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The analysis of golf swing as a kinematic chain using dual Euler angle algorithm

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Abstract

The manner in which anatomical rotation from an individual segment contributes to the position and velocity of the endpoint can be informative in the arena of many athletic events whose goals are to attain the maximal velocity of the most distal segment. This study presents a new method of velocity analysis using dual Euler angles and its application in studying rotational contribution from upper extremity segments to club head speed during a golf swing. Dual Euler angle describes 3D movement as a series of ordered screw motions about each orthogonal axis in a streamlined matrix form—the dual transformation matrix— and allows the translation and rotation component to be described in the same moving frame. Applying this method in biomechanics is a novel idea and the authors have previously applied the methodology to clinical studies on its use in displacement analysis. The focus of this paper is velocity analysis and applications in sports biomechanics. In this study, electrogoniometers (Biometrics, UK) with a frequency of 1000 Hz were attached to a subject during the execution of the swing to obtain the joint angles throughout the motion. The velocity of the club head was then analyzed using the dual velocity which specifies the velocity distribution of a rigid body in screw motion at any point in time as the dual vector. The contributions of each segment to the club-head velocity were also compared. In order to evaluate this method, the calculated position and velocity of the club head were compared to the values obtained from video image analysis. The results indicated that there is good agreement between calculated values and video data, suggesting the suitability of using the Dual Euler method in analyzing a kinematic chain motion.

Keywords: Dual vector; Arm segment rotation; Dual Euler angle; Dual velocity; Goniometer; Golf swing; Jacobian; Dual transformation matrix

1. Introduction

Sprigings et al. (1994) were among the pioneers in investigating end-effector velocity in the human kinematic chain. They also examined the contribution of segmental rotations of the arm to racquet head speed in tennis. Their theoretical development was based on a three-dimensional (3D) kinematic method of formulating vector equations from the results of the video image analysis. Sprigings et al. (1994) cautioned that there are certain inaccuracies in the component summation curve, originating from the inaccuracy of the measurements for pronation/supination of the forearm. The inaccuracies were largely attributed to the small diameter of the wrist and elbow. The errors would affect the accuracy of the reconstructed anatomical axes and further affect the accuracy of the computed vectors and angular velocities.

The joint coordinate system (Grood and Suntay, 1983) and Euler angle convention (Chao, 1980) are biomechanical standards for describing angular motion such as the joint movements in terms of anatomical rotations (e.g. flexion/extension, pronation/supination, and internal/external rotation). However, the joint coordinate system and Euler angle convention are

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unable to describe the kinematic chain movements of the human body with multi-joints, as there are segmental lengths involved. Euler angles require an additional 3D position vector to handle the translation component so that it can describe both rotation and translation in either moving or fixed reference system. Since Euler angles treat rotations and translations as separate entities, it lacks intuitive physical interpretation. On top of that, with the sequence-dependent Euler angles being non-vectors, it makes velocity analysis more complex to conduct and less intuitive to understand.

One common and conventional method to describe the kinematic chain is via a rotation matrix and a 3D position vector. The orientation of the moving link with respect to the reference link at any joint configuration can be expressed as the direction cosines between two sets of axes (Zatsiorsky, 1998), represented by an orthogonal rotation matrix. The combination of a rotation matrix and a position vector, as a 4×4 homogeneous transformation matrix, can fully describe the six-degree-of-freedom joint motion. However, there is no physical interpretation for the parameters in the rotation matrix. Furthermore, the transformation matrix does not handle velocity directly. When finding the velocity of the end effector, the transformation matrix is used to locate the position vector of the end effector ($\vec{P}$), which is then cross-producted with the angular velocity ($\vec{\omega}$) before using the transformation matrix again to express the velocity in terms of the end-effector coordinates (Craig, 1989). The angular velocity, however, cannot be calculated together with the linear velocity using the transformation matrix method.

Although the screw axis method could describe full six-degree-of-freedom spatial motions with parameters that are easy to interpret, the dual Euler angles method was preferred in this study because the technical screw axis method may be considered inappropriate to describe a somatic or anatomic motion. A modification of the Euler angle formulation which used dual angles has been applied to the study of clinical biomechanics by the authors (Ying and Kim, 2002, 2005; Ying et al., 2004; Wong et al., 2005). With the dual Euler angles, the gross motion at the joint is represented by three screw motions through the coordinate axes of the reference Cartesian coordinate system. In this way, the rotation and translation of a segment are combined and interpreted with respect to the same Cartesian coordinate system. The dual Euler angles method can be further applied to kinematic chain problem as the trigonometric functions are well defined and all identities of ordinary trigonometry hold true for dual angles (Fischer, 1999), rendering a straightforward formation of the transformation matrix. In this way, 3D movements can be described as a series of ordered screw motions about each orthogonal axis in a streamlined matrix form. This is where the dual Euler angles method stands out, especially for studies involving multi-segment sports biomechanics because it can provide intuitive physical interpretation.

In this study, the velocity of the club head was analyzed using the dual velocity, which specifies the velocity distribution of a rigid body in screw motion at any point in time as the dual vector. With the dual velocity formulation, linear and angular velocities of the end effector can be conveniently computed simultaneously because its angular and linear components are merged into dual numbers and treated as a single entity, which conventional tools could not accomplish. Moreover, since the dual velocity is specified as the dual vector, its analysis can be done in a similar manner as displacement analysis via the same dual transformation matrix. This feature, especially not requiring a separate formulation for velocities, might facilitate a practical analysis for practitioners because velocity and displacement analysis can be handled under a single rule. Finally the Jacobian of a spatial open chain concept (McCarthy, 2000) was addressed to relate the contributions of each segmental rotation (joint rates) to club head velocity (its end effector).

The objective of this study is to present a new method of velocity analysis using dual Euler angles, and its application in studying the rotational contribution from upper extremity segments to club-head speed during a golf swing. With this algorithm, the contributions of proximal segment motions to the kinematics of the distal segment also can be determined. The segmental rotations were captured using goniometer. By modeling the kinematic chain as a serially multi-link system connected by joints and using a dual transformation matrix, the displacement and the speed of the most distal segment can be computed concurrently.

2. Methods

2.1. Dual Euler angles

Euler angles are often used for describing joint kinematics; however, with Euler angles only an attitude is defined (Zatsiorsky, 1998). Therefore, in order to account for the segment length within the kinematic chain, dual Euler angles proposed by Ying and Kim (2002) were adopted as a representation of the anatomical rotation on joints (for a more detailed description of dual Euler angles, please refer to Appendix A of this paper). Similar to the Euler angles method, the dual Euler angle method is also sequence-dependent. The $Zy'x''$ sequence of three screw motions was used by adopting the typical Aerospace Sequence (Kuipers, 1999). In this case, the ordered sequence of screw motion begins first with respect to the Z-axis, then with respect to the new y-axis (y'-axis) which is called the floating axis, and finally with respect to the new x-axis (x''-axis). These local axes are denoted by single prime ($'$) and double prime ($''$) according to the number of preceding screw motions defining their positions.

2.2. Multi-link chains and transformation analysis

A human body in action can be modeled as a multi-link system comprising several body segments connected by joints (Vaughan et al., 1982). In this study, a golfer was depicted as a serially joined five segment model consisting of the torso, left upper arm, left forearm, left hand and club (Fig. 1). An orthogonal Cartesian frame was attached to each of the links. A local frame {1} was attached to the rotating torso at the glenohumeral joint, {2} was attached to the upper arm at the elbow joint, {3} was attached to the forearm at the wrist joint, {4} was attached to the hand at the end of the hand grip, and finally {5} was attached to the center of the club head. In addition, a virtual reference frame {0}, with axes coinciding with those of the other local frames at neutral positions and fixed in space at the waist level, was introduced to calculate the relative position of a body with respect to the global frame $\{G\}$. The directions of the three axes of each frame were assigned so as to approximate the different anatomical axes of rotation for each segment. It is important that the chosen axes are consistent with the functional axes of the joint examined in the study. The origins of these frames are located at the center of each relevant joint. The global frame, $\{G\}$, was fixed to the laboratory floor. The neutral positions of all local frames are chosen to be the positions when a subject assumes the classical anatomical posture (Fig. 1).

