Numerical study on the wrist action during the golf downswing

C.C. Chen, Y. Inoue and K. Shibara

Department of Intelligent Mechanical Systems Engineering, Kochi University of Technology, Kochi-prefecture, Japan

Abstract

The purpose of this paper was to examine whether the ball position and wrist action (different types of torque application) could be optimised to increase the horizontal golf club head speed at impact with the ball. A two-dimensional double pendulum model of the golf downswing was used to determine to what extent the wrist action affected the club head speed in a driver, and how this affected the optimum ball position. Three different patterns of wrist actions (negative, positive, and negative–positive torque at the wrist) were investigated; and two criteria (maximum and impact criteria) were used to assess their effectiveness in terms of the maximum horizontal club head speed, and the club head speed as the shaft becomes vertical when viewed 'face-on'. The simulation results indicated that the horizontal club head speed at impact could be increased by these patterns of wrist actions and the optimum ball position could be determined by the impact criterion. Based on the analysis of the energy flow from the input joints of shoulder and wrist to the arm and club head, the way the wrist action affects the club head speed has been discussed. The sensitivity of the results to small changes in model parameter values and initial conditions was investigated. The results were also examined under different torque patterns.

Keywords: wrist action, horizontal club head speed, golf swing model, optimum ball position, sensitivity

Introduction

The golf swing is highly dependent on the golfer's wrist action. From the 'natural release point', when the wrist joint turns freely due to the centrifugal torque of the golf club, even though the input torque about the wrist joint is zero, to impact of the club with the ball, the wrist action is generally divided into three patterns: active (using positive torque), passive (using negative torque) and passive–active (using negative–positive torque). Over the years, golfers and researchers have asked the question: which kind of wrist action can provide the optimum club head speed at impact? Based on the study of the stroboscopic photograph of Bobby Jones' swing, Williams (1967) found that no additional wrist torque was necessary after the 'natural release point', and the club freewheeled at the latter stage of the downswing. Jorgensen (1970) and Cochran & Stobbs (1999) believed that the passive wrist action (the so-called 'late hit') enhanced the club head speed at impact. Cochran & Stobbs (1999) also considered that

Correspondence address:
Dr Chaochao Chen
Department of Intelligent Mechanical Systems Engineering
Kochi University of Technology
Tosayamada-cho
Kochi-prefecture 782-8502
Japan
Tel: 81 887 57 2170
Fax: 81 887 57 2170
E-mail: kut_chen@yahoo.com

the club head speed at impact could be increased by the passive–active wrist action with an appropriately 'timed' wrist torque. Using a three-segment model, Sprigings & Neal (2000) also confirmed that the passive–active wrist action could provide an advantage in club head speed at impact. However, the golf ball position – the point where impact occurs – was assumed to be constant in these studies.

In contrast, research carried out by Pickering & Vickers (1999) proved that the golf ball position would influence the final club head speed. Therefore, the ball position in this paper was not determined in advance, but included in the optimisation models (maximum and impact criteria). Pickering & Vickers (1999) used the maximum criterion (the maximum criterion can determine the maximum horizontal club head speed at impact) to study the passive wrist action, yet the active and passive–active wrist actions were not considered. It has been reported by McLean (1999) that the shaft positions of many professional golfers are always maintained vertical at impact when viewed 'face-on'. The impact criterion (vertical club shaft at impact) was therefore applied to examine the role of the wrist action in this paper. The wrist action was separately investigated by the maximum criterion and impact criterion, and the computational results between both models were compared.

As far as the authors are aware, the way the wrist action alters the club head speed at impact has not been thoroughly investigated, although some researchers did include this point in their work. Jorgensen (1970) attributed the role of the passive wrist action to the change of a mysterious 'timing' term, yet an explicit explanation was not presented. Sprigings & Mackenzie (2002) used a three-segment model comprising the torso, arm and golf club to identify the mechanical sources of power that are responsible for the increase in club head speed. The ball position, however, remained constant in their simulation model and thus the influence of ball position on the energy transfer between the arm and golf club at impact was neglected. In the current study, it is demonstrated how the wrist action affects the club head speed from an energy-based analysis when the ball position is not constant, but determined by two criteria.

The aim of this study was to investigate which kind of wrist action could improve the horizontal club head speed at impact by two criteria in consideration of the golf ball position, and to determine the optimum ball position at impact for various types of wrist actions. Furthermore, the way the wrist action affects the club head speed was discussed by analysing the energy flow from the input joints of the shoulder and wrist to the arm and club head.

