The influence of moment of inertia on baseball/softball bat swing speed

Keith Koenig, Nan Davis Mitchell, Thomas E. Hannigan, J. Keith Clutter

Department of Aerospace Engineering, Mississippi State University, USA

Abstract

The speed at which a player can swing a bat is central to the games of baseball and softball, determining, to a large extent, the hit speed of the ball. Experimental and analytical studies of bat swing speed were conducted with particular emphasis on the influence of bat moment of inertia on swing speed. Two distinct sets of experiments measured the swing speed of college baseball and fast-pitch softball players using weighted rods and modified bats. The swing targets included flexible targets, balls on a tee and machine pitched balls. Internal mass alterations provided a range of inertial properties. The average measured speeds, from 22 to 31 m s^{-1}, are consistent with previous studies. Bat speed approximately correlates with the moment of inertia of the bat about a vertical axis of rotation through the batter's body, the speed generally decreasing as this moment of inertia increases. The analytical model assumes pure rotation of the batter/bat system about a vertical axis through the batter's body. Aerodynamic drag of the batter's arms and the bat is included in the model. The independent variable is bat moment of inertia about the rotation axis. There is reasonable agreement between the model and the measured speeds. Detailed differences between the two suggest the importance of additional degrees of freedom in determining swing speed.

Keywords: baseball, softball, bat, swing, speed, inertia

Nomenclature

Lower case	
$c_d(x)$	local drag coefficient
$d(x)$	local diameter
$\mathrm{d}F(x)$	local aerodynamic force
$\mathrm{d}M(x)$	local aerodynamic moment
t	time
x	axial location along bat measured from handle end
x_{cg}	axial location of bat centre of gravity from handle end
Upper case	
C_D	effective bat drag coefficient
D	effective bat diameter
I	moment of inertia of batter/bat system about effective batter axis of rotation = $I_{\text{batter}} + I_{\text{bat}}$
I_{bat}	moment of inertia of bat about batter axis of rotation; same as I_{body}

Correspondence address:
Keith Koenig
Department of Aerospace Engineering
Mississippi State University
Mississippi State, MS 39762, USA
Tel: +1-662-325-7286
Fax +1-662-325-7730
E-mail: koenig@ae.msstate.edu

I_{batter}	moment of inertia of batter about his/her axis of rotation
I_{body}	moment of inertia of bat about batter axis of rotation; same as I_{bat}
I_{cg}	moment of inertia of bat about bat centre of gravity
K_D	aerodynamic moment coefficient
M	net aerodynamic moment
L	length of bat
R	radius from axis of rotation to speed measurement location
S1, S2	speed sensor arrays in study 1 (Fig. 1)
T	tip location arrays in study 1 (Fig. 1)
T	batter torque
$V(x)$	local bat velocity
V_{bat}	swing speed, bat speed at measurement location
Greek	
θ	bat rotation angle
ρ	air density
ω	bat angular velocity

Introduction

The speed at which a ball leaves a bat, the hit speed, is a fundamental element in both the play and the risk of the games of baseball and softball. Hit speed depends in complex ways on numerous factors, among which the bat swing speed plays a prominent role. In particular, holding all other factors constant, hit speed is directly proportional to the speed of the bat contact point at impact, as demonstrated in Kirkpatrick (1963), Brody (1986) and Hester & Koenig (1993). Bat swing speed is also a factor in the ability of the player to properly position a bat, temporally and spatially, during a swing. Bat control depends largely on the ability of the player to accelerate (or decelerate) the bat. As the time integral of acceleration through the swing, bat speed thus represents a measure of how well a player can locate the bat at the proper place at the proper time. Clearly, the speed at which a player can swing a bat is central to baseball and softball.

Recent experimental studies of bat swing speed include Bahill & Karnavas (1989), Welch *et al.* (1995), Crisco *et al.* (1999) and Fleisig *et al.* (2002). The test subjects in these previous studies were skilled adult baseball and fast-pitch softball players. In most cases, speeds were measured at a point on the bat near its centre of gravity and at a position in the swing near that for bat/ball contact. Reported average speeds are generally in the range of 25 to 29 m s^{-1} for bats with masses close to typical playing mass. Bahill & Karnavas (1989) also include data for very lightweight bats for which average swing speeds greater than 31 m s^{-1} were observed.

Various studies have also addressed analysis and modelling of bat swing processes. Adair (1990) and Welch *et al.* (1995) present descriptions of the motions involved in swinging a bat and representative values for key physical features of a swing. Based on the measurements of Bahill and Karnavas (1989), Bahill and Freitas (1995) provide an empirical correlation between optimum bat mass and player size. Analysis of the momentum and energy transfer during a bat/ball collision and prediction of the subsequent final ball speed requires knowledge of the rotational and linear motions of the bat prior to the collision. Such analyses appear, for example, in Kirkpatrick (1963), Brody (1986), Brancazio (1987) and Watts & Bahill (2000). The mechanisms leading to bat speed and the role of bat speed in producing final ball speed are clear from these analyses.

The measurements, descriptions and correlations presently available concentrate mainly on effects of bat mass on swing speed. However, since the bat motion is primarily one of rotation, then moment of inertia must also be a fundamental factor in determining speed. It is the purpose of the present paper to examine the influence of bat moment of inertia on swing speed through swing speed measurements and an analytic model.

Two experimental swing speed studies – the first conducted in 1989 and the second in 1997 – will be described. These experiments served as the motivation for the present analytical model. These studies will be denoted from here on as 1 and 2 or as first and second, respectively. Some portions of study 1 appear in Clutter (1989) and Clutter & Koenig (1999). Similarly, Koenig *et al.* (1997) and Davis (1998) describe portions of study 2.

The analytic model presented here is a single degree-of-freedom representation of the swing process.

The purpose of this model is to provide a framework for visualizing the role played by the bat moment of inertia in determining bat swing speed. The model also includes the effects of aerodynamic drag. Although simplistic, the representation described here begins to bridge the gap between qualitative and empirical descriptions and fully predictive physical models.

Experiments

Studies 1 and 2 involved speed measurements of bats swung by experienced players in controlled hitting situations. In study 1 male collegiate baseball players used production bats and weighted rods. Female collegiate fast-pitch softball players as well as male collegiate baseball players participated in study 2 and used production and modified production bats. The basic measurement in both investigations was the time of flight of the bat between vertically oriented light beams and light sensors. Bat speeds reported here were calculated by dividing the distance between sensors by this time of flight.