The local coordinate system, x_i, y_i, z_i, is attached to each link i at the distal joint. The position of frame $\{j\}$ relative to frame $\{i\}$ is described by the 3×3 dual transformation matrix ${}^{i}_{j}\hat{M}$. The operator ${}^{i}_{j}\hat{M}$ allows any

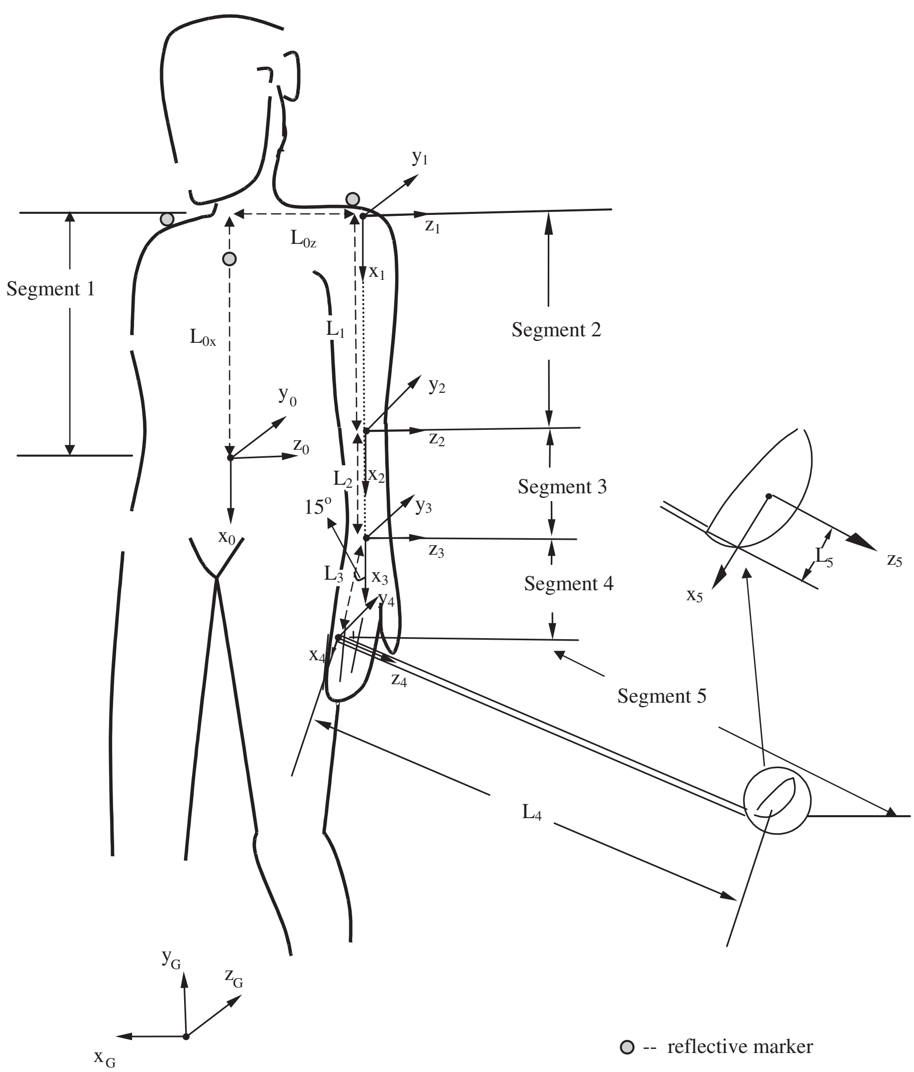


Fig. 1. The anatomical posture and locations of markers on the subject and the club-head.

line vector expressed in terms of the unit vectors of frame $\{j\}$ to be expressed in terms of the unit vectors of frame $\{i\}$. Considering the Denavit–Hartenberg parameters (Denavit and Hartenberg, 1955), the individual joint-link transformation matrices based on dual Euler angles can be expressed by concatenating the matrices: ${}^{G}_{0}\hat{M}$, ${}^{0}_{1}\hat{M}$, ${}^{1}_{2}\hat{M}$, ${}^{2}_{3}\hat{M}$, ${}^{3}_{4}\hat{M}$ and ${}^{4}_{5}\hat{M}$ with respect to the global frame $\{G\}$.

The transformation matrix of frame $\{0\}$ with respect to the global frame is represented by: ${}^{G}_{0}\hat{M}$, which can be calculated from ${}^{G}_{0}R$, the rotation matrix of frame $\{0\}$ with respect to frame $\{G\}$, and the displacement between frame $\{0\}$ and frame$\{G\}$.

The movement of torso, $\{1\}$, with respect to the virtual reference frame, $\{0\}$, is represented by

$$
\begin{aligned}
{}^{0}_{1}\hat{M} &= {}^{0}_{0'}\hat{M}\,{}^{0'}_{0''}\hat{M}\,{}^{0''}_{1}\hat{M} \\
&= \left[\hat{R}_Z(\hat{\gamma}_0 + \varepsilon L_{0z})\right]\left[\hat{R}_{y'}\left(\hat{\beta}_0\right)\right]\left[\hat{R}_{x''}(\hat{\alpha}_0 + \varepsilon L_{0x})\right],
\end{aligned} \tag{1a}
$$

where L_{0z} and L_{0x} are the respective lengths along the z- and x-axes measured from frame $\{1\}$ with respect to frame $\{0\}$. γ_i, β_i and α_i are the Euler angles for the $Zy'x''$ sequence taken with respect to the neutral position. Frame $\{0\}$ was selected with the origin at the waist, with the x-axis pointing inferiorly, the y-axis pointing posteriorly and the z-axis pointing laterally (to the left of the subject) with respect to the trunk.

The movement of the upper arm, $\{2\}$, with respect to the torso, $\{1\}$, at the shoulder joint is represented by

$$
\begin{aligned}
{}^{1}_{2}\hat{M} &= {}^{1}_{1'}\hat{M}\,{}^{1'}_{1''}\hat{M}\,{}^{1''}_{2}\hat{M} \\
&= \left[\hat{R}_Z(\hat{\gamma}_1)\right]\left[\hat{R}_{y'}\left(\hat{\beta}_1\right)\right]\left[\hat{R}_{x''}(\hat{\alpha}_1 + \varepsilon L_1)\right],
\end{aligned} \tag{1b}
$$

where L_1 represents the upper arm segment length, the distance between the glenohumoral joint and the elbow joint. Frame $\{1\}$ was assigned to the torso with its origin at the glenohumeral joint, with the x-axis pointing inferiorly, the y-axis pointing posteriorly and the z-axis pointing laterally (to the left of the subject) with respect to the torso.

The movement of the lower arm, $\{3\}$, with respect to the upper arm, $\{2\}$, at the elbow joint is represented by

$$
\begin{aligned}
{}^{2}_{3}\hat{M} &= {}^{2}_{2'}\hat{M}\,{}^{2'}_{3}\hat{M} \\
&= \left[\hat{R}_Z(\hat{\gamma}_2)\right]\left[\hat{R}_{x''}(\hat{\alpha}_2 + \varepsilon L_2)\right],
\end{aligned} \tag{1c}
$$

where L_2 represents the lower arm segment length, which is defined as the distance between the axes of flexion–extension of the elbow and the wrist joints at $\{2\}$ and $\{3\}$, respectively. Frame $\{2\}$ was constructed with its origin at the elbow joint, with the z-axis coinciding with the flexion–extension axis at the elbow and the x-axis defined as a common normal to the axes of retroversion–anteversion in the glenohumeral and the axes of flexion–extension in the elbow joint. The y-axis was constructed as orthogonal to both x- and z-axes.

