s, and 100s, and extending to numbers in the 1000s in grade 3.

Examples
The following are the Grade 2 fact strategies with examples and examples of the same facts applied to 10s, 100, and 1000s:

a) Doubles Facts (4 + 4, 40 + 40, 400 + 400, and 4000 + 4000)
b) Plus One (Next Number) Facts (5 + 1, 50 + 10, 500 + 100, 5000 + 1000)
c) 1-Apart (Near Double) Facts (3 + 4, 30 + 40, 300 + 400, 3000 + 4000)
d) Plus Two (Next Even/Odd) Facts (7 + 2, 70 + 20, 700 + 200, 7000 + 2000)
e) Plus Zero (No Change) Facts (8 + 0, 80 + 0, 800 + 0, 8000 + 0)
f) Make 10 Facts (9 + 6, 90 + 60, 900 + 600, 9000 + 6000
8 + 4, 80 + 40, 800 + 400, 8000 + 4000)
g) The Last 12 Facts with some possible strategies (may be others):
h) 2-Apart (Double Plus 2) Facts (5 + 3, 50 + 30, 500 + 300, 5000 + 3000)
i) Plus Three Facts (6 + 3, 60 + 30, 600 + 300, 6000 + 3000)
j) Make 10 (with a 7) Facts (7+ 4, 70 + 40, 700 + 400, 7000 + 4000)

Examples of Some Practice Items

40 + 40 = 90 + 90 = 50 + 50 =
300 + 300 = 7 000 + 7 000 = 2 000 + 2 000 =
70 + 80 = 50 + 60 = 7 000 + 8 000 =
3 000 + 2 000 = 40 + 60 = 50 + 30 =
700 + 500 = 100 + 300 = 7 000 + 9 000 =
8 000 + 6 000 = 3 000 + 5 000 = 4 000 + 2 000 =
55 + 0 = 0 + 47 = 376 + 0 =
5 678 + 0 = 0 + 9 098 = 811 + 0 =
70 + 20 = 30 + 20 = 60 + 20 =
800 + 200 = 100 + 200 = 4 000 + 2 000 =
Addition Facts Extended to 10s and 100s

1. Double Facts

There are only ten doubles facts from 0 + 0 to 9 + 9. They are: 0 + 0, 1 + 1, 2 + 2, 3 + 3, 4 + 4, 5 + 5, 6 + 6, 7 + 7, 8 + 8 and 9 + 9. They are a powerful strategy when learning number facts.

Review these basic double facts, and then extend to 10s and 100s.

Examples

a) If you know that 6 + 6 = 12, then applying the doubles strategy to adding numbers in the 10s, you know

   60 + 60 = 120.

b) If you know that 3 + 3 = 6, then applying the doubles strategy to adding numbers in the 100s, you know

   300 + 300 = 600.

Examples of Some Practice Items

a) Some items applying this strategy to adding numbers in the 10s are:

   40 + 40 = 20 + 20 =

   70 + 70 = 10 + 10 =

   90 + 90 = 80 + 80 =

   50 + 50 = 30 + 30 =

b) Some items applying this strategy to adding numbers in the 100s are:

   200 + 200 = 600 + 600 =

   400 + 400 = 800 + 800 =

   500 + 500 = 700 + 700 =

   900 + 900 = 100 + 100 =

2. Near-Doubles (1-Aparts) Facts

The near-doubles are also called the “doubles plus one” facts and include all combinations where one addend is one more than the other. The strategy is to double the smaller number and add one. For example: 6 + 7 is (6 + 6) + 1.

Review these near-doubles facts and then extend to 10s and 100s: 5 + 6, 2 + 3, 3 + 4, 9 + 10, 8 + 9, 6 + 7, 5 + 4, 2 + 1, 7 + 8.