Experiment 1 Details

In the first study, data were obtained from three array units of upward-looking light sensors illuminated by overhead sources. A schematic of one of the array units appears in Fig. 1 and a sketch of the three units in the system configuration appears in Fig. 2.

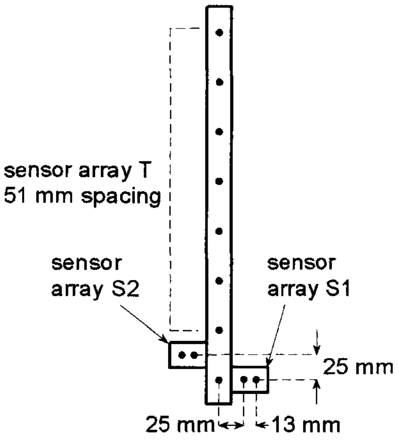

Figure 1 Sensor array for study 1

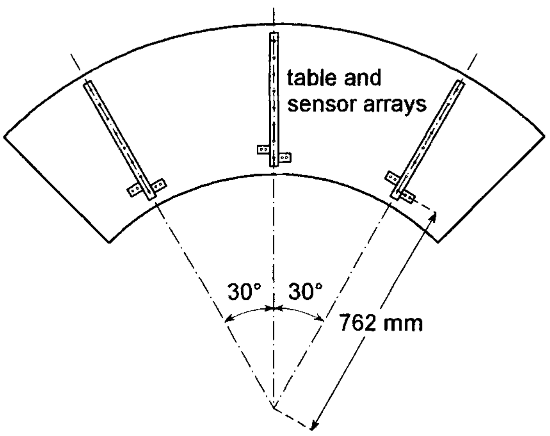

Figure 2 Sensor array layout for study 1

The light source consisted of a 600 W incandescent bulb and three 1.2 m parabolic mirrors mounted 1.5 m above the sensors. This light arrangement produced three sheets of nearly parallel light rays, one sheet for each sensor array unit. The sensors were semiconductor-based photodetectors. In each unit (Fig. 1), sensor array T provided information on the location of the tip of the bat, while arrays S1 and S2 measured bat speed and provided information on bat angle in the horizontal plane. All sensors had 3.7 mm receiving lens diameters and response times typically less than 50 ms, and were recessed 50 mm below the surface of the arrays. The passages to the sensors in array T were 4.8 mm in diameter, while those in the S arrays were 1.6 mm in diameter. The T sensors were sampled at 2 kHz and the S sensors were sampled at 250 kHz. Uncertainty in the time at which a bat began to cover or uncover a sensor was ± 1 sample period and the uncertainty in the measured transit time between two beams was ± 2 sample periods. The switching times of the various elements in the data acquisition circuit were less than 1 μs.

Overall uncertainty for the speed of an individual swing was the result of uncertainties due to sensor and circuit response times, discrete sampling and geometry of the beams and sensors. These sources were essentially independent. The overall uncertainty (root-sum-square) for the speed of an individual swing was 4.4% of the reported measurement.

Testing took place in an indoor laboratory. Players swung at a short, flexible target located near the centre sensor array. The height of this target was adjusted to allow each participant to swing naturally.

Thirteen bat or bat-like configurations were used. Three of these were production bats, one each constructed of aluminum, wood and graphite. The other ten test implements were based on a thin-wall aluminum tube to which various combinations of weights could be added. The tube and weights were designed so that any one of the bat's principal inertial properties – mass, centre of gravity and moment of inertia about the centre of gravity – could be varied while keeping the other two constant, or nearly constant.

The principal inertial properties of the bats are presented in Table 1. Bats 1–3 have nearly the same values for x_{cg} and I_{cg}, with mass being the variable property. Although the moment of inertia for bats 1–3 is clearly not constant, it varies by only 5.1% (highest to lowest values), while the mass varies by 32%. Similarly, bats 4–6 have variable x_{cg} with nominally constant values for mass and I_{cg}. Again, I_{cg} actually varies, this time by 15%, but the variation in centre of gravity location is much greater, at 50%.

Seven collegiate male baseball players participated in the tests. Players were typically tested in groups of two or three. For each bat, data for five swings by first one player and then the other (and then the third) were obtained. The production bats were used first in order to let the players become comfortable with the test environment. Measurements with the bare tube and then the tube with weights followed. This ordered, rather than random, sequence was adopted to minimize the time that the players were required to be in the laboratory.

The results reported here are bat speeds for a point approximately 0.15 m from the bat tip measured by the middle array unit (Fig. 2). Individual and group averages and standard deviations appear in Table 2.

Table 1 Bat properties in study 1

	Mass (kg)	x_{cg}(m)	I_{cg}(kg-m^2)	I_{body}(kg-m^2)
Wood	0.901	0.565	0.0565	0.739
Aluminum	0.851	0.540	0.0638	0.671
Graphite	0.798	0.514	0.0531	0.588
Test rod	0.350	0.432	0.0206	0.211
1	0.724	0.508	0.0691	0.547
2	0.838	0.508	0.0726	0.627
3	0.959	0.508	0.0708	0.705
4	0.839	0.406	0.0673	0.492
5	0.841	0.508	0.0718	0.628
6	0.843	0.610	0.0776	0.783
7	0.954	0.406	0.0678	0.551
8	0.950	0.406	0.0982	0.579
9	0.952	0.406	0.130	0.611

Experiment 2 Details

Seven 4.2 mW lasers provided the light for the second experiment. The lasers were positioned so that each illuminated a corresponding sensor mounted in a regulation-size home plate as shown in Fig. 3. The lasers were 2.4 m above the plate. Uncertainty in beam spacing was estimated to be ± 0.2 mm about a nominal value of 50 mm.

Semiconductor-based photodetectors like the ones in the first study were set in the home plate so that each receiving lens was approximately 6 mm below the plate surface and was aligned with the corresponding laser. Sensor outputs were sampled at 50 kHz by a PC-based digital oscilloscope and data acquisition system. Uncertainties in sampling and switching times were the same as those in study 1.