Mathematical model

As shown in Fig. 1, the mathematical model of the golf downswing was deemed to be a two-dimensional double pendulum. This model consists of a rigid arm link and a rigid club link. It was assumed that the swing took place in a plane tilted at an angle ϕ to the ground. The assumption of the planar movement of the downswing is well supported in the work of Cochran & Stobbs (1999) and Jorgensen (1994).

The downswing was separated into two phases. In phase 1, the two rigid bodies rotate as one body with a constant wrist-cock angle. In phase 2, the various types of wrist actions were employed.

The following notation was applied:

G_1, G_2 Torque on arm and club, respectively
m_1, m_2 Mass of arm and club, respectively
a_1, a_2 Length of arm and club, respectively

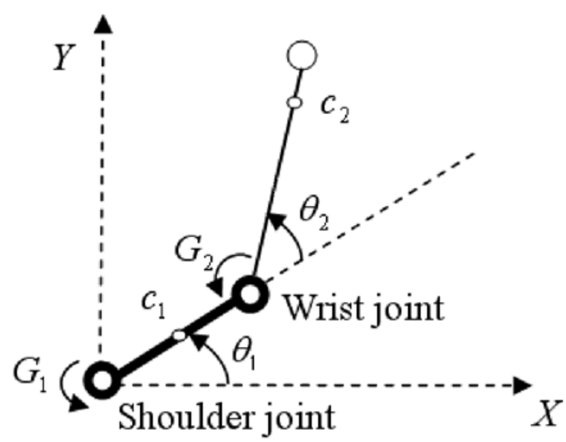

Figure 1 Double pendulum model of golf downswing. c_1, c_2 are the centres of the mass of arm and club, respectively. The *X* axis is parallel to the target line on the ground.

l_1, l_2 Length from shoulder joint to c_1 and from wrist joint to c_2, respectively
I Moment of inertia of arm about shoulder joint
J Moment of inertia of club about c_2
θ_1, θ_2 Angle of arm and club, respectively
ϕ Inclination of plane of downswing
g Acceleration due to gravity

The values of arm and club parameters used in the calculation were those given by Lampsa (1975), as shown in Table 1, together with club data which was considered to be appropriate for a driver.

Table 1 Parameter values of swing model

m_1	7.312 kg
a_1	0.615 m
l_1	0.326 m
I	1.150 kg m^2
m_2	0.394 kg
a_2	1.105 m
l_2	0.753 m
J	0.077 kg m^2
ϕ	60°

The equations of motion of the model were derived using the Lagrangian method. The detailed equations in two phases are given as follows.
In phase 1:

$$(I + J + m_2l_2^2 + m_2a_1^2 + 2m_2l_2a_1 \cos\theta_2)\ddot{\theta}_1 + (J + m_2l_2^2 + m_2l_2a_1\cos\theta_2)\ddot{\theta}_2 - (2\dot{\theta}_1 + \dot{\theta}_2)m_2l_2a_1\sin\theta_2\dot{\theta}_2 + g\sin\phi[m_2l_2\cos(\vartheta_1 + \theta_2) + (m_1l_1 + m_2a_1)\cos\theta_1] = G_1 \quad (1)$$

$$(J + m_2l_2^2)\ddot{\theta}_2 = 0 \quad (2)$$

In phase 2:

$$(I + J + m_2l_2^2 + m_2a_1^2 + 2m_2l_2a_1 \cos\theta_2)\ddot{\theta}_1 + (J + m_2l_2^2 + m_2l_2a_1\cos\theta_2)\ddot{\theta}_2 - (2\dot{\theta}_1 + \dot{\theta}_2)m_2l_2a_1\sin\theta_2\dot{\theta}_2 + g\sin\phi[m_2l_2\cos(\vartheta_1 + \theta_2) + (m_1l_1 + m_2a_1)\cos\theta_1] = G_1 \quad (3)$$

$$(J + m_2l_2^2 + m_2l_2a_1\cos\theta_2)\ddot{\theta}_1 + (J + m_2l_2^2)\ddot{\theta}_2 + m_2l_2a_1\sin\theta_2\dot{\theta}_1^2 + g\sin\phi m_2l_2\cos(\vartheta_1 + \theta_2) = G_2 \quad (4)$$

Simulation method

Two criteria were used in our simulation: the maximum criterion and the impact criterion. First, the criterion of maximum horizontal club head speed at impact (maximum criterion) was used to investigate the effects of different kinds of wrist actions on the golf downswing. The following equation indicates the maximum criterion.

$$-a_1(\sin\theta_1\ddot{\theta}_1 + \cos\theta_1\dot{\theta}_1^2) - a_2[\sin(\theta_1 + \theta_2)(\ddot{\theta}_1 + \ddot{\theta}_2) + \cos(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2)^2] = 0 \quad (5)$$

Eqn. 5 was obtained by differentiating the equation of the horizontal club head speed, v_h,

$$v_h = -a_1\sin\theta_1\dot{\theta}_1 - a_2\sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2) \quad (6)$$

with respect to time, and then equated to zero.