Examples

a) If you know that 6 + 6 = 12, then applying the near-doubles strategy to adding numbers in the 10s, you know 60 + 70 is (60 + 60) + 10 = 130.

b) If you know that 7 + 7 = 14, then applying the near-doubles strategy to adding numbers in the 100s, you know 800 + 700 is (700 + 700) + 100 = 1500.
Examples of Some Practice Items

a) Some items applying this strategy to adding numbers in the 10s are:

\[
\begin{align*}
30 + 40 &= 70 + 80 \\
50 + 60 &= 10 + 20 \\
80 + 90 &= 50 + 40 \\
30 + 20 &= 20 + 30 \\
60 + 70 &= 60 + 70 \\
70 + 80 &= 70 + 80 \\
10 + 20 &= 10 + 20 \\
50 + 40 &= 50 + 40
\end{align*}
\]

b) Some items applying this strategy to adding numbers in the 100s are:

\[
\begin{align*}
400 + 500 &= 300 + 200 \\
400 + 300 &= 600 + 500 \\
200 + 100 &= 700 + 800 \\
600 + 700 &= 800 + 900 \\
900 + 800 &= 900 + 800 \\
300 + 200 &= 600 + 700 \\
600 + 900 &= 600 + 900 \\
200 + 100 &= 200 + 100
\end{align*}
\]

3. Doubles Plus 2 (2-Aparts) Facts

For addends that differ by 2, such as 3 + 5, 4 + 6, or 5 + 7, there are two possible strategies each depending on knowledge of doubles.

a) One strategy is “doubling the smaller plus 2”. For example, 4 + 6 is double 4 and 2 more.

b) Another strategy, “doubling the number between”, can be used for addends that differ by 2. For example, 5 + 7 is (5 + 1) + (7 - 1) which is doubling the number between 5 and 7 which makes 6 + 6.

Review these doubles plus 2 facts: 7 + 9, 6 + 8, 5 + 7, 1 + 3, 4 + 2, 3 + 5, 6 + 4, and then extend to 10s and 100s.

Example

a) 30 + 50 = double 30 plus 20 = 60 + 20 = 80

b) 70 + 90 = (70 + 10) + (90 - 10) or 80 + 80 = 160

Examples of Some Practice Items

a) Some items applying this strategy to adding numbers in the 10s are:

\[
\begin{align*}
40 + 60 &= \quad 30 + 10 = \\
60 + 80 &= \quad 20 + 40 = \\
90 + 70 &= \quad 50 + 30 = \\
\end{align*}
\]

b) Some items applying this strategy to adding numbers in the 100s are:

\[
\begin{align*}
100 + 300 &= \quad 700 + 900 = \\
700 + 500 &= \quad 800 + 600 = \\
300 + 500 &= \quad 600 + 400 = \\
\end{align*}
\]
c) Some items applying this strategy to adding numbers in the 10s are:

\[
\begin{align*}
40 + 60 &= \\
90 + 70 &= \\
10 + 30 &= \\
30 + 50 &=
\end{align*}
\]

\[
\begin{align*}
80 + 60 &= \\
40 + 20 &= \\
30 + 50 &=
\end{align*}
\]

d) Some items applying this strategy to adding numbers in the 10s are:

\[
\begin{align*}
300 + 500 &= \\
700 + 500 &= \\
700 + 900 &= \\
600 + 800 &=
\end{align*}
\]

4. **Plus or Minus 0 – “No Change”**

Facts with zero do not require any strategy but rather a good understanding of the meaning of zero and addition. For example, 7 + 0 is still 7 because adding 0 makes “no change”.

If we apply the “plus 0 - no change” understanding to numbers in the 1s, 10s, and 100s we know:

**Examples of Some Practice Items**

\[
\begin{align*}
1s & 5 + 0 = \\
& 0 + 9 = \\
& 6 + 0 = \\
10s & 50 + 0 = 50 \\
& 90 + 0 = \\
& 0 + 40 = \\
100s & 800 + 0 = 800 \\
& 300 + 0 = \\
& 0 + 400 =
\end{align*}
\]

5. **Make 10 or 100**

These facts all have one addend of 8 or 9. The strategy for these facts is to build onto the 8 or 9 up to 10 and then add on the rest.