Overall uncertainty for the speed of an individual swing was the result of uncertainties due to sensor and circuit response times, discrete sampling and

Table 2 Speeds measured in study 1

Player/ bat	Wood	Alum	Graph	Rod	1	2	3
A	17.2	17.5	20.9	26.8	21.1	19.8	18.9
B	26.1	26.2	28.1	36.7	24.8	24.5	22.9
C	24.8	25.0	25.8	28.9	26.3	24.1	22.8
D	26.0	26.3	27.1	31.0	25.8	25.4	25.0
E	26.9	28.3	30.2	34.2	26.1	25.6	28.5
F	25.3	26.5	25.8	29.4	24.5	23.6	22.4
G	28.4	27.9	31.5	33.7	27.3	25.4	25.1
Average	25.0	25.4	27.1	31.5	25.2	24.1	23.7
s	3.6	3.6	3.5	3.5	2.0	2.0	3.0

Player/ bat	4	5	6	7	8	9
A	25.1	22.8	18.9	19.3	16.8	20.2
B	27.2	24.6	22.2	24.7	25.5	25.7
C	27.5	24.9	21.6	25.2	22.9	23.0
D	27.1	24.7	23.4	25.9	25.1	25.3
E	33.0	28.9	26.1	28.9	28.8	28.7
F	24.7	22.5	20.3	24.5	24.1	23.7
G	28.7	26.6	24.4	27.9	29.2	26.9
average	27.6	25.0	22.4	25.2	24.6	24.8
s	2.8	2.2	2.5	3.1	4.2	2.8

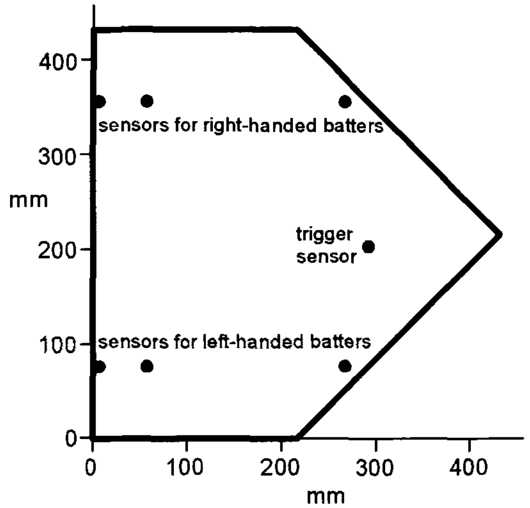

Figure 3 Sensor locations in plate for study 2 (sensor size exaggerated). Sensors at 7 and 58 mm horizontally give speed. Sensors near 270 mm horizontally give bat inclination and data acquisition trigger information.

geometry of the beams and sensors. These sources were essentially independent. The overall uncertainty (root-sum-square) for the speed of an individual swing was 3.3% of the reported measurement.

Testing took place in an indoor batting facility. Players hit off a tee and also hit machine-pitched balls. For the men, the pitching machine was 14.6 m from the plate and the average pitch speed was 28.6 m s^{-1}. The corresponding values for the women were 12.2 m and 22.4 m s^{-1}. These conditions gave ball flight times of 0.51 s for the baseballs and 0.55 s for the softballs. Based on the distance from the pitcher's rubber to home plate – 18.4 m for collegiate baseball and 14.6 m for collegiate softball – the test flight times were approximately equivalent to pitch speeds of 36 m s^{-1} and 27 m s^{-1} on actual baseball and softball fields, respectively.

Six bat configurations were used for each of the baseball and softball tests. The bats were production aluminum models to which copper weights were added to the handle or barrel ends to adjust net mass and moment of inertia. Principal inertial properties of the bats appear in Table 3. The nomenclature follows that of Fleisig *et al.* (2002) since five of the bats in study 2 were also used in the experiments conducted by Dr Fleisig and colleagues. The bat labelled 'team' is an unmodified bat actually used by the participants.

Twenty collegiate male baseball players and ten collegiate female fast-pitch softball players participated in the tests. Players were typically tested in pairs. For each bat and each ball setting (tee or pitch), data for five acceptable swings by first one player and then the other were obtained. The order of bats used was random. Data were recorded from each player for five good swings for a particular bat and ball setting. The high and low values within these five were discarded and then the average across all participants for each bat/ball combination was determined.

The results reported here are from the sensor pairs in Fig. 3 for right- and left-handed batters. Individual and group averages and standard deviations appear in Table 4.

Experimental results

To interpret the measured speed as the swing speed required the assumption that during a given swing the same part of the bat crossed each light sensor. This was not directly verified in these experiments. However, in both studies the pairs of sensors used to measure speed were spaced less than one bat barrel diameter. Adair (1990) and Fleisig *et al.* (2002) showed that within any azimuthal translation of one bat diameter during a swing, the radial motion of a bat is quite small. Treating the quotient of sensor spacing and time of flight as swing speed is therefore a reasonable interpretation.

Table 3 Bat properties in study 2

	Mass, kg	**Length, m**	x_{cg}(m)	I_{cg}(kg-m^2)	I_{body}(kg-m^2)
Baseball					
team	0.811	0.836	0.531	0.0556	0.622
B-30	0.853	0.864	0.531	0.0733	0.669
B-Hand++	0.924	0.856	0.490	0.0918	0.676
B-Hand+	0.899	0.856	0.508	0.0843	0.678
B-End+	0.918	0.859	0.561	0.0720	0.761
B-End++	0.947	0.859	0.569	0.0803	0.803
Softball					
team	0.694	0.838	0.406	0.0556	0.407
S-Hand+	0.697	0.831	0.411	0.0554	0.413
S-23	0.646	0.838	0.462	0.0424	0.423
S-Hand++	0.757	0.838	0.409	0.0606	0.446
S-End+	0.714	0.841	0.493	0.0560	0.510
S-End++	0.768	0.841	0.516	0.0605	0.577

Table 4 Speeds measured in study 2

Baseball/pitch setting

Player/ bat	E+	H+	30	H++	E++	team
A	–	–	–	–	–	–
B	25.3	23.9	20.7	29.2	22.9	31.0
C	–	–	–	–	–	–
D	26.4	26.6	29.9	29.7	29.0	27.6
E	27.6	27.9	29.8	28.3	25.6	30.3
F	29.9	30.8	29.5	33.1	25.0	34.4
G	20.7	19.0	24.3	24.3	16.5	31.0
H	33.4	29.1	31.6	31.8	29.0	31.4
I	22.2	22.4	20.3	24.2	23.1	23.4
J	25.5	32.1	31.0	30.3	28.0	28.6
K	22.0	29.9	18.6	33.5	21.4	20.7
L	34.1	33.8	31.0	30.4	29.6	31.0
M	29.8	29.4	27.3	32.3	21.0	–
N	17.4	26.2	28.8	24.9	19.1	28.0
O	27.7	30.0	27.4	22.7	22.3	22.4
P	20.5	17.9	29.1	21.3	21.0	29.7
Q	17.2	23.4	33.9	31.7	27.7	25.0
R	31.2	25.0	31.5	30.9	27.6	–
S	15.5	30.2	29.8	30.4	30.2	31.5
T	32.4	32.8	31.4	32.8	33.2	33.9
Average	25.5	27.2	28.1	29.0	25.1	28.7
σ	5.8	4.6	4.3	3.8	4.5	4.0
Number	54	54	54	54	54	48