The movement of the hand, $\{4\}$, with respect to the lower arm, $\{3\}$, at the wrist joint is represented by

$$
\begin{aligned}
{}^{3}_{4}\hat{M} &= {}^{3}_{3'}\hat{M}\,{}^{3'}_{3''}\hat{M}\,{}^{3''}_{4}\hat{M} \\
&= \left[\hat{R}_Z(\hat{\gamma}_3)\right]\left[\hat{R}_{y'}\left(\hat{\beta}_3 + 15^\circ\right)\right]\left[\hat{R}_{x''}(\varepsilon L_3)\right],
\end{aligned} \tag{1d}
$$

where L_3 represents the length between the wrist joint to the grip. The additional 15° at β_3 arises from the way the subject (golfer) held the club. Frame $\{3\}$ was constructed with its origin at the wrist joint, with the z-axis coinciding with the flexion–extension axis at the wrist, and the x-axis defined as common normal to the axes of flexion/extension in the elbow and the wrist joint. The y-axis was constructed as orthogonal to both x- and z-axes.

The movement of the club head, $\{5\}$, with respect to the hand, $\{4\}$, is represented by

$$
\begin{aligned}
{}^{4}_{5}\hat{M} &= {}^{4}_{4'}\hat{M}\,{}^{4'}_{5}\hat{M} \\
&= \left[\hat{R}_Z(\varepsilon L_4)\right]\left[\hat{R}_{x''}(-\varepsilon L_5)\right],
\end{aligned} \tag{1e}
$$

where L_4 represents the total sum of the grip length, shaft length and hosel length of the club. L_5 represents the distance from the end of the hosel to the center of the club head. Frame $\{4\}$ was constructed with its origin at the grip end, with the z-axis coinciding with the shaft of the club and the x-axis pointing in the opposite direction as the club-head, perpendicular to the shaft. The y-axis was constructed as orthogonal to both x- and z-axes. Frame $\{5\}$ was constructed with its origin at the middle of the club-head and its orientation the same as Frame $\{4\}$.

For the algorithm presented, the following assumptions are made: (a) the constructed orthogonal axes for the three segments represent their anatomical axes; (b) the valgus/varus rotation at the elbow joint is assumed as zero; (c) the longitudinal rotation of the hand at the wrist joint is also assumed to be zero; and (d), the hand and club are treated as one single rigid body.

For simplicity, the inter-joint offsets (defined as the distance between the origins of frames $\{i\}$ and $\{i+1\}$ measured along the joint axis, z_i,) are assumed to be negligible. The twisted angles, which differs from zero when the axes of flexion/extension in two joints have the same link, for example, the elbow and wrist of the lower arm segment are not exactly in the same (frontal) plane. In this study these angles are small enough to be ignored.

2.3. Position analysis

We have seen that the algebra of dual vectors has all the properties of conventional vector algebra. The "origin-displacement equation" (see Appendix B for an introduction of the origin-displacement equation) is a relationship in dual-number mathematics which allows the relative position of one coordinate frame with respect to the other to be recovered from the dual-number coordinate-transformation matrix between them (Fischer, 1999). It was developed by Hsia and Yang (1981) who referred to it as the "principle of transference"—the connection that it provides between the geometry of points and the geometry of lines.

Using the "origin-displacement equation" method, the origin of the relevant frames attached to the elbow, wrist and club-head can be located. In this study, the following notation for position vector was adopted:

${}^{i}P_{\mathrm{e}}$ = position vector of an element e with regard to coordinate frame i.

Therefore, the position vectors of the various link-ends in terms of the global coordinate are

$$\begin{aligned}&\text{Elbow}: \ {}^{G}P_{\text{elbow}}={}^{G}_{1}R^{1}\,D_{\text{elbow}}+{}^{G}P_{\text{shoulder}},\\&\text{Wrist}: \ {}^{G}P_{\text{wrist}}={}^{G}_{1}R^{1}\,D_{\text{wrist}}+{}^{G}P_{\text{shoulder}},\\&\text{Grip}: \ {}^{G}P_{\text{grip}}={}^{G}_{1}R^{1}\,D_{\text{grip}}+{}^{G}P_{\text{shoulder}},\\&\text{Club-head}: \ {}^{G}P_{\text{club-head}}={}^{G}_{1}R^{1}\,D_{\text{club-head}}+{}^{G}P_{\text{shoulder}},\end{aligned} \quad (2)$$

where ${}^{G}P_{\text{shoulder}}$, the location of the shoulder joint, is taken from the camera system. ${}^{G}_{1}R={}^{G}_{0}R^{0}_{1}R$, the rotation matrix, is calculated using the algorithm—least-square estimation of transformation matrix between two sets of point patterns—proposed by Umeyama (1991) using the camera data. ${}^{i}D_{j}$ is the location of the origin of frame $\{j\}$ in terms of coordinate frame $\{i\}$ calculated from ${}^{i}_{j}\hat{M}$.

2.4. Dual velocity analysis

It has been shown that the movement of a rigid body from one position to another constitutes a screw motion that involves the simultaneous translation along, and rotation about, a screw axis. The velocity distribution of a rigid body in screw motion at any given time is specified by the dual vector. Now, the dual velocity $\hat{V}$ is obtained by multiplying (dual multiplication) the dual scalar and the components of the dual vector such as

$$\hat{V}=(\Omega+\varepsilon V)\hat{u}. \quad (3a)$$

The unit line vector $\hat{u}$ specifies the screw axis by

$$\vec{u}=\vec{u}+\varepsilon\left(\overrightarrow{OP}\times\vec{u}\right), \quad (3b)$$

where the symbol $\vec{u}$ represents a unit vector and the vector $\overrightarrow{OP}$ extends from the origin of the coordinate system to any point on the screw axis.

The dual scalar $\Omega+\varepsilon V$specifies the magnitude of (i) Ω = angular speed about the screw axis, which is given by its primary part and (ii) V = linear speed along the screw axis, which is given by its dual part. Just as the dual transformation matrices are used to describe line vectors in different frames, the same transformation matrices can also be used to determine the dual velocity of any point on a body if the dual velocity at the screw axis is given by Eq. (3a). The following section exemplifies how dual velocities are treated in conjunction with the dual transformation matrix. We adopt a notation for dual velocity such that ${}^{R}\hat{V}^{p}_{ij}$ = dual velocity at a point P of body i relative to body j in terms of the unit vectors of frame $\{R\}$.

Since the actions at the segmental level involve only rotational actions and not translational ones, the dual velocities are simply left with the primary component only. Accordingly, the velocities of the torso rotations and those at the shoulder, elbow and wrist joints were obtained by differentiating the angular displacement time curves of the respective rotations (γ_i, β_i and α_i) and their angular velocities are denoted as $\dot{\gamma}_i$, $\dot{\beta}_i$, and $\dot{\alpha}_i$, respectively.

$$\text{Torso}: \ {}^{0}\hat{V}^{o_0}_{0G}=\begin{Bmatrix}0\\0\\\dot{\gamma}_0\end{Bmatrix}, {}^{0'}\hat{V}^{o_0}_{0'0}=\begin{Bmatrix}0\\\dot{\beta}_0\\0\end{Bmatrix}, {}^{0''}\hat{V}^{o_0}_{0''0'}=\begin{Bmatrix}\dot{\alpha}_0\\0\\0\end{Bmatrix}, \quad (4a)$$

where ${}^{0}\hat{V}^{o_0}_{0G}$, ${}^{0'}\hat{V}^{o_0}_{0'0}$ and ${}^{0''}\hat{V}^{o_0}_{0''0'}$ represent the dual velocity at origin, o_0, of frame $\{0\}$ caused by trunk flexion/extension, side bending and rotations in terms of its preceding reference frames $\{0\}$, $\{0'\}$, and $\{0''\}$, respectively.