**Examples**

a) For 9 + 6, think: 9 + 1(from the 6) is 10, and 10 + 5 (the other part of the 6) is 15 or

\[
9 + 6 = (9 + 1) + 5 = 10 + 5 = 15
\]

The “make 10” strategy can be extended to facts involving 7.

b) For 7 + 4, think: 7 and 3 (from the 4) is 10, and 10 + 1 (the other part of the 4) is 11 or

\[
7 + 4 = (7 + 3) + 1 = 10 + 1 = 11.
\]
Examples of Some Practice Items

Some items applying this strategy to adding numbers in the 1s are:

8 + 5 = 9 + 5 =
9 + 2 = 5 + 7 =
7 + 6 = 9 + 4 =
8 + 6 = 4 + 8 =

Some items applying this strategy to adding numbers in the 10s and therefore using ‘make 100’ are:

40 + 30, think: 4 tens and 3 tens are 7 tens or 70.
80 + 40, think: 8 tens and 4 tens are 12 tens or 120
20 + 60 =
60 + 50 =
40 + 40 =
90 + 20 =
60 + 80 =

6. Plus 2s or “Next Even/Odd” Number

When adding 2 to a single digit number, for example 5 + 2 use the “plus 2s” strategy. This strategy involves jumping to the “next even or odd number”. For example, 5 + 2 is 7 because the next odd number after 5 is 7; 6 + 2 is 8 because the next even number after 6 is 8.

Now try these: 4 + 2, 7 + 2, 3 + 2, 8 + 2, 6 + 2, 1 + 2, 5 + 2, 2 + 2, 9 + 2.

If we apply the “plus 2s” or “next even/odd” strategy to numbers in the 10s, and 100s, we know:

Examples of Some Practice Items

<table>
<thead>
<tr>
<th>10s</th>
<th>100s</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 + 20 = 90</td>
<td>200 + 700 = 900</td>
</tr>
<tr>
<td>60 + 20 =</td>
<td>600 + 200 =</td>
</tr>
<tr>
<td>30 + 20 =</td>
<td>300 + 200 =</td>
</tr>
<tr>
<td>40 + 20 =</td>
<td>200 + 400 =</td>
</tr>
<tr>
<td>50 + 20 =</td>
<td>800 + 200 =</td>
</tr>
<tr>
<td>10 + 20 =</td>
<td>500 + 200 =</td>
</tr>
<tr>
<td>20 + 20 =</td>
<td>100 + 200 =</td>
</tr>
<tr>
<td>80 + 20 =</td>
<td></td>
</tr>
</tbody>
</table>

7. Plus 3s or “Adding 2 and Then 1”

When adding 3 to a single digit number, a fact such as 6 + 3 can be solved by using the “plus 3s” strategy of “adding 2 and then 1”, thinking “next even/odd and then next number”.

<table>
<thead>
<tr>
<th>Examples of Some Practice Items</th>
<th>100s</th>
</tr>
</thead>
<tbody>
<tr>
<td>10s</td>
<td></td>
</tr>
<tr>
<td>70 + 20 = 90</td>
<td></td>
</tr>
<tr>
<td>60 + 20 =</td>
<td></td>
</tr>
<tr>
<td>30 + 20 =</td>
<td></td>
</tr>
<tr>
<td>40 + 20 =</td>
<td></td>
</tr>
<tr>
<td>50 + 20 =</td>
<td></td>
</tr>
<tr>
<td>10 + 20 =</td>
<td></td>
</tr>
<tr>
<td>20 + 20 =</td>
<td></td>
</tr>
<tr>
<td>80 + 20 =</td>
<td></td>
</tr>
</tbody>
</table>
Examples

6 + 3 is (6 + 2) + 1 or 8 + 1 = 9.

Now try these: 8 + 3, 5 + 3, 9 + 3, 3 + 3, 1 + 3, 7 + 3, 2 + 3, 4 + 3.

Examples of Some Practice Items

If we apply the “plus 3s” or “adding 2 and then 1” strategy to numbers in the 10s and 100s we know:

- **10s**
  - 50 + 30 =
  - 30 + 60 =
  - 90 + 30 =
  - 20 + 30 =

- **100s**
  - 400 + 300 =
  - 700 + 300 =
  - 300 + 800 =
  - 200 + 300 =
B. Addition — Mental Calculations

Using Addition Facts for 10s and 100s
This strategy applies to calculations involving the addition of two numbers in the tens, hundreds or thousands with only one non-zero digit in each number.