Baseball/tee setting

Player /bat	E+	H+	30	H++	E++	team
A	29.4	31.6	30.8	31.7	32.1	31.5
B	27.0	29.4	30.0	31.7	30.4	29.5
C	33.3	34.2	34.1	31.2	33.0	34.2
D	32.1	34.5	–	–	–	–
E	–	24.9	24.5	27.9	24.6	27.0
F	28.7	31.7	30.1	33.7	31.6	30.8
G	20.9	22.9	23.4	21.5	23.5	25.5
H	35.8	31.8	32.9	31.9	31.2	33.3
I	23.8	20.3	18.6	23.8	23.0	23.3
J	30.0	31.4	30.1	27.4	29.3	29.9
K	32.8	30.2	28.7	34.6	29.6	30.8
L	25.5	26.1	25.8	25.6	25.9	25.3
M	32.1	34.2	34.2	32.7	31.4	32.9
N	28.8	25.6	27.2	27.2	29.9	29.6
O	24.1	24.8	25.5	27.1	23.8	23.9
P	28.8	30.8	28.0	27.4	29.4	28.3
Q	35.0	33.0	37.6	43.5	39.7	36.7
R	35.1	32.1	34.6	27.2	31.5	31.7
S	30.4	34.2	31.7	27.4	28.3	31.6
T	32.1	33.1	29.3	37.3	31.1	34.2
Average	29.8	29.8	29.3	30.1	29.4	30.0
σ	4.1	4.2	4.6	5.1	4.0	3.7
Number	57	60	57	57	57	57

Softball/pitch setting

Player/ bat	E+	H+	23	H++	E++	team
A	–	19.7	16.1	–	19.2	24.2
B	21.3	20.7	21.8	19.8	20.4	21.9
C	–	–	–	–	–	–
D	29.0	30.0	29.6	28.7	26.8	27.9
E	24.0	25.6	26.2	25.2	23.8	23.3
F	19.2	22.8	17.9	–	25.0	20.1
G	26.8	26.9	24.8	24.3	27.4	28.3
H	26.6	25.1	26.1	27.2	24.4	29.2
I	25.0	25.7	26.7	25.4	24.8	26.7
J	20.1	21.1	–	23.3	18.9	–
Average	24.0	24.2	23.7	24.8	23.4	25.2
σ	3.5	3.3	4.7	2.9	3.2	3.3
Number	24	27	24	21	27	24

Softball/tee setting

Player/ bat	E+	H+	23	H++	E++	team
A	25.4	26.3	25.8	25.2	25.4	23.9
B	21.9	29.9	24.4	25.8	24.8	24.6
C	19.4	20.7	18.4	18.1	21.4	–
D	26.3	26.7	25.1	27.0	29.9	29.0
E	23.1	22.3	22.3	21.8	20.5	21.0
F	27.4	26.7	26.6	29.6	25.2	25.8
G	24.6	24.8	25.1	26.3	23.7	23.3
H	26.6	28.7	27.6	26.3	28.3	27.6
I	24.2	25.0	24.9	24.6	23.2	25.8
J	24.0	25.8	25.7	24.8	23.9	–
Average	24.3	25.7	24.6	24.9	24.6	25.1
σ	2.4	2.7	2.6	3.1	2.9	2.5
Number	30	30	30	30	30	24

Fig. 4 presents the men's results from studies 1 and 2 plus those of Fleisig *et al.* (2002). Fig. 5 presents the women's results from the second study and from Fleisig *et al.* (2002). In both figures I_{bat} is the moment of inertia of the bat about an assumed axis of rotation. For all of the results presented here, this axis of rotation is taken to be 0.3 m from the bat handle end toward the player, and is based on observations during studies 1 and 2.

The velocity magnitudes presented in Tables 2 and 4 and Figs. 4 and 5 range from 22 to 31 m s^{-1}. These are consistent with speeds observed in previous studies, such as Bahill & Karnavas (1989), Welch *et al.* (1995), Crisco *et al.* (1999) and Fleisig *et al.* (2002). The basic trend in Figs. 4 and 5 is a general, but non-monotonic, decrease in swing speed as bat inertia increases. For both women and men, speeds were slower for swings at pitched balls than for swings at balls on the tee. There was more deviation from the general trend for swings at pitched balls for both groups of players. These slower and more variable bat

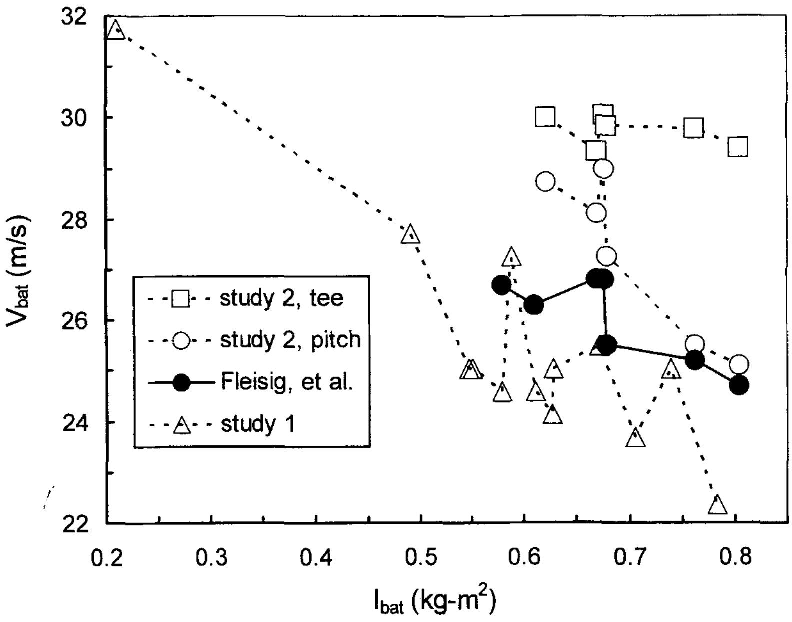

Figure 4 Men's results from studies 1 and 2 and Fleisig *et al.* (2002)