$$\text{Shoulder joint}: {}^{1}\hat{V}^{o_1}_{10''}=\begin{Bmatrix}0\\0\\\dot{\gamma}_1\end{Bmatrix}, {}^{1'}\hat{V}^{o_1}_{1'1}=\begin{Bmatrix}0\\\dot{\beta}_1\\0\end{Bmatrix}, {}^{1''}\hat{V}^{o_1}_{1''1'}=\begin{Bmatrix}\dot{\alpha}_1\\0\\0\end{Bmatrix}, \quad (4b)$$

where ${}^{1}\hat{V}^{o_1}_{10''}$, ${}^{1'}\hat{V}^{o_1}_{1'1}$ and ${}^{1''}\hat{V}^{o_1}_{1''1'}$ represent the dual velocity at origin, o_1, of frame $\{1\}$ caused by upper arm anteversion/retroversion, abduction/adduction and internal/external rotation in terms of its preceding reference frames $\{1\}$, $\{1'\}$, and $\{1''\}$, respectively.

$$\text{Elbow joint}: \quad {}^{2}\hat{V}^{o_2}_{21''}=\begin{Bmatrix}0\\0\\\dot{\gamma}_2\end{Bmatrix}, \ {}^{2'}\hat{V}^{o_2}_{2'2}=\begin{Bmatrix}\dot{\alpha}_2\\0\\0\end{Bmatrix}, \quad (4c)$$

where ${}^{2}\hat{V}^{o_2}_{21''}$ and ${}^{2'}\hat{V}^{o_2}_{2'2}$ represent the dual velocity at origin, o_2, of frame $\{2\}$ caused by lower arm flexion/extension and forearm pronation/supination in terms of its preceding reference frames $\{2\}$ and $\{2'\}$, respectively.

$$\text{Wrist joint}: \quad {}^{3}\hat{V}^{o_3}_{32'}=\begin{Bmatrix}0\\0\\\dot{\gamma}_3\end{Bmatrix}, \ {}^{3'}\hat{V}^{o_3}_{3'3}=\begin{Bmatrix}0\\\dot{\beta}_3\\0\end{Bmatrix}, \quad (4d)$$

where ${}^{3}\hat{V}^{o_3}_{32'}$ and ${}^{3'}\hat{V}^{o_3}_{3'3}$ represent the dual velocity at origin, o_3, of frame $\{3\}$ caused by hand flexion/extension and ulnar/radius abduction in terms of its preceding reference frames $\{3\}$ and $\{3'\}$, respectively.

The contribution of individual joint rotation to the club-head velocity in biomechanics will be explained in the context of the Jacobian in robotic research (Craig, 1989). First, we note that the Jacobian matrix relates joint velocities to Cartesian velocities. For example:

$$\mathbf{V} = \mathbf{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \tag{5a}$$

where $\boldsymbol{\theta}$ is the vector of joint angles of the upper extremity segments, and $\mathbf{V}$ is a vector of Cartesian velocities of the end effector. Elements of the Jacobian, $\mathbf{J}(\boldsymbol{\theta})$, are partial derivatives that relate the infinitesimal displacement of the end point to the infinitesimal joint displacement at the present joint configuration. Note that for any given configuration of the kinematic chain, joint rates are related to velocity of the end effector in a linear fashion. Finding Jacobians is not a simple matter for robotic researchers as well as biomechanical researchers, even if it could be obtained by directly differentiating the kinematics equations of the given mechanism. Hence, in this work, a novel approach was introduced to derive Jacobian in human kinematic chains by using the dual velocities and dual transformation matrix that have been previously derived in the multi-link chain and transformation analysis.

The angular velocities of each segment relate to both rotational and linear velocity at the club head. In order to find out how each segmental rotation contributes to the club-head velocity, the angular velocity of the segments about their respective axes were used to multiply with the corresponding dual Euler transformation matrices to determine the dual velocity at the club-head.

Using the wrist as an example, the two segmental actions that contribute to the club-head movement are classified as the hand flexion/extension and ulnar/radius abduction. To determine the contribution of these two actions on the club-head velocity, the segmental velocities were multiplied with the corresponding dual transformation matrices from its corresponding frame to the club-head frame:

Hand ulnar/radius abduction contribution: ${}^{5}_{3'}\hat{M}\,{}^{3'}\hat{V}^{o_3}_{3'3}$, where the dual transformation matrix ${}^{5}_{3'}\hat{M}$ transforms ${}^{3'}\hat{V}^{o_3}_{3'3}$ to the terminal point (club-head frame). The matrix is referred to as the structural equation of the kinematic chain moving from the wrist to the end point.

Hand flexion/extension contribution: ${}^{5}_{3}\hat{M}\,{}^{3}\hat{V}^{o_3}_{32'}$, where ${}^{5}_{3}\hat{M}$ transforms ${}^{3}\hat{V}^{o_3}_{32'}$ to the terminal point (club-head frame).

The total wrist contribution to the club head is simply the sum of the two actions:

$$\text{wrist contribution}: \quad {}^{5}_{3}\hat{M}\,{}^{3}\hat{V}^{o_3}_{32'} + {}^{5}_{3'}\hat{M}\,{}^{3'}\hat{V}^{o_3}_{3'3}. \tag{5b}$$

Note that Eq. (5b) states that the first element of the Jacobian for the wrist joint equals the dual transformation matrix between frame {3} caused by hand flexion/extension and the club-head frame. The second element of this Jacobian equals the dual transformation matrix between frame {3′} caused by the ulnar/radius abduction and the club-head frame. This example illustrates that the Jacobian is defined simply by writing the dual transformation matrix between the acting joint axis and club-head frame.

Therefore, the dual velocity of the end point (i.e. the point on the center of the club head) is given by the sum of the contribution of each segmental rotation such that

$$\begin{aligned}{}^{5}\hat{V}^{o_5}_{50} = {} & {}^{5}_{0}\hat{M}\,{}^{0}\hat{V}^{o_0}_{0G} + {}^{5}_{0'}\hat{M}\,{}^{0'}\hat{V}^{o_0}_{0'0} + {}^{5}_{0''}\hat{M}\,{}^{0''}\hat{V}^{o_0}_{0''0'} \\ & + {}^{5}_{1}\hat{M}\,{}^{1}\hat{V}^{o_1}_{10''} + {}^{5}_{1'}\hat{M}\,{}^{1'}\hat{V}^{o_1}_{1'1} \\ & + {}^{5}_{1''}\hat{M}\,{}^{1''}\hat{V}^{o_1}_{1''1'} + {}^{5}_{2}\hat{M}\,{}^{2}\hat{V}^{o_2}_{21''} \\ & + {}^{5}_{2'}\hat{M}\,{}^{2'}\hat{V}^{o_2}_{2'2} + {}^{5}_{3}\hat{M}\,{}^{3}\hat{V}^{o_3}_{32'} + {}^{5}_{3'}\hat{M}\,{}^{3'}\hat{V}^{o_3}_{3'3},\end{aligned} \tag{5c}$$

where ${}^{5}\hat{V}^{o_5}_{50}$ represents the dual velocity at origin, o_5, of frame {5} with respect to frame {0} in terms of reference frame {5}. The club head velocity can be written in terms of the global frame $\{G\}$ as shown in (5) to facilitate the comparisons between other systems.

$${}^{G}\hat{V}^{o_5}_{5G} = {}^{G}_{5}M_{\mathrm{p}}\,{}^{5}\hat{V}^{o_5}_{50}, \tag{6}$$

where matrix ${}^{G}_{5}M_{\mathrm{p}}$ is the primary component of the dual transformation matrix of ${}^{G}_{5}\hat{M}$, which equals the rotation matrix. Because the dual velocity is not a line vector, the translation between different frames in which it is expressed is not a factor.