The function **NDSolve** in MATHEMATICA 5.2 was used to drive the simulation model. To obtain the swing motions of the arm and club at impact, including $\theta_1(t_{imp})$, $\theta_2(t_{imp})$, $\dot{\theta}_1(t_{imp})$, $\dot{\theta}_2(t_{imp})$ by the maximum criterion, the root of eqn. 5 at the time of impact, t_{imp}, was solved using the function of **FindRoot** which makes the left-hand side of eqn. 5 less than 10^{-6} in magnitude. The impact time t_{imp} was then applied in the function **NDSolve** to achieve the angles and angular velocities of the arm and club at impact.

After obtaining the swing motion of the arm and club at impact, the maximum horizontal club head speed v_m was determined from eqn. 7.

$$v_m = -a_1\sin[\theta_1(t_{imp})]\dot{\theta}_1(t_{imp}) - a_2\sin[\theta_1(t_{imp}) + \theta_2(t_{imp})][\dot{\theta}_1(t_{imp}) + \dot{\theta}_2(t_{imp})] \quad (7)$$

The optimum ball position dx at impact is given by

$$dx = a_1\cos[\theta_1(t_{imp})] + a_2\cos[\theta_1(t_{imp}) + \theta_2(t_{imp})]. \quad (8)$$

The different patterns of wrist action (passive, active and passive–active) were studied using the maximum criterion. Six positive constant wrist torques were employed: 5 N m; 10 N m; 15 N m; 20 N m; 25 N m and 30 N m. Here, the maximum wrist torque (30 N m) was given by Neal *et al.* (1999), who measured this from a low handicap amateur golfer, using inverse dynamics.

For the passive wrist action (PW), the arm release angle, θ_1^r, given by eqn. 9, which denotes when the wrist joint can be opened, was delayed in every degree of freedom from the ‘natural release point’ until the point at which the set negative wrist torque was reached. The absolute value of the regulated negative wrist torque is the same as that of the positive wrist torque but with the opposite sign.

$$\theta_1^r = \theta + p1, \quad p1 = 1°, 2° \ldots, \theta_1^p \quad (9)$$

θ is the integer part of θ_1^n, where θ_1^n is the arm rotational angle when the ‘natural release point’ is reached and θ_1^p is the arm rotational angle when the regulated negative wrist torque is reached.

For the active wrist action (AW), the onset and end of the positive wrist torque were determined when the arm rotational angle satisfied eqn. 10 and eqn. 11, respectively.

$$\theta_1^o = \theta + p2, \quad p2 = 1°, 2° \ldots, \theta_1°(t_{imp}) - 1° \quad (10)$$

$$\theta_1^t = \theta_1^o + p3, \quad p3 = 1°, 2° \ldots, \theta_1°(t_{imp}) \quad (11)$$

where θ is the integer part of θ_1^n. θ_1^o and θ_1^t are the arm rotational angles when the positive wrist torque is activated and deactivated respectively; $\theta_1°(t_{imp})$ is the arm rotational angle when impact occurs.

For the passive–active wrist action (PAW), the negative wrist torque was applied and then followed by a positive wrist torque. The application of the negative wrist torque was the same as that in the PW, in which the negative torque kept the wrist-cock angle constant until the desired arm release angle θ_1^r was reached. The onset and termination of the positive wrist torque, following the passive wrist action, were given by eqns. 12 and 13, respectively.

$$\theta_1^{o1} = \theta_1^r + p4, \quad p4 = 0°, 1° \ldots, \theta_1°(t_{imp}) - 1° \quad (12)$$

$$\theta_1^{t1} = \theta_1°(t_{imp}) \quad (13)$$

Where θ_1^{o1} and θ_1^{t1} are the arm rotational angles as the positive wrist torque is activated and deactivated respectively and $\theta_1°(t_{imp})$ is the arm rotational angle when impact occurs.

The natural release wrist action (NW), where no wrist torque is employed after the ‘natural release point’, was also studied for the purpose of comparison with the three wrist actions as mentioned above.