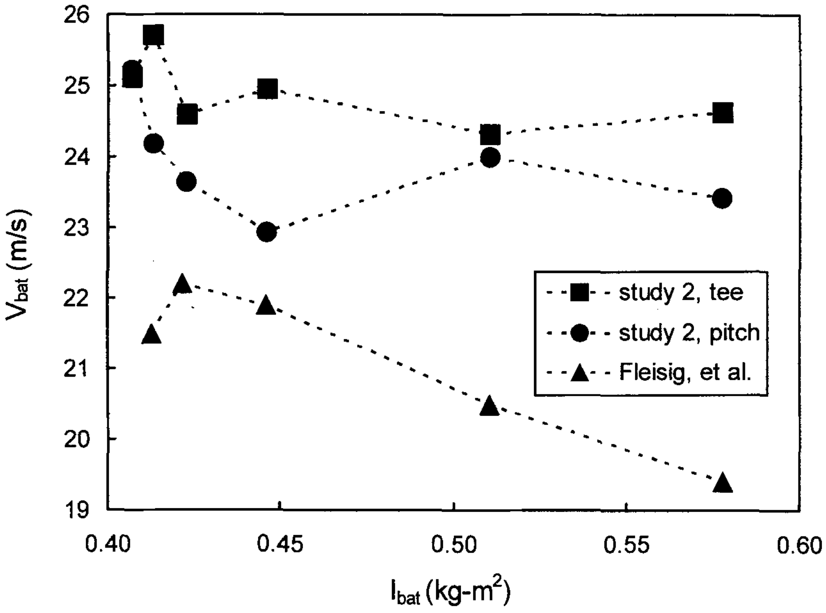

Figure 5 Women's results from study 2 and Fleisig *et al.* (2002)

speeds would appear to be largely a consequence of the greater amount of decision-making and adjustment required in order to hit a moving ball. The rather large variations in speed for swings at the pitched balls also suggest that bat properties in addition to I_{bat} may be playing a role in determining swing speed.

Analytical model

A framework for visualizing the role of bat inertial properties during the swing was the goal of the analysis here. For this purpose the swing was approximated by a pure rotation of the body and the bat about a vertical axis through the batter. This is admittedly a highly simplistic representation, but, as will be seen, captures important aspects of the swing. A torque is applied by the batter to accelerate his/her body and the bat. Inertia and aerodynamic drag impede the motion. Conservation of angular momentum applied to this situation is

$$I\frac{d\omega}{dt} = T - K_D\omega^2 \tag{1}$$

where $I = I_{\text{batter}} + I_{\text{bat}}$ is the moment of inertia of the batter/bat system, ω is the angular velocity, t is time and the batter applies torque T to the batter/bat system. The moment of inertia and torque are approximated in the present analysis as constants through the swing.

The quantity K_D is an aerodynamic parameter, which is shown below to be given by

$$K_D = \frac{1}{2}\rho C_D D \frac{L^4}{4} \ . \tag{2}$$

Here ρ is air density and C_D, D and L are representative values for drag coefficient, diameter and length of the arm/bat system (the principal components subject to aerodynamic drag) respectively. The parameter K_D is obtained from integration along the bat of the differential moments due to aerodynamic drag. The drag acting on an axial differential section, dx, is

$$dF(x) = \frac{1}{2}\rho V(x)^2 c_d(x) d(x) dx$$

where V, c_d and d are the local air velocity, section drag coefficient and diameter respectively, and x is the distance from the axis of rotation. Under the assumption of pure rotation with angular velocity ω, the linear velocity is

$$V(x) = \omega x \ .$$

Although the local section drag coefficient and diameter change along the arm/bat system, their variations are relatively small and in the opposite sense (d increasing, c_d decreasing with increasing x) over the outer portion of this system. Consequently, it is not too severe an approximation to let constants C_D and D replace $c_d(x)$ and $d(x)$, and this is done here.

The differential moment due to the force is

$$dM(x) = x\, dF(x) \ .$$

With the approximations above the differential moment is

$$dM(x) = \frac{1}{2}\rho\omega^2 x^3 C_D D\, dx$$

Integration from $x = 0$ to $x = L$, the distance from the axis of rotation to the end of the bat, gives

$$M = \frac{1}{2}\rho\omega^2 C_D D \frac{L^4}{4} = K_D\omega^2$$

This moment resists the rotation of the arm/bat system and appears as the negative quantity on the right-hand side of Eqn. (1).

Two integrations of Eqn. (1) with motion starting from rest yield ω and θ, the angle travelled, as functions of time. Eliminating time in these results yields

$$\omega = \sqrt{\frac{T}{K_D}} \tanh\left(\cosh^{-1}\left(\exp\left(\frac{K_D}{I_{\text{batter}} + I_{\text{bat}}}\theta\right)\right)\right) \tag{3}$$

When aerodynamic drag is negligible, then $K_D \approx 0$ and Eqn. (3) reduces to

$$\omega = \sqrt{\frac{2\theta T}{I_{\text{batter}} + I_{\text{bat}}}} \ . \tag{4}$$

The bat linear velocity is obtained by multiplying ω by a representative radius from the axis of rotation to the measurement station, that is, $V = \omega R$. For the use of Eqns. (3) and (4) here, the independent variable is the bat moment of inertia, I_{bat}. The quantities θ, K_D, T and I_{batter} are treated as parameters.

For the calculations here, θ is 135° for all cases. This is based on observations during the experiments and the examples shown in Adair (1990) and Fleisig *et al.* (2002).