Since the directional contributions of individual segments relative to the club head velocity are different, the magnitudes of the vector sums of the dual vectors in Eq. (5c) do not immediately represent the contributions of each part. Hence, the projections of the dual vectors to the direction of the actual resultant velocity of the club head must be determined first before the percentage of the resultant velocity generated by individual joints is determined.

3. Experimental setup

The subject involved for this study was a single handicap, semiprofessional golfer and the experiment was conducted at the Singapore Sports Council biomechanics laboratory. The approval of the Nanyang Technological University's School Ethics Committee was obtained prior to the study. The electrogoniometer (EGM) (Biometrics, UK) was attached to the left arm of the subject by double-sided adhesive tape to quantify the joint rotation during the swing (Fig. 2).

Two EGMs were used to record movement about the shoulder joint, one 2D-EGM (anterversion/retroversion, abduction/adduction) and one torsionmeter (external/internal rotation). The goniometers were attached to the acromion process and to the upper arm, just below the deltoid attachment. Another goniometer (2D-EGM, one motion only: extension/flexion) was attached to the dorsolateral side of the upper arm and forearm to record the movement about the elbow. The goniometer

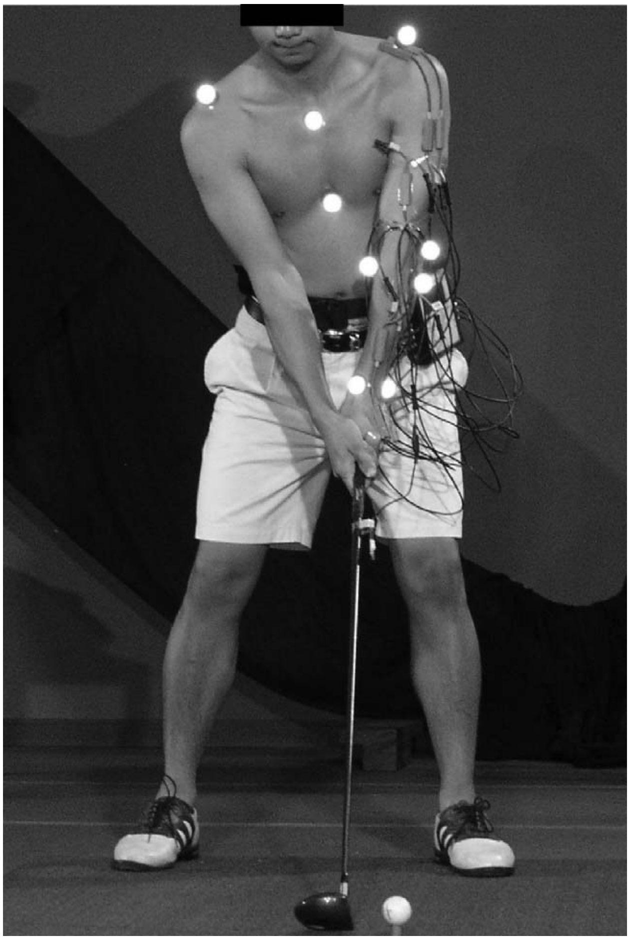


Fig. 2. Attachments of the EGMs and the markers.

for the wrist (2D-EGM, extension/flexion, radial/ulnar abduction) and the torsionmeter for the forearm (pronation/supination) were connected to the dorsal sides of the hand and forearm. Additionally, an accelerometer, equipped with a 1D-fingergoniometer, was attached to the side of the club shaft to determine the impact in the time course of the swing. The data sampling frequency of the EGMs was set at the maximum of 1000 Hz. Savitzky–Golay smoothing filters (Savitzky and Golay, 1964), with 99 points and an order of 8, were first used to smooth the angular displacement time curves using a standard routine in Matlab (The Math Works, Inc., Natick, USA) before the curves were differentiated to determine the velocity. The velocity curves are then smoothed again with Savitzky–Golay smoothing filters, with 99 points and an order of 8.

The angular velocity for the torso rotation, club-head position and velocity were obtained by using the 3D video analysis method—PEAK MOTUS (Peak Performance Technologies, Inc., USA). Three high-speed video cameras operating at 200 Hz were used. Three reflective markers were placed on the shoulders and jugular notch of the subjects (Fig. 1). The object space calibration errors were 0.1311%, 0.1161%, 0.2176% in X, Y, and Z directions, respectively.

4. Results

In order to evaluate the validity of this proposed method, the computed club-head linear velocity (using the Dual Euler angles and Dual velocities algorithm) was compared with measured values obtained from video analysis. The computed and measured club-head linear velocities are presented in Fig. 3. Both the computed and measured velocities were resolved in negative x-direction of the laboratory frame, where the target was located. The computed club-head velocity time trace was in good agreement with the measured velocity time curve. The closeness between the two traces indicates the viability of this algorithm in the study. The comparison of the calculated club-head displacement time curve, in global frame, using position vector analysis with that of the video measured displacement time curve (Fig. 4) also indicated similarity in trend and magnitude. The obvious difference between the measured and calculated velocities and displacements occurred at about 0.1 s before impact. The possible reasons for the difference will be discussed in the following section.

Time course of angular velocity of the different joints are presented in Fig. 5. At the impact (club–ball contact), the wrist extension has the highest angular velocity of $11.8\,\mathrm{rad\,s^{-1}}$. The external rotation of the upper arm also had a notable angular velocity of $9.6\,\mathrm{rad\,s^{-1}}$ followed by forearm supination ($8.2\,\mathrm{rad\,s^{-1}}$).

The computed results indicated that, for the single-handicap subject at impact, the torso rotation contributed most significantly to the final resultant linear

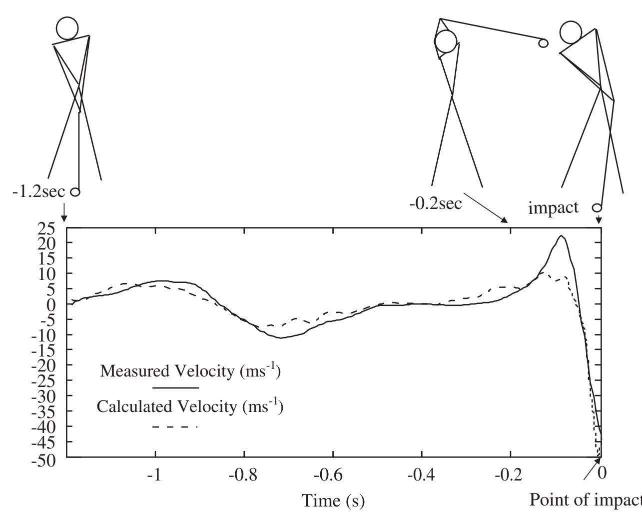


Fig. 3. The comparison of the calculated and measured club-head speed in global frame $\{G\}$ and the sequence of swing.

velocity of the club-head speed at $16\,\mathrm{m\,s^{-1}}$. The wrist extension contributes with $7.6\,\mathrm{m\,s^{-1}}$, while the upperarm abduction is $6\,\mathrm{m\,s^{-1}}$ (this is presented in Fig. 6 and Table 1.). These movements were the three main contributions to the final club-head speed at impact, and together they accounted for over 60% of the final velocity.

5. Discussion

The proposed method provides comprehensive analytical tools for studying human movement using video or film kinematic approaches. This method also provides details of individual segmental contributions to the studied movement. With similarity of club-head position and velocity between the calculated and measured curves demonstrated in Figs. 3 and 4, the suitability of the proposed method can be ascertained. The magnitude of peak club-head velocity calculated by the algorithm also agrees well with the findings from other studies (e.g., Penner, 2003). The positions of the elbow, wrist and hand were plotted to give visual feedback during the swing (Fig. 7). These plots can also be animated to show the actual swing motion. The angular velocities of the club-head about the axes of the club-head coordinate system were shown (Fig. 8). The kinematics of the golf swing can be conveniently obtained with this algorithm. The results of such analyses should be useful for coaches and researchers.

