

Simple models of walking and jumping

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Abstract

Alexander, R.McN., 1992. Simple models of walking and jumping. *Human Movement Science* 11, 3–9.

Many biomechanical models of human movement are complex by the standards of physical mechanics, but this paper presents two that are exceedingly simple. The first helps to explain the transition from walking to running by showing that a straightlegged style of walking would become impossible above a certain speed. The second uses the force-velocity properties of muscle to explain why high jumpers run up much more slowly than long jumpers. Simple models are particularly useful in identifying basic principles because the simpler the model, the easier it is to discover which of its features gives rise to the observed effect.

The human body is far too complicated to be imitated in detail by any model. All models of it that have been devised by biomechanicists are gross simplifications, but many are complex by the standards of physical mechanics. For example, the model used by Yeadon (1990) for his brilliant analysis of twisting somersaults represents the body as an assembly of eleven rigid segments each with appropriate dimensions, mass and moments of inertia, and Seireg and Arvikar's (1973) classic analysis of standing represents each leg as an assembly of three segments operated by 29 muscles.

I admire those two models, which are well adapted to their functions, but am concerned by the feeling that seems to be common among biomechanicists, that because the human body is complex they should always reproduce as much as possible of its complexity in their models. My aim in this paper is to encourage the use of simple models by showing how two exceedingly simple ones have contributed to our understanding of human movement.

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A speed limit for walking

People walk to go slowly and run to go fast. Adults of normal stature make the change at a speed of about 2.3 metres per second, but small children start running at lower speeds (Alexander 1984). Two features of walking that distinguish it from running are that each foot remains on the ground for more than half the duration of the stride (so that there is always at least one foot on the ground) and that, while it is on the ground, the knee of the same leg remains almost straight. This straight-legged style of walking is a human peculiarity, not practised by any animal (Alexander 1991b). It has been represented by one of the simplest of all biomechanical models, which was devised to explain why we do not walk at high speeds (Alexander 1976, 1984).

The model (fig. 1a) has just two segments, a rigid trunk and a straight, rigid leg. All its mass is located in the trunk. When the foot is on the ground the hip must move along a circular arc around it, with radius r equal to the length of the leg. If the trunk remains vertical, its centre of mass also must move along an arc of radius r .

A body moving in a circle has an acceleration towards the centre. If the speed of the trunk is v this acceleration is v^2/r . At the stage of the stride when the leg is vertical, this acceleration is directed vertically downwards. It cannot be greater than g , the acceleration of a body falling freely under gravity

$$v^2/r \leq g,$$

$$v \leq (gr)^{1/2}.$$

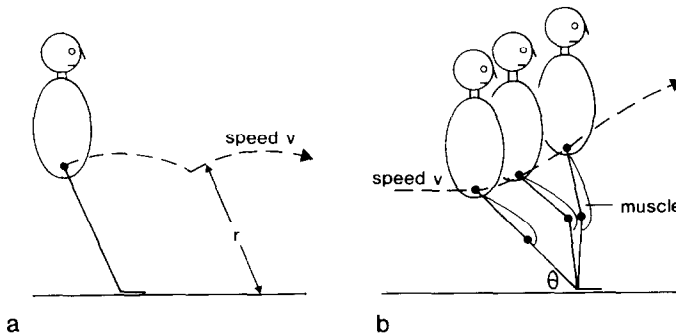


Fig. 1. The models of (a) walking and (b) take-off for a jump.

This inequality sets a speed limit for walking. On earth, g is about 10 ms^{-2} . Normal adults have leg lengths r of about 0.9 m, so the inequality tells us that we cannot walk faster than $(10 \times 0.9)^{1/2} = 3 \text{ ms}^{-1}$. This is only a little faster than the speed at which we normally start running. It is admittedly less than the maximum speeds of about 4 ms^{-1} that good athletes attain in walking races, but they make peculiar movements of the lower back and pelvis that increase the radius of curvature of the path of the centre of mass, making it greater than leg length (Alexander 1984). Thus r in the inequality above is increased, making higher speeds (v) possible.

So simple a model will not always be appropriate. For example, in another study of walking Mochon and McMahon (1980) wanted to investigate the possibility that each leg could swing forward passively while its foot was off the ground. For that they needed a model which had thighs, shanks and feet, each with appropriate masses and moments of inertia. For other problems, models considerably more complicated than that may be needed, but it will generally be best to keep the model as simple as is consistent with its task.

Athletic jumping

Athletes making high or long jumps take off from a run. At the last footfall before the jump, the foot is set down with the knee straight, and with the leg sloping at some angle θ to the horizontal (fig. 1b). The knee bends and extends again, and the athlete is thrown into the air. Good long jumpers run up fast, at near-maximal sprinting speeds of about 10 metres per second, and set down the leg at an angle θ of $60\text{--}65^\circ$. High jumpers run up more slowly, at about 7 metres per second, and use a smaller angle θ of about $45\text{--}50^\circ$ (for references, see Alexander 1990). Can a model help us understand the differences of technique?

One might think of using a model that had springs instead of legs, like the models of running that have been used to great effect by McGeer (1990) and others. The spring would be compressed (simulating the bending of the take-off leg) and recoil, throwing the athlete into the air. That would imply that nearly all the kinetic energy of the run-up could be converted to potential energy, enabling the athlete to jump unrealistically high.