Table 5 Parameter values for analytical model

Batter properties	**Study 1 men**	**Study 2 men**	**Study 2 women**	**Fleisig *et al.* (2002) men**	**Fleisig *et al.* (2002) women**
I_{batter}, kg-m^2	0.444	0.444	0.311	0.444	0.311
Pitch torque, N-m	–	316	164	282	130
Tee torque, N-m	246	375	178	–	–
Miscellaneous					
R* for *V	0.762	m			
θ	2.36	rad			
Cylinder length	0.914	m			
Diameter (study 1)	0.0508	m			
Diameter (study 2)	0.0699	m			
K_D (study 1)	0.00544	N-m-s^2			
K_D (study 2)	0.00748	N-m-s^2			
***R* handle**	0.305	m			
ρ	1.22	kg/m^3			
C_D	1.00	–			

The moment due to aerodynamic drag, K_D, is determined by approximating the batter's arms and the bat by a 0.91 m long constant diameter cylinder. The effective diameter of this cylinder is 0.051 and 0.070 m for studies 1 and 2 respectively. The drag coefficient is 1.0 (Hoerner, 1965) and air at standard sea level conditions is assumed. The radii from the axis of rotation to the handle end of the bat and to the measurement site are 0.31 m and 0.76 m respectively. Inserting all of these properties into Eqn. (2) gives K_D = 0.0054 and 0.0075 kg-m^2 for the first and second studies respectively.

At this point, the batter torque and moment of inertia are still to be determined. The approach here is to find values for these parameters from least square curve fits of Eqn. (3) to the experimental results. Values can be found for each experimental setting, for example study 2, men, tee or study 2, women, pitch. For the results to be presented here, only one value for I_{batter} is used for the men (studies 1 and 2) and only one value is used for the women. The men's moment of inertia is obtained by fitting Eqn. (3) to the data of study 1 with torque and batter moment of inertia as the unknown parameters. The value of I_{batter} obtained from this fit, 0.44 kg-m^2, is used as the moment of inertia for all men. The moment of inertia for the women is chosen as 70% of this, 0.31 kg-m^2, since the women weighed, on average, approximately 70% as much as the men. Study 1 is used as the basis for moment of inertia since this study covered a considerably larger range of bat moments of inertia and therefore provides greater resolution than study 2.

A different value of torque is used for each batting situation. The two-parameter fit of study 1 to Eqn. (3) that provides I_{batter} also gives the torque for study 1. For the other cases, a one-parameter curve fit of Eqn. (3) to the experimental data is performed with torque as the unknown parameter.

The parameters are summarized in Table 5. (Table 5 also includes parameters for the data of Fleisig *et al.* (2002) for later comparison.) Certainly, most of these quantities vary for each person, bat and individual swing. In fact, many of these vary through each swing as well, particularly the arm/bat system moment of inertia, which may change significantly as the batter's hands move radially during the swing. However, because the purpose of Eqn. (3) is to provide a framework for visualizing the role of bat inertial properties during the swing, the assumption of constant values for these parameters may not be too critical.

Analytical results

Data from the first study and the corresponding application of Eqn. (3) appear in Fig. 6 overleaf. The agreement between the measurements and the analytic prediction is amazingly good, with

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - y_i')^2}{\sum_{i=1}^{n} y_i^2 - n \cdot \bar{y}^2} = 0.9993 \qquad (5)$$

where the y_i are the measured values, the y_i' are the values from Eqn. (3) and $\bar{y}$ is the average of the measured data. That the computed curve passes through the lightest bat point is a result of the curve fit. Eqn. (3) is not forced to go through this point.

The scatter of the measured speeds about the theoretical curves in Fig. 6 is partly a consequence of degrees of freedom not included in the present model. Among the motions not included in the present analysis are rotation of the bat about the batter's wrist, rotations at the shoulders and elbows, and translation of the entire batter/bat system. An accounting for these motions requires inclusion of bat mass and bat moment of inertia about the wrist axis as additional independent variables.

Speeds predicted by Eqn. (3) for study 2 appear in Fig. 7. The analytical prediction generally follows the trend of the measurements, although there are significant detailed variations of the measured speeds about the computed curves. As with Fig. 6, these detailed variations may arise from contributions of body translation and rotation about the wrist that are not included here. On a larger scale, for both men and women Eqn. (3) slightly overestimates the effects of bat inertia for swings at the ball on a tee. The opposite appears to be the case for the men swinging at pitched balls with Eqn. (3) slightly under-predicting the inertia effect. For the women swinging at pitched balls the comparison is not clear. Additional measurements with bat inertias beyond the range included here are needed to properly assess the fidelity of the model for this case. Overall, however, Eqn. (3) does a reasonably good job of estimating the consequences of relatively large changes in I_{bat} on bat swing speed.

Eqn. (3) is compared to the data of Fleisig *et al.* (2002) in Fig. 8. The trend for the men is the same as displayed in Fig. 7. For the women, there is much better agreement between theory and measurements than in Fig. 7.

With respect to aerodynamic drag, Fig. 9 compares Eqns. (3) and (4) for study 2. The torque in