Examples
For 30 + 20, think: 3 tens and 2 tens are 5 tens or 50.
This strategy involves combining the single non-zero digits as if they were single-digit addition facts and then attaching the appropriate place value name to the result.

Examples of Some Practice Items
Some examples of applying this strategy to adding numbers in the 10’s are:
Add your own examples:

| 58 + 6 = | 18+4 = |
| 5 + 49 = | 79+6 = |
| 17 + 4 = | 6+27 = |
| 29 + 3 = | 47+5 = |
| 38 + 5 = | 88+3 = |

Some examples of applying this strategy to adding numbers in the 100’s are:
Add your own examples:

| 680 + 28 = |
| 490 + 15 = |
| 170 + 40 = |
| 560 + 89 = |
| 870 + 57 = |
| 780 + 67 = |

Quick Addition – No Regrouping
This pencil-and-paper strategy is used when there are more than two combinations in the calculations, but no regrouping is needed and the calculations are presented visually instead of orally. It is included here as a mental math strategy because students will do all the combinations in their heads starting at the front end. It is important to present these addition questions both horizontally and vertically.
Examples
a) For 56 + 23, simply record, starting at the front end, 79.
b) For 543 + 256, simply record, starting at the front end, 799.

Examples of Some Practice Items
a) Some practice items for numbers in the 10s are:

Add your own items:

\[
\begin{array}{cccc}
71 + 12 &=& 34 & 56 & 25 \\
63 + 33 &=& +62 & +31 & +74 \\
44 + 53 &=& \\
37 + 51 &=& \\
15 + 62 &=& \\
66 + 23 &=& \\
43 + 54 &=& 
\end{array}
\]

b) Some practice items for numbers in the 100s are:

Add your own items:

\[
\begin{array}{cccc}
385 + 673 + 532 &=& \\
+413 + 312 + 301 &=& 
\end{array}
\]

385 + 673 + 532 =
144 + 333 =
507 + 201 =
623 + 234 =
770 + 129 =
534 + 435 =
450 + 238 =

Front End Addition

This strategy involves adding the highest place values and then adding the sums of the next place value(s).

Examples

For 45 + 17, think 40 + 10 and 5 + 7 or 50 + 12 = 62
For 24 + 12 + 31, think: 20 + 10 + 30 is 60 and (4 + 2 + 1) is 7 and 60 + 7 = 67.
Examples of Some Practice Items

Some practice items for numbers in the 10s are:

Add your own items:

15 + 66 =
74 + 19 =
33 + 21 + 42 =
25 + 11 + 43 =
41 + 13 + 24 =

Finding Compatibles

This strategy for addition involves looking for pairs of numbers that add to powers of ten to make the addition easier.

Examples

For 3 + 8 + 7, think: 3 + 7 is 10 plus 8 is 10 + 8 = 18.
20 + 40 + 80, think: 20 and 80 is 100, so 100 and 40 is 140.

Examples of Some Practice Items

Some items applying this strategy to numbers in the 1s are:

Add your own items:

6 + 9 + 4 = 3 9 5 8 5
2 + 3 + 8 = 7 8 9 3 3
4 + 6 + 2 = +4 +1 +5 +2 +7
1 + 9 + 5 =
5 + 6 + 5 =

Some items applying this strategy to numbers in the 10s are:

10 + 60 + 90 = 60 40 90
30 + 50 + 70 = 30 70 80
80 + 30 + 20 = +70 +60 +10
60 + 30 + 40 =
80 + 20 + 79 =

Break Up and Bridge

This strategy for addition involves starting with the first number and adding the values in the place values, starting with the larger of the second number. The 100s chart is an effect tool to teach this strategy.
Examples
For 45 plus 38, think: 45 and 30 (from the 33) is 75 and 75 plus 8 (the rest of the 38) is 83.

Examples of Some Practice Items
Some items applying this strategy to numbers in the 10s are:

- $37 + 45 = \underline{82}$
- $72 + 28 = \underline{100}$
- $25 + 76 = \underline{101}$

Compensation
This strategy for addition involves changing one number to a ten; carrying out the addition and then adjusting the answer to compensate for the original change.