Concerning the club shaft position at impact, both observations made by Mclean (1999), an acclaimed professional golf instructor, and many swing photographs of professional golfers such as Dai Rees, Nicklaus and Woods, clearly indicate that the shaft is held vertical at impact when viewed face-on. Thus, in the second simulation we adopted the impact criterion (vertical club shaft at impact) to investigate the role of the wrist action. The impact criterion was given by

$$\theta_1(t_{imp}) + \theta_2(t_{imp}) - 270° = 0. \quad (14)$$

The function **FindRoot** was used to obtain the impact time, t_{imp}, by making the left-hand side of eqn. 14 less than 10^{-6} in magnitude. Then the club head speed and ball position at impact could be determined using the same method as that in the maximum criterion.

We assumed that the simulation process commenced when the golfer had just completed his backswing and was about to begin his downswing. Lampsa (1975) believed that a pause usually occurs at this moment, indicating that the angular velocities of arm and club are zero. Thus the following initial conditions for the downswing were chosen:

$$\theta_1(0) = 90°, \dot{\theta}_1(0) = 0; \theta_2(0) = -90°, \dot{\theta}_2(0) = 0. \quad (15)$$

It has been noted that there are many different types of torque functions of the shoulder joint that have been applied in previous studies: Jorgensen (1994) and Pickering & Vickers (1999) considered that the shoulder input torque was constant during the downswing; Milne & Davis (1992) used a ramp torque function and Suzuki & Inooka (1999) set the torque function as a trapezoid. In the present study, the input torque of the shoulder joint G_1 was assumed to be constant during the downswing as in Jorgensen (1994) and Pickering & Vickers (1999). The value of G_1 was chosen as 110 N m which makes the swing similar to that of a professional golfer (the horizontal club head speed at impact can reach 47.0741 m s^{-1} using NW).

Results and discussion

Fig. 2 shows the maximum horizontal club head speed at impact using three patterns of wrist actions (PW, AW and PAW), plotted against the wrist torque by the maximum criterion. The figure clearly shows that the larger wrist torque gives the largest horizontal club head speed at impact. We also note that the increase in speed can be gained for all kinds of wrist action compared to NW. It is clear that PAW results in a higher club head speed at impact when compared to

PW and AW with the same wrist torque. For example, when the wrist torque is 15 N m, PAW gives the highest horizontal club head speed at impact, 50.3 m s^{-1}, which is 6.9% greater than that of NW; while PW and AW result in increases of 1.4% and 5.6%, respectively.

The optimum golf ball position at impact given by the maximum criterion is shown in Fig. 3. The ball position is defined for a right-handed golfer with horizontal displacement BC, as shown in Fig. 4. As we can see from Fig. 3, the optimum ball position is determined by the various types of wrist action and wrist torque. Figs. 2–3 show that for PW, an increase in the horizontal club head speed is achieved when the ball is far away from the centre of the golfer's stance (large ball position). This result is consistent with the conclusion of Pickering & Vickers (1999). It is known that the large ball position needs a relatively large arm release angle (the so-called 'late hit') to achieve maximum club head speed when the passive wrist action is used (Pickering & Vickers, 1999). Fig. 5 shows that a large arm release angle requires a high wrist torque which even exceeds the limit of 30 N m (Neal *et al.*, 1999). It therefore appears impractical for golfers using passive wrist action to obtain an improvement in horizontal club head speed when the ball position is too large, since it is not possible for humans to provide such a large wrist torque, which would be required to achieve the desired arm release

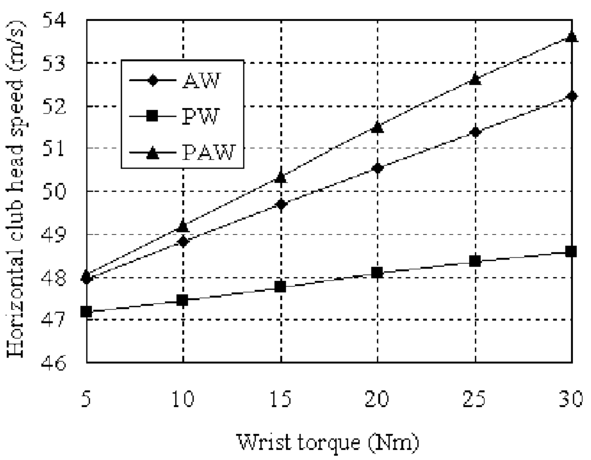

Figure 2 Maximal horizontal club head speed at impact by maximum criterion.