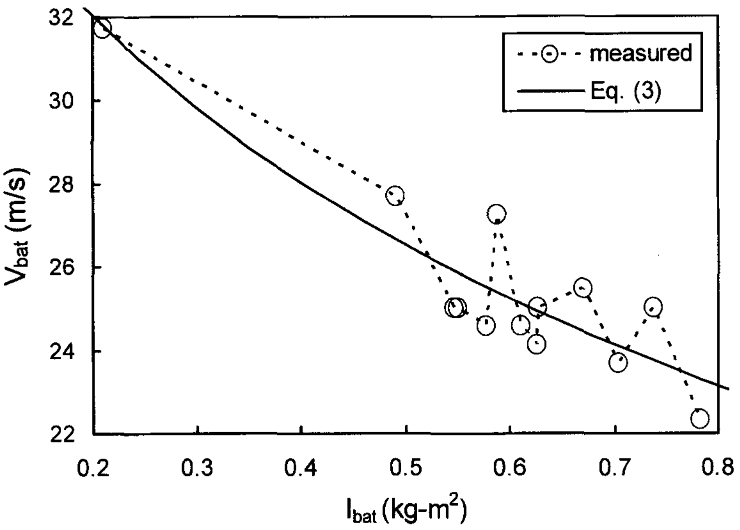

Figure 6 Eqn. (3) applied to results from study 1

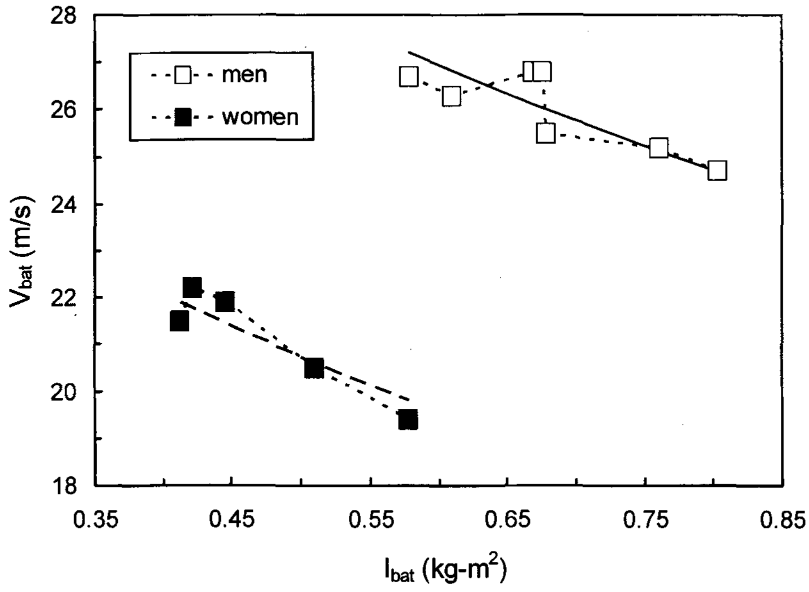

Figure 7 Eqn. (3) applied to results from study 2

Table 5 is used in Eqn. (4) so that Fig. 9 compares the swing speed with and without drag for the same batter input. Clearly, there is little difference between the no drag and drag cases. In all cases there is less than 1.5% difference between the no drag and drag speeds. This is less than half of the speed improvement reportedly obtained in tests for the patent of the dimpled bat (DiTullio, 1994). Aerodynamic drag apparently plays only a minor role in determining bat swing speed.

In view of the observation from Fig. 9 that aerodynamic drag is not an important factor, then Eqn. (4) represents the essential result of the analysis. Eqn. (4) was derived here through conservation of angular momentum. It can also be simply obtained by realizing that the final kinetic energy of the arm/bat system, $\frac{1}{2}I\omega^2$, is equal to the work done as the bat is swung. This work, for constant torque T, is $T\theta$ where θ is the angle through which the bat is swung. Replacing ω by V/R and solving for V gives Eqn. (4).

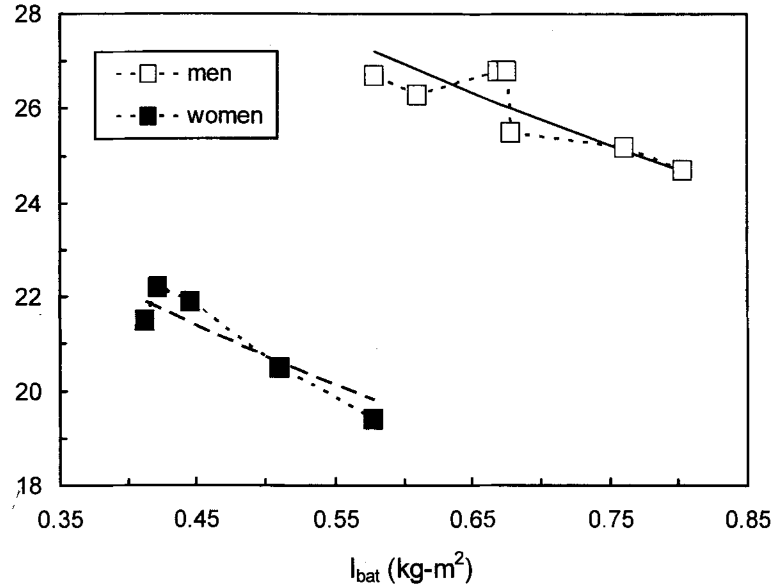

Figure 8 Eqn. (3) applied to results from Fleisig *et al.* (2002)

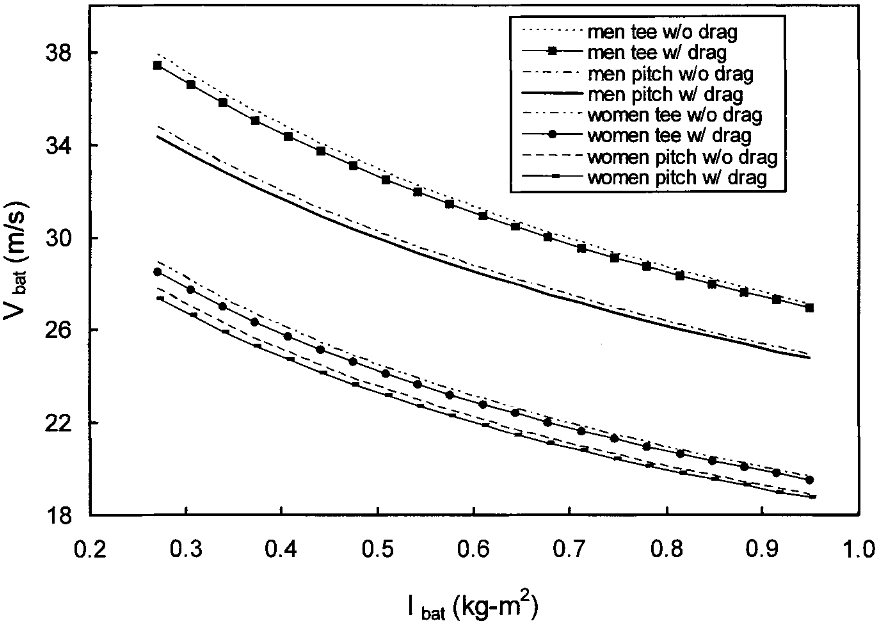

Figure 9 Theoretical results for study 2 without and with drag

Here the bat and batter share the energy input, $T\theta$, which is independent of the bat. A similar concept was presented by Adair (2003), but with the work done producing translational kinetic energy. The motion of the arm/bat system is perhaps more closely approximated by a rotation so that the present analysis may be preferable to that of Adair.

Fig. 10 presents excerpts from Figs. 4 and 5 along with results from Watts and Bahill (2000). The independent variable is bat mass rather than moment of inertia. The results from Watts and Bahill (2000), for two major league baseball players, are displayed here by simple empirical linear fits to their data. Although the five data sets in Fig. 10 differ in magnitude, they have similar overall slopes. Bat speed drops approximately 1 to 2 m s^{-1} as the mass increases from 0.8 to 1 kg.