Examples
For 52 + 9, think 52 plus 10 is 62, but I added 1 too many, to take me to the next 10 to compensate, so I subtract one from my answer, 62, to get 61 or $52 + 9 = (52 + 10) - 1 = 61$.

For 65 + 29, think 65 + 30 = 95, but I added one too many, so I subtract 1 to get $95 - 1 = 94$.

Some items applying this strategy to numbers in the 10s are:

- $43 + 9 = \underline{52}$
- $56 + 8 = \underline{64}$
- $79 + 2 = \underline{81}$
- $48 + 5 = \underline{53}$
C. Subtraction— Fact Learning

Facts and the Fact Learning Strategies

Due to the use of addition facts in the subtraction strategies it is important that you ensure students have a reach a reasonable response time in all addition facts before moving on to subtraction facts. The main strategy for Subtraction at this level is to ‘think addition’.

First, review the subtraction facts to 18 and the fact learning strategies addressed in previous grades, as outlined below. Students will use these facts and strategies when doing mental math subtraction in the 10s and 100s.

1. Doubles

This strategy uses the addition double facts to help find the answers to related subtraction combinations.

**Examples**

For example: For $12 - 6$, think; $6$ plus what makes $12$? $6 + 6 = 12$, so $12 - 6 = 6$

**Examples of Some Practice Items**

Some practice examples are:

$$10 - 5 = \quad 6 - 3 = \quad 14 - 7 = \quad 4 - 2 =$$

$$16 - 8 = \quad 8 - 4 = \quad 18 - 9 =$$

2. Near-Doubles (1-Apart) Facts

This strategy also uses the addition double facts and near-double facts to help find the answers to related subtraction combinations. When the part being subtracted is close to half of the total, we can think of an addition double fact, and then adjust it by $1$ to find the answer.

**Example**

For example: For $9 - 4$, think: $4$ plus $4 = 8$, $4 + 5 = 9$, so $9 - 4 = 5$

**Examples of Some Practice Items**

Some practice examples are:

$$13 - 6 = \quad 11 - 5 = \quad 15 - 7 =$$

$$17 - 8 = \quad 7 - 3 = \quad 16 - 7 =$$

Subtraction Facts Extended to 10s and 100s

1. Doubles

This strategy uses the addition double facts to help find the answers to related subtraction combinations.
Examples

a) For 12 – 6, think: 6 plus what makes 12? 6 + 6 = 12, so 12 – 6 = 6
b) If you know that 8 – 4 = 4, then 80 – 40 = 40
c) If you know that 6 – 3 = 3, then 600 – 300 = 300

Examples of Some Practice Items

a) Some practice items are:
   
   \[
   \begin{align*}
   10 – 5 &= \\
   6 – 3 &= \\
   14 – 7 &= \\
   4 – 2 &= \\
   16 – 8 &= \\
   8 – 4 &= \\
   18 – 9 &= \\
   
   \end{align*}
   \]

b) Some practice items for numbers in the 10s are:
   
   \[
   \begin{align*}
   60 – 30 &= \\
   100 – 50 &= \\
   20 – 10 &= \\
   40 – 20 &= \\
   120 – 60 &= \\
   180 – 90 &= \\
   140 – 70 &= \\
   160 – 80 &= \\
   
   \end{align*}
   \]

c) Some practice items for numbers in the 100s are:
   
   \[
   \begin{align*}
   200 – 100 &= \\
   800 – 400 &= \\
   1000 – 500 &= \\
   1200 – 600 &= \\
   1600 – 800 &= \\
   1800 – 900 &= \\
   
   \end{align*}
   \]

2. Near-Doubles (1-Aparts) Facts

This strategy also uses the addition double facts and near-double facts to help find the answers to related subtraction combinations. When the part being subtracted is close to half of the total, we can think of an addition double fact, and then adjust it by 1 to find the answer.