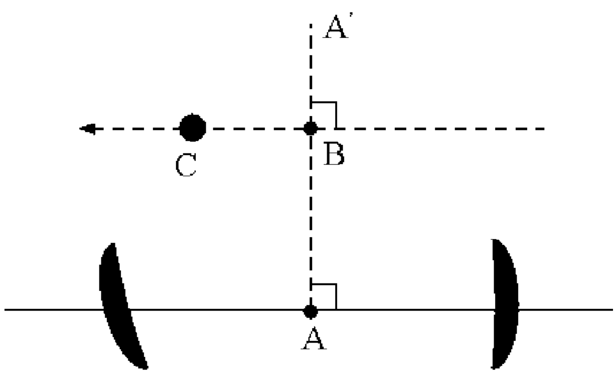

Figure 4 Golf ball position for a right-handed golfer when viewed overhead. Point C is the ball position and line AA' goes through the centre of stance.

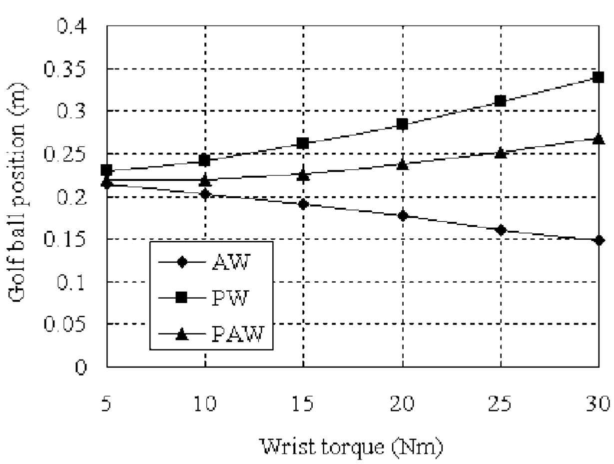

Figure 3 Optimum golf ball position by maximum criterion.

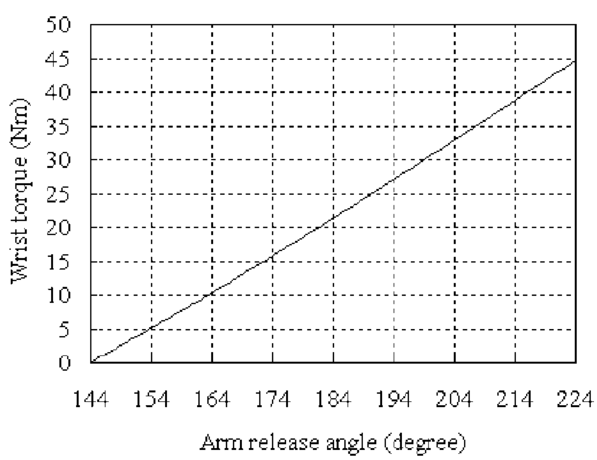

Figure 5 Required wrist torque when the arm release is delayed (the so-called 'late hit').

angle. For PAW, Figs. 2–3 show that the large ball position also gives an increased horizontal club head speed. However, it should be noted that the optimum ball position is obviously smaller than that for passive wrist action. Figs. 2–3 also show that the relatively small ball position is able to provide an improvement of horizontal club head speed for AW.

For AW and PAW, the arm rotational angle is used to describe when the optimum timing for the activation of positive wrist torque occurs (Fig. 6). As we can see, the optimum timing for the activation occurs when the arm link approximately reaches the angle of 210°. This result agrees well with that from Jorgensen (1994) and Sprigings & Neal (2000). We also observe that the optimum activation changes slightly with the wrist torque.

As reported by Cochran & Stobbs (1999), the timing for the activation of positive wrist torque was able to influence the club head speed at impact. We examined this point in our simulation by advancing and delaying the optimum activation of positive wrist torque (Fig. 7). For AW, if the activation of positive wrist torque is advanced as the arm angle arrives at 180°, the horizontal club head speed at impact is reduced by 0.8%; when the activation is delayed until the arm angle reaches 230°, the reduction is 0.3%. For PAW, the decreases are 0.8% and 0.2%, respectively.

The simulation results also show that the horizontal club head speeds at impact obtained using the maximum and impact criteria are almost the same for NW and PW. The maximum speed difference between them is merely 0.0002 m s^{-1} for NW and 0.0008 mm s^{-1} for PW. The optimum ball positions are also almost equal for both NW and PW: the maximum difference of ball position is 4 mm and 8 mm for NW and PW respectively. On the basis of these results, it can be concluded that for the golfers whose wrist actions belong to NW or PW, the simple way to achieve the optimum ball position is to put the ball at the position where the shaft is vertical at impact when viewed face-on. This theoretical finding is consistent with the actual shaft position at impact observed from the numerous swing photographs of professional golfers such as Hogan, Lietzke, Nicklaus, Norman, Nelson, Peete, Price, Snead and Woods (McLean 1999).