As in the previous figures, detailed variations appear in Fig. (10). The numbers beside the data from study 1 for bats approximately 2.3 kg in mass and beside the data from study 2 for bats approximately 2.5 kg in mass are the corresponding values of I_{bat} (kg-m^2) for those bats. At least for these subsets, variations in the moment of inertia track the data spread. The speed clearly decreases as moment of inertia increases even though bat mass does not appreciably change.

Concluding remarks

The present measurements illustrate the complexity of the bat swing process. Multiple bat inertial properties, moment of inertia about a body axis and mass principal among them, play important roles in determining bat speed. Although there are not yet sufficient data available to accurately quantify these roles, the present results along with others cited here provide general measures of these roles.

The present analysis predicts relatively well the decrease in swing speed that occurs for moderately large increases in bat moment of inertia about a body axis. Inclusion of body translation and rotation about a wrist axis are important future steps. More accurate representations of player inertial properties and strength capabilities should also be pursued. Nevertheless, this model, based on pure rotation about a body axis, provides a useful framework for estimating bat swing speed.

Acknowledgements

Funding for this study was provided by the National Collegiate Athletic Association, T. Breidenthal monitor, by the Mississippi/NASA Space Grant Program and by the Department of Aerospace Engineering, Mississippi State University.

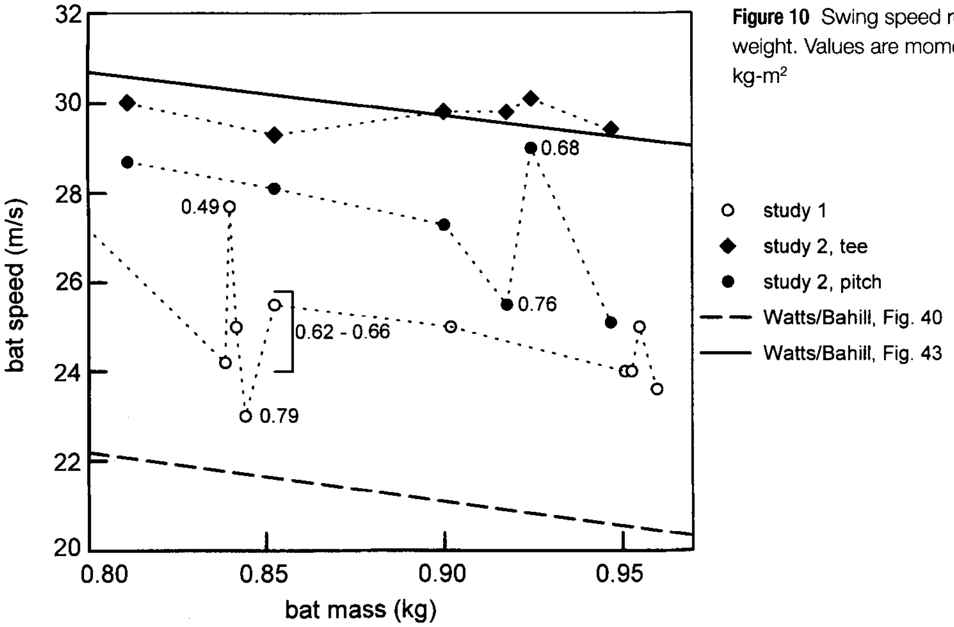

Figure 10 Swing speed relation to bat weight. Values are moments of inertia in kg-m^2

References

Adair, R.K. (2002) *The Physics of Baseball.* Harper & Row, NY.

Bahill, A.T. & Karnavas (1989) Determining ideal baseball bat weights using muscle force–velocity relationships. *Biological Cybernetics*, **72**, 89–97.

Bahill, A.T. & Freitas, M.M. (1995) Two methods for recommending bat weights. *Annals of Biomedical Engineering*, **23**, 436–444.

Brancazio, P.J. (1987) private communication. Department of Physics, Brooklyn College, City University of New York.

Brody, H. (1986) The sweet spot of a baseball bat. *American Journal of Physics*, **54**(7), 640–643.

Clutter, K. (1989) *Development and implementation of a baseball-bat swing testing system*, M.S. Thesis, Department of Aerospace Engineering, Mississippi State University.

Clutter, J.K. & Koenig, K. (1999) The effect of baseball-bat properties on swing speed. In: *Proceedings of the 1999 Bioengineering Conference (bound)*, BED-Vol. 42 (eds V.J. Goel, R.L. Spilker, G.A. Atheshian & L.J. Soslowsky), pp. 627–628. ASME, NY.

Crisco, J.J. (1997) *Final Report: NCAA Research Program on Bat and Ball Performance.* Providence, RI for the National Collegiate Athletic Association, Overland Park, KS.

Crisco, J.J., Greenwald, R.M. & Penna, L.H. (1999) *Baseball Bat Performance: A Batting Cage Study (Draft Report).* National Institute for Sports Science and Safety, Providence, RI.

Davis, E.N. (1998) *Effects of moment of inertia on swing speed of baseball and softball bats*, M.S. Thesis, Department of Aerospace Engineering, Mississippi State University.

DiTullio, J.C. (1994) *Reduced aerodynamic drag baseball bat*, United States Patent 5,284,332, February 8.

Fleisig, G.S., Zheng, N., Stodden, D.F. & Andrews, J.R. (2002) Relationship between bat mass properties and bat velocity. *Sports Engineering*, **5**, 1–8.

Hester, L.R. & Koenig, K. (1993) Performance measurements of baseball bats. *Journal of the Mississippi Academy of Sciences*, **38**, August/September, 7–10.

Hoerner, S.P. (1965) *Fluid-Dynamic Drag*. Published by the author, Brick Town, NJ.

Kirkpatrick, P. (1963) Batting the ball. *American Journal of Physics*, **31**, 606–613.

Koenig, K., Hannigan, T., Davis, N., Hillhouse, M. & Spencer, L. (1997) Inertial effects on baseball bat swing speed. In: *NCAA Research Program on Bat and Ball Performance, Final Report*, J.J. Crisco, Program Director, National Collegiate Athletic Association, Overland Park, KS.

Watts, R.G. & Bahill, A.T. (2000) *Keep Your Eye on The Ball.* W.H. Freeman And Co., New York.

Welch, C.M., Banks, S.A., Cook, F.F. & Draovitch, P. (1995) Hitting a baseball: a biomechanical description. *Journal of Orthopaedic Sports Physical Therapy*, **22**(5), 193–201.