The findings also reveal the importance of wrist "uncocking" in achieving a high club-head speed. This result is consistent with the findings of Sprigings and Neal (2000). However, the study also reveals the importance of the external rotation of the upper arm and the supination of the forearm in a golf swing. These rotations along the longitudinal axes cannot be detected using traditional 2-dimensional approaches and are not often emphasized in conventional golf instruction.

There are many advantages in using the algorithm proposed in this paper. Angular velocities taken from the first derivative of the angular displacement data when directly measured is more straightforward and less prone to error as compared to the method used by Sprigings et al. (1994). The errors introduced during the process (Sprigings et al., 1994) that provides the angular velocities by formulating vector equations, which involved the position and velocity vectors taken form

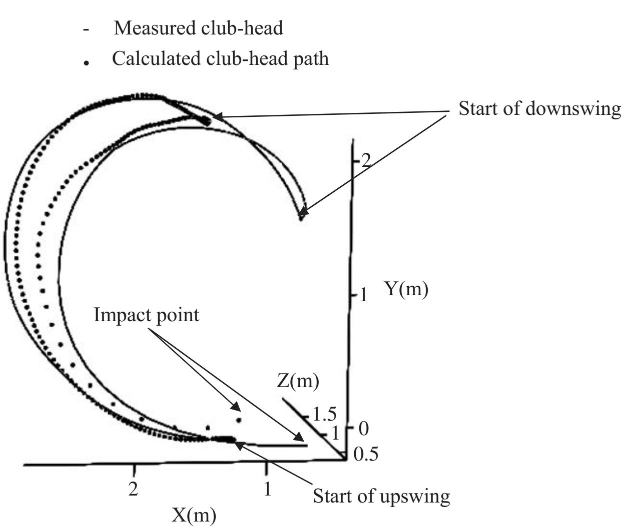


Fig. 4. The comparison of the calculated and measured club-head path in the global frame $\{G\}$.

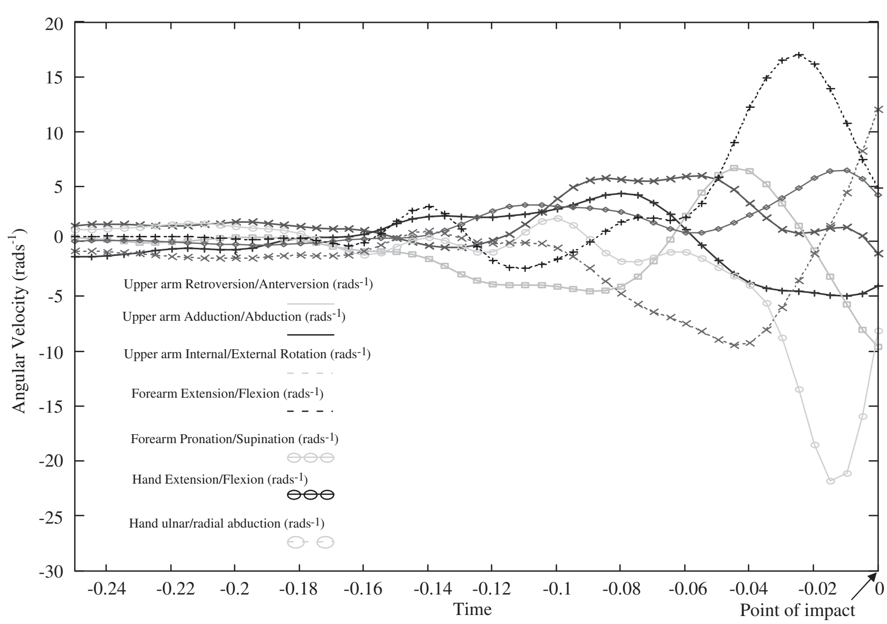
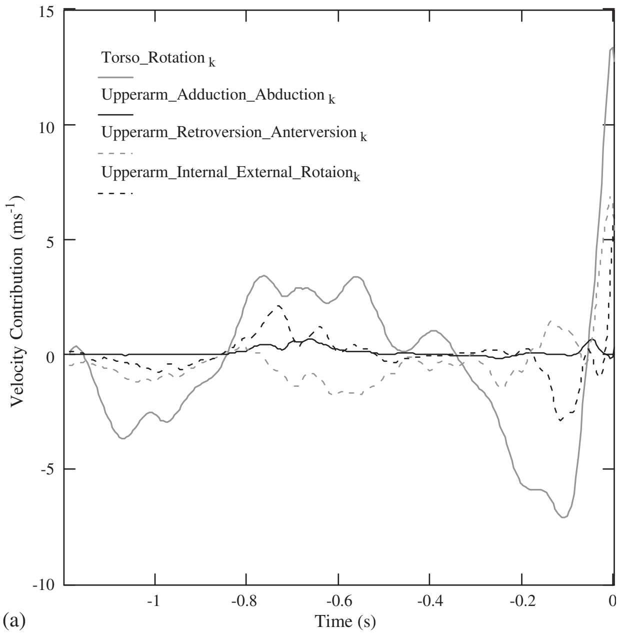


Fig. 5. Angular velocity of different arm segment rotations.



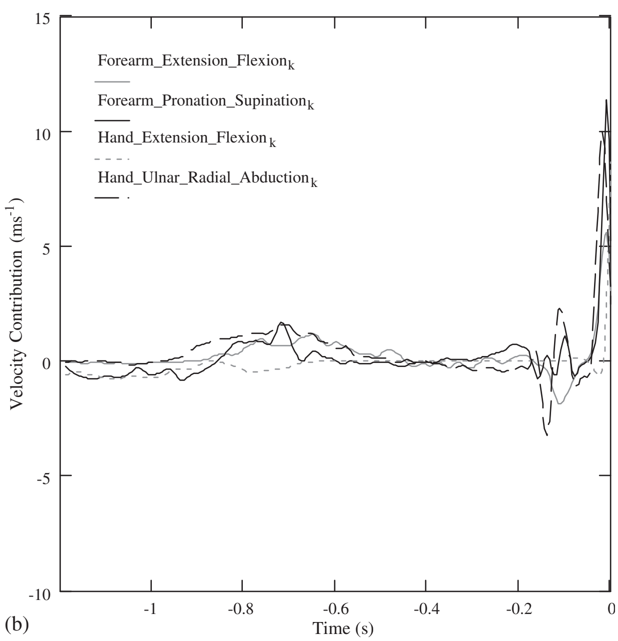


Fig. 6. Contribution of (a) torso and upper arm rotation, (b) forearm rotation and hand to the forward velocity of club-head speed.

the video data, may be larger than the errors introduced by the simpler process in this study, which involved instrumentation and subsequent differentiation only. It is also easier for the coach/athlete to understand joint angular velocities as compared to acceleration. A subject's quantified swing can be compared to expert golfers', thus enabling identification of areas for remedy. The present method is also simpler and makes result extraction easier than the image methods used by Sprigings et al. (1994). Therefore, the present method can serve as a viable method of ascertaining the

Table 1
Angular velocity and individual contributions to the resultant velocity at ball impact

	Resultant club-head velocity	Torso	*Upper arm: retroversion+/ anteversion–*	*Upper arm: adduction +/ abduction–*	*Upper arm: internal rotation+/ external rotation–*	*Forearm: extension+/ flexion-*	Forearm: pronation+/ supination–	Hand: extension+/ flexion –	Hand: ulnar abduction+/ radial abduction–
Average of angular velocity (rad s^{-1})	38.8	11.8	−1.1	−4.0	−9.6	4.2	−8.2	12.0	4.9
Average of individual contribution of linear velocity on the club-head resultant velocity ($m s^{-1}$)(%)	47.4	16.0	−0.1	6.0	5.5	4.2	4.6	7.6	3.5
	(100%)	(33.8%)	(−0.2%)	(12.7%)	(11.6%)	(8.9%)	(9.7%)	(16.0%)	(7.4%)