For AW and PAW, the maximum difference in speed between the two criteria is 0.0797 m s^{-1} and 0.0785 m s^{-1}, respectively, and the maximum difference of ball position is 78 mm and 73 mm, respectively. The optimum ball position at impact for the two criteria is shown in Fig. 8. We note that the ball position given by the maximum criterion is larger than that given by the impact criterion, and the position difference becomes larger with increased wrist torque. However, due to the small difference in the horizontal club head speed at impact for the two criteria (the maximum difference is only 0.0797 m s^{-1}), the impact criterion can be regarded as a reliable reference to obtain the optimum ball position for AW and PAW.

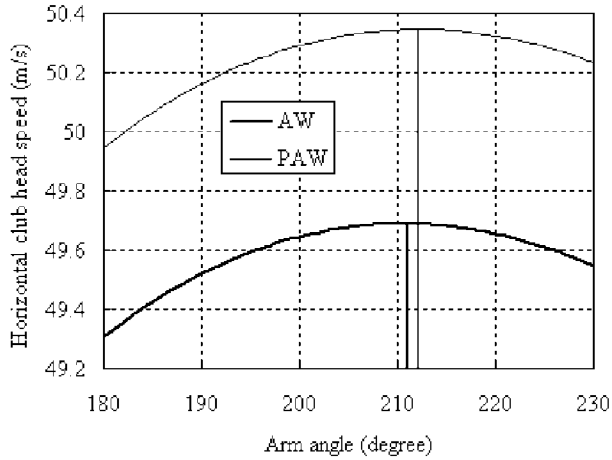

Figure 7 Horizontal club head speed at impact with different 'timed' activation of positive wrist torque (15 N m). The thick vertical line shows the optimum activation for AW; the thin vertical line indicates the optimum activation for PAW.

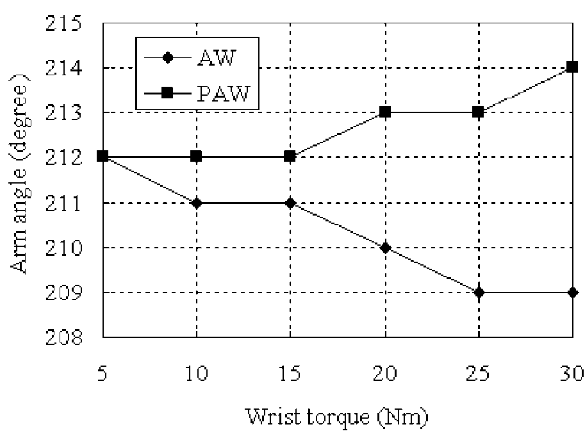

Figure 6 Arm angle when the optimum activation of positive wrist torque occurs.

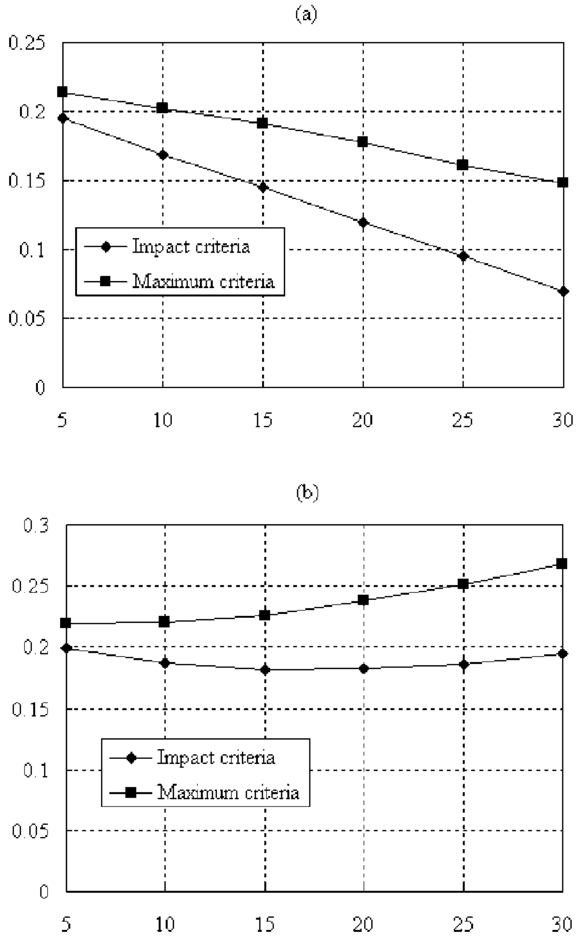

Figure 8 Optimum ball position by impact and maximum criteria **(a)** AW; **(b)** PAW.