Unlike springs, muscles cannot exert as much force when shortening (doing work) as when being stretched (acting as brakes). If the knee muscles, for example, are fully activated throughout take-off, they will degrade more energy to heat (as the knee bends) than they do work (as it extends again). Therefore, mechanical energy is lost.

For this reason, it seemed useful to devise a model of jumping that took account of the physiological properties of muscle (Alexander 1990). The body was treated as a rigid trunk with massless, two-segment legs (fig. 1b). Only one muscle was represented, an extensor of the knee, but it was given realistic physiological properties. Because there were no muscles at the hip, the force exerted on the ground was always in line with the hip, which is reasonably realistic. The centre of mass of the model was placed at the hip: in real people it is about 5 cm higher.

The behaviour of the model was investigated by computer simulation. Initially, it was travelling horizontally at a chosen run-up speed. The foot was set down with the knee almost straight (not quite straight, to avoid infinite forces) and with the leg sloping at a chosen angle. The muscle was assumed to be fully active, while the foot remained on the ground. The forces and movements as the knee bent and extended again were calculated, and the path of the centre of mass was followed after the foot left the ground. The height of the jump was taken to be the maximum height to which the centre of mass rose. (Good athletes using the style of jumping known as the Fosbury flop may even pass the centre of mass under the bar.) The length of the jump was the horizontal distance that the centre of mass travelled, before hitting the ground. When realistic initial speeds and angles were used, the simulations gave patterns of force on the ground very similar to those recorded in real jumps (but it must be admitted that one of the parameter values had been chosen specifically to get the magnitudes of the forces right) and gave realistic jump heights and distances.

Fig 2a shows how long jump performance depended on the initial speed and leg angle. The speed axis has been truncated at 11 metres per second, which is approximately the highest speed that good male sprinters can attain. The contours show that the jump is longest when the athlete runs up as fast as possible, setting down his leg at about 70° . In contrast, fig. 2b shows that a high jumper should not run at maximum speed. The highest jumps are obtained when he runs up at

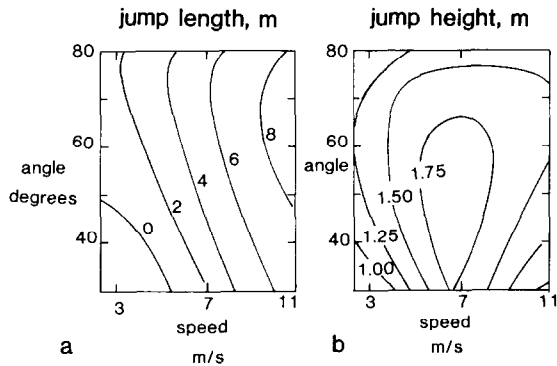


Fig. 2. Jumping performance related to run-up speed (ms^{-1} , horizontal axis) and the angle at which the leg is set down (degrees, vertical axis). The contours show (a) the length (m) of a long jump and (b) the height (m) of a high jump, as predicted by the model. Alexander (1990) gave results in dimensionless terms which have been translated, in this figure, to refer to an athlete of 1.8 m stature.

about 7 metres per second, setting down his leg at $45\text{--}50^\circ$. The optimum speeds and angles suggested by the graphs are close to those used by excellent athletes. Their precise values depend, of course, on the values chosen for the parameters of the model, but large changes in the parameters move the optima only a little.

The reason why too high a speed reduces the height of a high jump seems to be this. The upward momentum given to the body equals the vertical impulse (force multiplied by time) exerted on the ground. The faster the athlete runs up, the less time he can keep the foot on the ground. However fast he goes he cannot exert more than a certain force, so if he goes too fast he cannot exert as large an impulse. The force that a spring can exert is not limited in this way, so a model with a spring instead of a muscle would suggest falsely that high jumpers should run up as fast as possible.

Conclusion

The models that have been presented in this paper may seem ludicrously simple. The model of walking has rigid, massless legs with no muscles, and the model of jumping has massless legs with only one muscle. I claim nevertheless that these models help us to understand

human movement and that their simplicity is their great virtue. They highlight basic principles by showing that the conclusions drawn from them depend on very few assumptions.

The model of walking had no muscles and the model of running jumps only one, so they tell us nothing about sequencing and timing of muscle activity. However, they do suggest that similar models with a few more muscles may have potential value in studies of sequencing and timing.

A model with two muscles has indeed been used, in a brief discussion of standing jumps (Alexander 1989). When jumping from a crouched position, we start extending our knees before our ankles. The model however showed no advantage in sequential extension of joints: it jumped highest when its two leg muscles extended their joints simultaneously. It seems possible that it was too simple, in having no mass in its legs.

I have recently developed two-muscle models of throwing (Alexander 1991a). Different throwing techniques are used in different sports (putting the shot is a very different action from pitching a baseball) but they have one prominent feature in common: joints move in sequence from proximal to distal (see, for example, Atwater 1979). Models with different geometrics have been developed to represent different styles of throwing, each with just two muscles that are activated in sequence. Like real throwers, the models project their missiles fastest if the proximal muscles are activated before the distal ones, with an optimal delay. Herring and Chapman (1988) reported similar results in a brief preliminary account of another simple model.

The study of jumping and of throwing are examples of fields in which models seem likely to prove helpful in discovering basic principles that govern the sequencing of muscle action. The simpler the model, the easier it is to discover which of its features are essential to the observed effect.

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