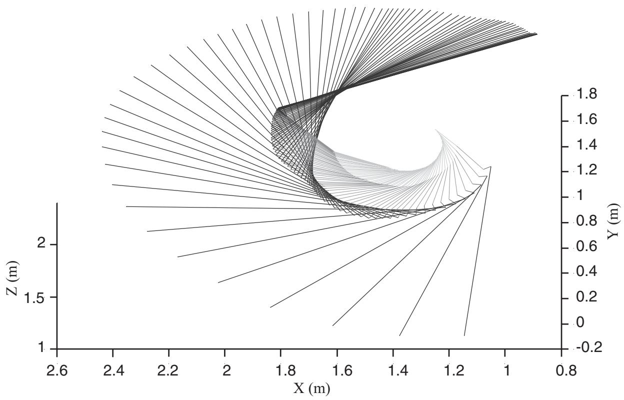


Fig. 7. Swing action (consisting of upperarm, forearm, hand and club) of the golfer from downswing to impact.

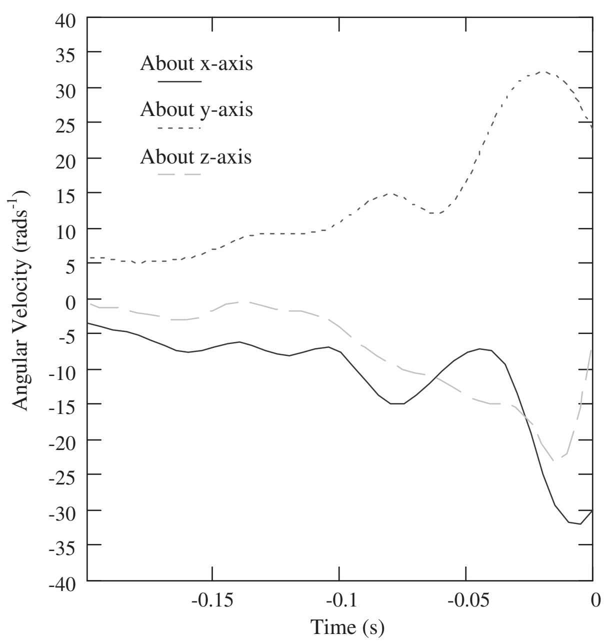


Fig. 8. Angular velocity of the club-head in the club-head coordinate system.

contribution of segmental rotations to the club-head velocity.

Although the method gives satisfactory results, there were some differences between the measured club-head velocity and the calculated one. The main discrepancy occurs at 0.1 s before the impact. This may be due to the placement of the goniometer at the shoulder. For the measurement of the upper arm adduction/abduction, a goniometer was attached to the acromion process and to the upper arm, below the deltoid attachment. This placement provides the measurement of the upper arm adduction/abduction, an action due to the contraction of the deltoid and suprasinatus muscles. At about 0.1 s before the impact, the outward rotation of the scapula was initiated to abduct the arm above the shoulder joint to execute the swing. This movement was not detected by the goniometer but the abduction was noted in the video analysis. Therefore, in Fig. 3, the two curves match initially but when the scapula rotation kicked in, it was not detected by the goniometer and hence the deviation in the two curves became apparent. This problem (illustrated in Fig. 4.) can be alleviated if the goniometer is attached to the lower side of the arm, from the medial side of the right upper arm to the breastbone, under the armpit. In this way, the goniometer reading will provide a measurement of the adduction/abduction that is generated by the deltoid and suprasinatus muscles as well as the scapula rotation. These results indicated somewhat fundamental limitations on usage of goniometers in capturing human motion even though they are considered as convenient tools, whose reliability largely depends on testers' skills to yield consistent results.

The other small differences may also be due to the errors of the measured velocity that was processed with image analysis using cameras of 200 Hz. The fact that the shaft of the club, which flexed considerably but was modeled as a rigid rod in the algorithm, may also contribute to the deviation of the computed and measured displacements and velocities of the club head. The contribution of the torso rotation, also calculated using the video analysis, may also introduce some errors to the analysis. Another limitation of the study is that the EGMs were attached to the subject's skin, and as Lu and O'Connor (1999) discovered, the skin movement may affect accuracy of the data significantly. Besides the skin movements, the muscles could also displace goniometers. Hence, for the use of the above-mentioned EGM method to acquire accurate rotation measurements, detailed calibration and careful positioning of the goniometers are required.

6. Conclusion

In our study, the presented method provides a convenient assessment of golf-swing effectiveness. The presented method can also be applied to other sports to examine segmental rotations. In general, this method facilitates the study of human motion with relative ease. The use of a biomechanical model in conjunction with the dual-number coordinate transformation for motion analysis was shown to provide accurate and reliable results. In particular, the advantage of using the dual Euler angles based on the dual-number coordinate transformation approach is that they allow for a complete 3D motion representation using six parameters for each anatomical joint. This method has proved to be an effective means to examine high-speed movement in 3D space. It also provides an option in assessing the contributions of the individual segmental rotations in production of the relevant velocity of the end-effector.

Acknowledgements

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Appendix A. Algebra for dual angles and dual transformation

Consider a rigid body moving from the initial position, where the local coordinate system M on the rigid body coincides initially with the global coordinate system G, to another position in space by rotating about and translating along the X-axis of G as shown in Fig. A.1. The screw motion displacement of the rigid body through the X-axis can be expressed in the form as $\hat{\alpha} = \alpha + \varepsilon a$, called dual angle, in which 'α' represents the rotation angle about the X-axis and 'α' represents the translation distance along the X-axis. In the above study, the rotation angle is taken from goniometer data and the translation distance is either zero or the segmental length.

At the initial position, a line vector on the rigid body is expressed in the form $\hat{\mathbf{V}}_0 = \mathbf{V}_0 + \varepsilon \mathbf{W}_0$, where $\mathbf{V}_0$ represents the magnitude and direction of the vector with respect to G, and $\mathbf{W}_0 = \mathbf{r} \times \mathbf{V}_0$, where $\mathbf{r}$ is a vector connecting the origin of G to any point on the line on which the vector lies. After the screw motion through the X-axis, the same vector moves to position 2. At the final position, the vector can be represented similarly as $\hat{\mathbf{V}} = \mathbf{V} + \varepsilon \mathbf{W}$. Then the vector satisfies the following dual transformation relationship:

$$\hat{\mathbf{V}} = \left[\hat{R}_X(\hat{\alpha})\right]\hat{\mathbf{V}}_0, \tag{A.1}$$

where

$$\left[\hat{R}_X(\hat{\alpha})\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\hat{\alpha} & -\sin\hat{\alpha} \\ 0 & \sin\hat{\alpha} & \cos\hat{\alpha} \end{bmatrix}$$

is the dual transformation matrix.