A set of comparisons in work and kinetic energy were undertaken among NW, PW, AW and PAW, using a 15 N m wrist torque. It should be noted that other wrist torques give rise to different comparative results, but the qualitative manner in which the wrist action affects the club head speed is not altered. Table 2 shows the work and kinetic energy of the arm and club head at impact given by the maximum criterion. It can be seen that the total work for PW, AW and PAW are all increased compared to that for NW. For PW, the higher total work obviously results from the increase of work produced by the shoulder joint because the negative wrist torque maintains a constant wrist-cock angle and thus offers zero work. For AW and PAW, the sum of work exerted by the shoulder and wrist joints gives an increase in total work, although the work done by the shoulder joint is smaller than that for NW. The work done due to the gravitational force is almost the same, even though various patterns of wrist actions are employed. Table 2 also shows that the ratio of the club head kinetic energy to total work is enhanced for PW, AW and PAW. This means that the efficiency of the swing is improved, especially for AW and PAW where the positive wrist torque is used.

Through analysing the energy transfer from the input joints of the shoulder and wrist to the club head, we find that two factors determine the club head speed at impact: (1) the work produced by the golfer; and (2) the energy transfer ratio (the ratio of club head kinetic energy to total work). It is evident that the larger the two factors, the faster the club head speed is at impact. For the wrist action using the positive torque (AW and PAW), both factors are enhanced as compared with those for NW, and thus an improvement of club head speed at impact can be achieved.

Table 3 shows comparisons of the work and kinetic energy for the two criteria. It can be observed that there is only a small difference in kinetic energy, meaning that the energy flowing into the swing system and the energy distribution at impact are almost the same for the two criteria. Thus the club head speeds at impact are very similar for the two criteria.

Table 2 Work and kinetic energy at impact (J).

Work and energy	NW		PW with 15 N m		AW with 15 N m		PAW with 15 N m	
Total work	431.5	%total	438.4	%total	437.7	%total	444.0	%total
Work by shoulder joint	386.0	(89.5)	392.9	(89.6)	373.2	(85.3)	379.9	(85.6)
Work by wrist joint	0	(0)	0	(0)	18.2	(4.2)	18.2	(4.1)
Work by gravity	45.5	(10.5)	45.5	(10.4)	46.3	(10.5)	45.9	(10.3)
Kinetic energy of arm	142.7	(33.1)	140.8	(32.1)	118.7	(27.1)	116.5	(26.2)
Kinetic energy of club head	227.3	(52.7)	234.3	(53.4)	253.2	(57.8)	260.3	(58.6)

It should be noted that the values of maximum horizontal club head speed and optimum ball position are obviously affected by different values of model parameters and initial conditions. To examine the influence of small changes in model parameter values on the above results, the equations of motion of the golf swing are re-written in another pattern (Jorgensen, 1994).

$$(SMA + SMC + MC\,LA^2 + 2FMC\,LA\cos\theta_2)\ddot{\theta}_1 + (SMC + FMC\,LA\cos\theta_2)\ddot{\theta}_2 - (2\dot{\theta}_1 + \dot{\theta}_2)FMC\,LA\sin\theta_2\dot{\theta}_2 + g\sin(\text{IP})(FMC\cos(\vartheta_1 + \theta_2) + (FMA + MC\,LA)\cos\theta_1 = G_1 \quad (16)$$

$$(SMC + FMC\,LA\cos\theta_2)\ddot{\theta}_1 + SMC\ddot{\theta}_2 + FMC\,LA\sin\theta_2\dot{\theta}_1^2 + g\sin(\text{IP})(FMC\cos(\vartheta_1 + \theta_2) = G_2 \quad (17)$$

Where seven new parameters are denoted as: SMA = second moment of arm about shoulder joint (I); FMA = first moment of arm about shoulder joint (m_1l_1); LA = length of arm (a_1); SMC = second moment of club about wrist joint ($\mathcal{J} + m_2l_2^2$); FMC = first moment of club about wrist joint (m_2l_2); MC = mass of club (m_2); IP = inclination of plane of downswing (ϕ).

The maximum horizontal club head speed and optimum ball position were investigated again by the two criteria, increasing only one parameter, such as SMA, by 10%, and maintaining the other parameters at their original values.

The numerical results demonstrate that both the maximum horizontal club head speed and optimum ball position change slightly with 10% increases in parameter values. For example, when the wrist torque is increased 10% from 15 N m, the maximum changes in club head speed and ball position are no more than 1.2913 m s^{-1} (an increase of 2.6%) and 78 mm, respectively. It is of great interest that the impact criterion can still determine the optimum ball position even though the relative model parameter values are increased by 10% (Table 4). It is clear that the differences in club head speed and ball position between the two criteria are very small for all of the seven increased parameters. For NW, the maximum difference in speed is merely

Table 3 Comparison of work and kinetic energy at impact between two criteria (J).The results in this table are obtained by which values from the maximum criterion minus those from the impact criterion.