Examples

a) For 9 – 4, think: 4 plus 4 = 8, 4 + 5 = 9, so 9 – 4 = 5
b) If you know that 9 – 4 = 5, then for 90 – 40, think: 40 plus 40 = 80; 40 + 50 = 90, so 90 – 40 = 50
c) If you know that 7 – 3 = 4, then for 700 – 300, think: 300 plus 300 = 600; 300 + 400 = 700, so 700 – 300 = 400

Examples of Some Practice Items

a) Some practice items are:
   
   \[
   \begin{align*}
   13 – 6 &= \\
   11 – 5 &= \\
   15 – 7 &= \\
   17 – 8 &= \\
   7 – 3 &= \\
   16 – 7 &= \\
   
   \end{align*}
   \]

b) Some practice items are:
   
   \[
   \begin{align*}
   70 – 30 &= \\
   50 – 20 &= \\
   150 – 70 &= \\
   130 – 60 &= \\
   
   \end{align*}
   \]
c) Some practice items are:

\[
\begin{align*}
900 - 400 &= 500 \\
500 - 200 &= 300 \\
1300 - 600 &= 700 \\
1500 - 700 &= 800 \\
1700 - 800 &= 900 \\
1100 - 500 &= 600
\end{align*}
\]

3. Make 10s and 100s

This strategy applies to calculations involving the subtraction of two numbers in the tens or hundreds with only one non-zero digit in each number. The strategy involves subtracting the single non-zero digits as if they were the single-digit subtraction facts.

**Examples**

If we know that \(8 - 3 = 5\), then for \(80 - 30\), think: 8 tens subtract 3 tens is 5 tens, or 50.

For \(500 - 200\), think: 5 hundreds subtract 2 hundreds is 3 hundreds, or 300.

**Examples of Some Practice Items**

Some practice items for numbers in the 10s are:

\[
\begin{align*}
90 - 10 &= 80 \\
60 - 30 &= 30 \\
70 - 60 &= 10
\end{align*}
\]

Some practice items for numbers in the 100s are:

\[
\begin{align*}
700 - 300 &= 400 \\
400 - 100 &= 300 \\
200 - 100 &= 100 \\
500 - 300 &= 200
\end{align*}
\]
D. Subtraction — Mental Calculations

Using “Think Addition” in Subtraction

This strategy demonstrates how students can use their knowledge of addition facts to find the answers to subtraction equations. Students will be able to look at a subtraction fact, such as 9 – 4, and think “4 plus what equals 9?” and determine the missing part.

Examples of Some Practice Items

10 – 4 = __  (4 plus what equals 10?)
8 – 2 = __  (2 plus what equals 8?)
8 – 5 = __  (5 plus what equals 8?)
10 – 3 = __  (3 plus what equals 10?)
9 – 3 = __  (3 plus what equals 9?)
6 – 4 = __  (4 plus what equals 6?)
80 – 20 = __  (20 plus what equals 80?)
60 – 40 = __  (40 plus what equals 60?)
90 – 30 = __  (30 plus what equals 90?)
80 – 50 = __  (50 plus what equals 80?)
100 – 30 = __  (30 plus what equals 100?)
70 – 40 = __  (40 plus what equals 70?)
800 – 200 = __  (200 plus what equals 800?)
900 – 300 = __  (300 plus what equals 900?)
1000 – 300 = __  (300 plus what equals 1000?)
600 – 400 = __  (400 plus what equals 600?)
800 – 500 = __  (500 plus what equals 800?)
700 – 400 = __  (400 plus what equals 700?)
Back Through 10

This strategy involves subtracting a part of the subtrahend to get to the nearest ten and then subtracting the rest of the subtrahend. *This strategy is most effective when the two numbers involved are quite far apart.*

**Examples**

a) For 15 – 8, think: 15 subtract 5 (one part of the 8) is 10 and 10 subtract 3 (the other part of the 8) is 7.

b) For 74 – 6, think: 74 subtract 4 (one part of the 6) is 70 and 70 subtract 2 (the other part of the 6) is 68.