In the same manner, the dual transformation matrices for a screw motion with respect to the Y-axis with a dual angle $\hat{\beta} = \beta + \varepsilon b$ and with respect to the Z-axis with a dual angle $\hat{\gamma} = \gamma + \varepsilon c$ are

$$\left[\hat{R}_Y(\hat{\beta})\right] = \begin{bmatrix} \cos\hat{\beta} & 0 & \sin\hat{\beta} \\ 0 & 1 & 0 \\ -\sin\hat{\beta} & 0 & \cos\hat{\beta} \end{bmatrix}$$

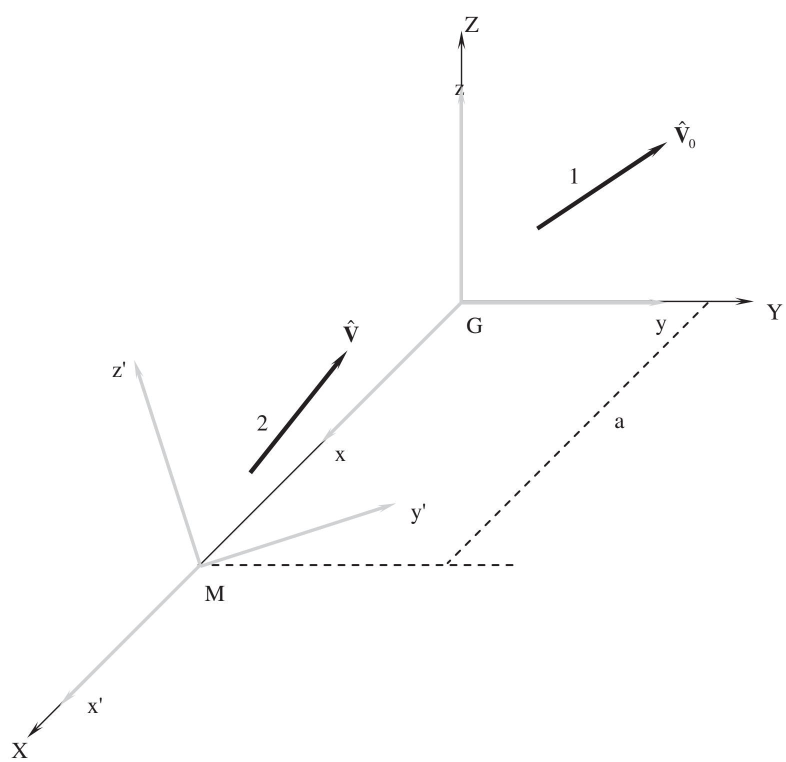

Fig. A.1. Screw motion through the X-axis.

and

$$\left[\hat{R}_Z(\hat{\gamma})\right] = \begin{bmatrix} \cos\hat{\gamma} & -\sin\hat{\gamma} & 0 \\ \sin\hat{\gamma} & \cos\hat{\gamma} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

respectively.

A general spatial motion of a rigid body moving in space can be considered as three successive screw motions about the axes of the coordinate system on the rigid body or the global coordinate system. Any sequence of screw motions about the axes can be chosen as long as the same axis is not repeated consecutively. The resultant dual transformation matrix $[\hat{R}]$ is the combination of all three dual transformation matrices. In this study, screw motions about the coordinate system which are fixed on the moving segment are used to describe joint motion, and the three dual angles expressing the screw motions about the axes of the moving segment coordinate system are called dual Euler angles.

Similar to the Euler angles method, the sequence of screw motions is important in the dual Euler angles method. For example, if the sequence of screw motions is chosen first with respect to the z-axis, then with respect to the new y-axis, and finally with respect to the new x-axis, the resultant dual transformation matrix can be obtained by

$$\left[\hat{R}\right] = \left[\hat{R}_z(\hat{\gamma})\right]\left[\hat{R}_{y'}(\hat{\beta})\right]\left[\hat{R}_{x''}(\hat{\alpha})\right]. \tag{A.2}$$

Similar to the ordinary transformation matrix in the Euler angles method, the dual transformation matrix is orthogonal, that is $[\hat{R}][\hat{R}]^{\mathrm{T}} = I$.

Appendix B. Origin-displacement equation

Consider two frames {A} and {B}, let

$\mathrm{R} = 3 \times 3$ primary-number matrix describing a change in orientation of frame {B} relative to frame {A}.

$\vec{D} = 3 \times 1$ primary-number vector describing displacement of the origin of frame {B} relative to frame {A}.

$[D] = 3 \times 3$ cross-product matrix of translational vector $\vec{D}$.

$[I] = 3 \times 3$ identity matrix.

From the definition of a dual vector, the dual transformation matrix of frame {B} relates to frame {A} as follows:

${}^{\mathrm{B}}_{\mathrm{A}}\hat{M} = ([I] + \varepsilon[D])R$

which can also be decomposed into primary and dual components

${}^{\mathrm{B}}_{\mathrm{A}}\hat{M} = U + \varepsilon V.$

Equating the primary and dual components, we obtain

$U = [R],$

$V = [D][R]$

which implies

$$[D] = VU^{\mathrm{T}}. \tag{B.1}$$

Therefore, given the dual transformation matrix, $\hat{M} = U + \varepsilon V$ between two frames, we can recover a 3×3 cross-product matrix, which forms the displacement vector between the origins of two frames.

References

Chao, E.Y.S., 1980. Justification of the triaxial goniometer in the measurement of joint rotation. Journal of Biomechanics 13, 989–1006.

Craig, J.J., 1989. Introduction to Robotics: Mechanics and Control. Addison-Wesley, Reading, MA.

Denavit, J., Hartenberg, R.S., 1955. A kinematic description for lower pair mechanisms based on matric. Journal of Applied Biomechanics 22, 215–221.

Fischer, I.S., 1999. Dual-Number Methods in Kinematics, Statics, and Dynamics. CRC Press, Boca Raton, FL.

Grood, E.S., Suntay, W.J., 1983. A joint coordinate system for the clinical description of three-dimensional motions: application to the knee. ASME Journal of Biomechanical Engineering 105, 136–144.

Hsia, L.M., Yang, A.T., 1981. On the principle of transference in three-dimensional kinematics. ASME Journal of Mechanical Design 103, 652–656.

Kuipers, J.B., 1999. Quaternion and Rotation Sequences. Princeton University Press, Princeton, NJ.

Lu, T.W., O'Connor, J.J., 1999. Bone position estimation from skin marker coordinates using global optimization with joint constraints. Journal of Biomechanics 32, 129–134.

McCarthy, J.M., 2000. Geometric Design of Linkages. Springer, New York.

Penner, A.R., 2003. The physics of golf. Reports on Progress in Physics 66, 131–171.

Savitzky, A., Golay, M., 1964. Smoothing and differentiation of data by simplified least square procedures. Analytical Chemistry 36, 1627.

Sprigings, E.J., Neal, R.J., 2000. An insight into the importance of wrist torque in driving the golfball: a simulation study. Journal of Applied Biomechanics 16, 356–365.

Sprigings, E., Marshall, R., Elliott, B., Jennings, L., 1994. A three-dimensional kinematic method for determining the effectiveness of arm rotations in producing racquet-head speed. Journal of Biomechanics 27, 245–254.

Umeyama, S., 1991. Least-squares estimation of transformation parameters between two points patterns. IEEE Transactions on Pattern Analysis and Machine Intelligence 13, 376–380.

Vaughan, C.L., Hay, J.C., Andrews, J.G., 1982. Closed loop problems in biomechanics. Part 1, A classification system. Journal of Applied Biomechanics 15, 197–200.

Wong, Y., Kim, W., Ying, N., 2005. Passive motion characteristics of the talocrural and the subtalar joint by dual Euler angles. Journal of Biomechanics in print.

Ying, N., Kim, W., 2002. Use of dual Euler angles to quantify the three-dimensional joint motion and its application to the ankle joint. Journal of Biomechanics 35, 1647–1657.

Ying, N., Kim, W., 2005. Determining dual Euler angles of the ankle complex in vivo using 'Flock of Birds' electromagnetic tracking device. Journal of Biomechanical Engineering 127, 98–107.

Ying, N., Kim, W., Wong, Y., Kam, B.H., 2004. Analysis of passive motion characteristics of the ankle joint complex using dual Euler angle parameters. Clinical Biomechanics 19, 153–160.

Zatsiorsky, V.M., 1998. Kinematics of Human Motion. Human Kinetics, Champaign, IL, USA.