Work and energy	NW	PW with 15 N m	AW with 15 N m	PAW with 15 N m
Total work	0.14	0.71	1.68	1.62
Work by shoulder joint	0.15	0.19	1.44	1.39
Work by wrist joint	0	0	0.31	0.32
Work by gravity	-0.01	0.52	-0.07	-0.09
Kinetic energy of arm	0.02	0	0.24	-0.02
Kinetic energy of club head	0.09	0.14	1.14	1.31

Table 4 Comparison between two criteria with a varied model parameter (15 N m wrist torque is used).

	SMC	FMC	MC	SMA	FMA	LA	IP
Original parameter values	0.3004 kg m^2	0.2967 kg m	0.3940 kg	1.1500 kg m^2	2.3837 kg m	0.6150 m	60.0000 deg
Parameter changes (per cent)	10	10	10	10	10	10	10
Difference in club head speed (m s^{-1})	0.0001 (NW) 0 (PW) 0.0215 (AW) 0.0204 (PAW)	0 (NW) 0.0003 (PW) 0.0228 (AW) 0.0241 (PAW)	0.0002 (NW) 0.0004 (PW) 0.0263 (AW) 0.0263 (PAW)	0.0002 (NW) 0.0004 (PW) 0.0279 (AW) 0.0278 (PAW)	0.0002 (NW) 0.0004 (PW) 0.0258 (AW) 0.0258 (PAW)	0 (NW) 0.0001 (PW) 0.0244(AW) 0.0243 (PAW)	0.0002 (NW) 0.0004 (PW) 0.0259 (AW) 0.0259 (PAW)
Difference in ball position (mm)	0 (NW) 0 (PW) 42 (AW) 40 (PAW)	2 (NW) 5 (PW) 42 (AW) 42 (PAW)	4 (NW) 5 (PW) 46 (AW) 44 (PAW)	4 (NW) 6 (PW) 47 (AW) 47 (PAW)	4 (NW) 5 (PW) 47 (AW) 44 (PAW)	1 (NW) 3 (PW) 44 (AW) 44 (PAW)	4 (NW) 5 (PW) 47 (AW) 44 (PAW)

0.0002 m s^{-1} and the maximum difference in ball position is only 4 mm. For PW, the maximum difference in speed and ball position are 0.0004 m s^{-1} and 6 mm, respectively. For AW and PAW, relatively large differences are exhibited but cannot influence the effectiveness of the impact criterion because the maximum difference in club head speed (0.0279 m s^{-1}) is no more than 0.057% as compared to that of the maximum criterion.

The same procedures were repeated when the initial angles of the arm and club were changed by 10% respectively [$\theta_1(0) = 99°$ or $\theta_2(0) = -99°$]. Similar results are observed: the maximum changes in club head speed and ball position are no more than 1.1746 m s^{-1} (2.5% compared to the original) and 35 mm, respectively; the maximum difference in club head speed between the two criteria is only 0.0282 m s^{-1}.

We should also note that the simulation results, including maximum club head speed and optimum ball position, depend on what is fed into the downswing model. So far, the simulation has been mainly concerned with a constant torque pattern. Different torque patterns were also carried out. For example, the shoulder torque was applied as a ramp function with a rise time of 110 ms and a maximum magnitude of 110 N m and the positive wrist torque was increased linearly in time from zero with a constant slope of 93.7 N m s^{-1} (the maximum wrist torque was 15 N m). The results are very similar to those of the constant torque pattern: the impact criterion still works well in determining the optimum ball position for various types of wrist action (the maximum difference in club head speed between the two criteria is only 0.0277 m s^{-1}); PAW gives the highest club head speed compared to NW, PW and AW; the positive wrist torque enhances both factors in determining the club head speed.

Conclusions

Our simulations show that a properly 'timed' wrist action with either passive, active or passive–active torques, with consideration of the golf ball position, all provide an improvement of horizontal club head speed at impact. The passive–active wrist action gives the maximum club head speed at impact. Golfers who turn wrists freely or use a negative wrist torque (the so-called 'late hit') can achieve the optimum ball position by the impact criterion, where the club shaft is vertical at impact when viewed face-on. Golfers who apply negative or negative–positive wrist torque can nearly achieve the optimum ball position by considering the impact criterion as a reference. The application of a positive wrist torque can increase two factors that facilitate the club head speed: (1) the work produced by the golfer; and (2) the energy transfer ratio.

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