**Examples of Some Practice Items**

Some practice items for numbers in the 10s are: (Add your own examples)

\[
\begin{align*}
15 - 6 &= 82 - 6 = 14 - 6 = \\
42 - 7 &= 17 - 8 = 85 - 7 = \\
34 - 7 &= 63 - 6 = 74 - 7 = \\
13 - 4 &= 13 - 6 = 97 - 8 = \\
61 - 5 &= 15 - 7 = 53 - 5 = 
\end{align*}
\]

Up Through 10

This strategy involves counting the difference between the two numbers by starting with the smaller; keeping track of the *distance* to the nearest ten; and adding to this amount the rest of the *distance* to the greater number. *This strategy is most effective when the two numbers involved are quite close together.*

**For example**

a) For 12 – 9, think: It is 1 from 9 to 10 and 2 from 10 to 12; therefore, the difference is 1 plus 2, or 3 or \((9 + 1 = 10 \text{ and } 2 + 10 = 12 \text{ so the difference is } 1 + 2 = 3)\).

b) For 84 – 77, think: It is 3 from 77 to 80 and 4 from 80 to 84; therefore, the difference is 3 plus 4, or 7.

**Examples of Some Practice Items**

Some practice items for numbers in the 10s are:

Add your own items:

\[
\begin{align*}
15 - 8 &= 12 - 8 = 58 - 49 = \\
14 - 9 &= 15 - 6 = 34 - 27 = \\
16 - 9 &= 16 - 7 = 71 - 63 = \\
11 - 7 &= 95 - 86 = 88 - 79 = \\
17 - 8 &= 67 - 59 = 62 - 55 = \\
13 - 6 &= 46 - 38 = 42 - 36 = 
\end{align*}
\]
**E. Addition and Subtraction — Computational Estimation**

**Front End**
Note: This strategy involves combining only the values in the highest place value to get a “ball-park”. Such estimates are adequate in many circumstances.

**Examples**

a) To estimate $43 + 54$, think: $40 + 50$ is $90$.

b) To estimate $92 - 53$, think: $90$ subtract $50$ is $40$.

c) To estimate $437 + 541$, think: $400$ plus $500$ is $900$.

d) To estimate $534 - 254$, think: $500$ subtract $200$ is $300$.

**Examples of Some Practice Items**

a) Some practice items for estimating *addition* of numbers in the 10s are:

- $62 + 31 = 34 + 42 = 96$
- $21 + 43 = 54 + 33 = 84$
- $44 + 23 = 12 + 51 = 59$
- $13 + 82 = 71 + 14 = 85$
- $73 + 12 = 24 + 73 = 97$

b) Some practice items for estimating *subtraction* of numbers in the 10s are:

- $93 - 62 = 32 - 23 = 70$
- $91 - 42 = 72 - 33 = 39$
- $64 - 23 = 84 - 61 = 23$
- $43 - 12 = 54 - 21 = 33$
- $81 - 54 = 73 - 44 = 29$

c) Some practice items for estimating *addition* of numbers in the 100s are:

- $234 + 432 = 703 + 241 = 937$
- $741 + 138 = 423 + 443 = 1164$
- $341 + 610 = 816 + 111 = 927$
- $647 + 312 = 512 + 224 = 736$
- $632 + 207 = 534 + 423 = 957$
d) Some practice items for estimating _subtraction_ of numbers in the 100s are:

\[
\begin{align*}
327 - 142 &= 718 - 338 = \\
928 - 741 &= 248 - 109 = \\
804 - 537 &= 823 - 240 = \\
516 - 234 &= 431 - 206 = \\
639 - 426 &= 743 - 519 = \\
\end{align*}
\]

### Adjusted Front End

This strategy begins by getting a Front End estimate and then adjusting that estimate to get a better, or closer, estimate by either (a) considering the second highest place values or (b) by clustering all the values in the other place values to “eyeball” whether there would be enough together to account for an adjustment.

**Examples**

a) To estimate 23 + 48, think: 20 plus 40 is 60 and 3 plus 8 would account for about another 10; therefore, the adjusted estimate is 60 + 10 or 70.

b) To estimate 42 - 19, think: 40 subtract 10 is 30; but 9 would account for about another 10; therefore, the adjusted estimate is 20.

c) Sometimes the numbers in the ones do not account for another ten and therefore, do not affect the estimation.

To estimate 31 + 22, think: 30 plus 20 is 50, and 1 plus 2 would not account for another 10, so the estimate is 30 + 20 or 50, with no adjustment.

d) Sometimes the number in the ones in the subtrahend does not account for another ten and therefore, does not affect the estimation.

To estimate 89 – 31, think: 80 subtract 30 is 50; and 1 would not account for another 10, so the estimate is 50 with no adjustment.

### Examples of Some Practice Items

a) Some practice items for estimating _addition_ of numbers in the 10s are:

\[
\begin{align*}
28 + 33 &= 20 + 30 + 11(\text{which is about 10}) = 60. \\
76 + 13 &= 62 + 29 = \\
39 + 64 &= 38 + 34 = \\
48 + 25 &= 82 + 17 = \\
54 + 28 &= 29 + 53 = \\
47 + 31 &= \\
\end{align*}
\]

b) Some practice items for estimating _subtraction_ of numbers in the 10s are:

\[
\begin{align*}
73 - 58 &= 70 - 50 - 8(\text{which is about another 10}) = 10 \\
92 - 48 &= 54 - 27 = \\
61 - 29 &= 93 - 19 = \\
82 - 47 &= 84 - 28 = \\
31 - 19 &= 63 - 18 = \\
\end{align*}
\]
c) Sometimes the numbers in the ones do not account for another ten and therefore, do not affect the estimation.

Some other practice items for estimating addition of numbers in the 10s demonstrating this are:

\[
\begin{align*}
51 + 33 &= 50 + 30 + (4 \text{ which would not account for another 10}) = 80 \\
42 + 22 &= 82 + 11 \\
63 + 11 &= 63 + 31 \\
42 + 31 &= 34 + 10 \\
72 + 22 &= 23 + 71 \\
21 + 33 &=
\end{align*}
\]

d) Sometimes the number in the ones in the subtrahend does not account for another ten and therefore, does not affect the estimation.

Some other practice items for estimating subtraction of numbers in the 10s demonstrating this are:

\[
\begin{align*}
76 - 53 &= 70 - 50 - (3 \text{ which would not account for another 10}) = 20 \\
45 - 12 &= 74 - 32 \\
94 - 23 &= 55 - 21 \\
62 - 51 &= 36 - 11 \\
91 - 14 &= 64 - 23 \\
83 - 42 &=
\end{align*}
\]

Rounding

This strategy involves rounding each number to the highest, or the highest two, place values and adding the rounded numbers.

Examples

a) For example to estimate 27 + 31, think: 27 rounds to 30 and 31 rounds to 30, so 30 plus 30 is 60.

b) For example to estimate 87 - 32, think: 87 rounds to 90 and 32 rounds to 30, so 90 subtract 30 is 60.

c) For example, to estimate 348 + 230, think: 348 rounds to 300 and 230 rounds to 200, so 300 plus 200 is 500.

d) For example to estimate 594 - 203, think: 594 rounds to 600 and 203 rounds to 200, so 600 subtract 200 is 60
Examples of Some Practice Items

a) Some practice items for rounding addition of numbers in the 10s are:
   
   \[ 48 + 23 = 50 + 20 = 70 \]
   \[ 34 + 59 = \]
   \[ 61 + 48 = \]
   \[ 18 + 22 = \]
   \[ 97 + 12 = \]

b) Some practice items for rounding subtraction of numbers in the 10s are:

   \[ 57 – 14 = 60 – 10 = 50 \]
   \[ 84 – 9 = \]
   \[ 82 – 59 = \]
   \[ 36 – 22 = \]
   \[ 43 – 8 = \]
   \[ 54 – 18 = \]
   \[ 68 – 34 = \]
   \[ 99 – 47 = \]
   \[ 93 – 12 = \]

 c) Some practice items for rounding addition of numbers in the 100s are:

   \[ 326 + 590 = \]
   \[ 218 + 411 = \]
   \[ 520 + 679 = \]
   \[ 384 + 310 = \]
   \[ 137 + 640 = \]
   \[ 698 + 180 = \]
   \[ 223 + 580 = \]
   \[ 290 + 570 = \]
   \[ 680 + 124 = \]
   \[ 530 + 360 = \]

d) Some practice items for rounding subtraction of numbers in the 100s are:

   \[ 420 – 198 = \]
   \[ 970 – 430 = \]
   \[ 870 – 399 = \]
   \[ 594 – 301 = \]
   \[ 260 – 98 = \]
   \[ 840 – 715 = \]
   \[ 830 – 580 = \]
   \[ 940 – 642 = \]
   \[ 780 – 270 = \]