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Optimization of pole characteristic in pole vaulting using three-dimensional vaulter model

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Abstract

Pole vaulting has progressed rapidly in terms of mechanical properties and appearance as a result of the use of a flexible fiberglass pole. Various researchers have modeled the vaulter body as a series of rigid-links, limited to a two-dimensional plane. In this research, a combination system consisting of a flexible pole and three-dimensional vaulter model which consists of a rigid-body link with a twist motion is presented. Here, the simulation analysis optimizes the pole characteristics and the joint torque with the application of a Genetic Algorithm. The control parameters were the joint torque and pole stiffness. From the results, it was clarified that there is optimal pole characteristic which is linked to the vaulter's initial velocity.

Keywords: Pole vaulting; Elastica; Simulation; Optimization; Genetic Algorithm

1. Introduction

Pole vaulting is a sport in which an athlete clears the highest jump heights achievable in a competition. A vaulter takes off and reaches a given height due to their kinetic energy and joint torque generated during the vault which is applied to the pole, and, as a result, gets height. Therefore, the height obtained changes greatly with combination of various parameters, such as takeoff speed, the pole characteristic, and torque of each joint produced by a vaulter. The purpose of this research is to determine the optimal solution of such a combination problem. However, since the joint torque actions change during pole vaulting, modeling the equations and using them as control parameters of an optimization problem is a difficult task to handle. One of the approaches taken to tackle the problem is the inverse approach. The analysis of inverse dynamics presumes each joint torque which should be generated by a pole vaulter using the information obtained from recordings of camera images of a pole vaulting competition and vaulter model [1]. These results were then used in order to identify the difference in the techniques used among pole vaulters. However, the torque histories obtained from inverse dynamics analysis is not necessarily the optimal torque, and it

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is not necessarily the optimal torque for all other vaulters as their physique and takeoff velocity differ appreciably. Moreover, the vaulter wants to know not only the torque but also how to select their pole.

Several authors have simulated the pole vaulting process in two-dimensional model [2][3][4]. In this paper, a three-dimensional model and a genetic algorithm are used to solve the optimization problem of the joint torque and pole stiffness distribution of pole vaulting. A complete system model is used to study the effects of different initial conditions and the effect of time variations in the control torques during the vault. The optimal joint torque history that a vaulter should generate and the optimal stiffness distribution are taken as an optimization problem that uses the height as the objective function using direct dynamics and a genetic algorithm.

2. Model

2.1. Model of pole-vaulter system

The schematic configuration of the pole-vaulter system model is given in Figure 1. The pole-vaulter model was represented by the large deflection pole and eight rigid links. In this paper, motion of the vaulter is assumed to be three-dimensional motion but the deformation plane of the pole is assumed to be two-dimensional for simplification. In addition the complexity of pole vaulting widens due to the loop formed by the vaulter's arms and the pole between his hand grip position. Since the torque generated by the vaulter's spread arms must be acting on the pole, the pole model must include this effect. The reaction forces acting at the left hand were replaced by a force-couple system acting at the upper end of the pole. Furthermore, since the mass of the pole is relatively small (2 - 4%) compared to that of the vaulter, the mass of a pole is neglected.

2.2. Large deformed multi-segments pole model

The pole model (shown in Figure 1) consists of large deformed multi-segments (segment number : N). The model was analyzed using nonlinear large deflection theory for a thin rod. The moment M_B and compressive force P generated by a vaulter are simultaneously applied to the pole (length L) at the upper end. In denoting the deflection

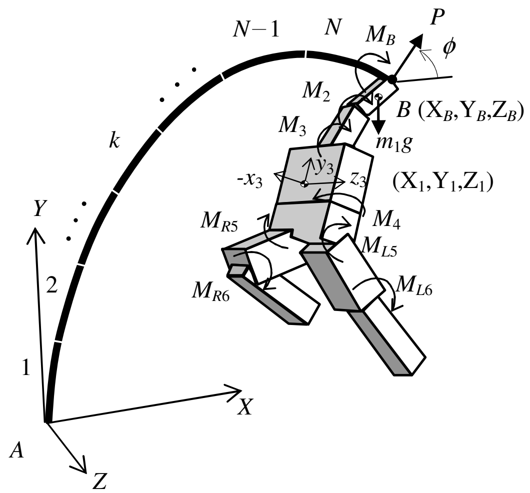

Figure 1. Schematic configuration of pole–vaulter system.

angle by θ, the arc length by s_k, the radius of curvature by R and the bending moment by M, the relationship between these parameters are given by:

$$1/R = M/(EI)_k = -d\theta/ds, \quad dx = ds\cos\theta, \; dy = ds\sin\theta \tag{1}$$

where $(EI)_k$ is stiffness of kth segment ,and E is the Young's modulus and I is the second area moment of the cross section about the neutral axis. In order to facilitate the discussion of large deformation behavior, the following non-dimensional variables are introduced:

$$\zeta_k = s_k/L, \; \xi = x/L, \; \eta = y/L, \; \lambda = PL^2/(EI)_N, \; \kappa_k = (EI)_1/(EI)k \tag{2}$$

The basic nonlinear equation for large deformation is derived as follows.

$$(d\theta/d\zeta_k)^2 = 2\,\kappa_k\,\lambda\cos(\theta-\phi) + C_k \tag{3}$$

Considering the boundary conditions at the upper end of the pole, i.e. the bending moment $d\theta/d\zeta_B = M_B L/(EI)_N = \beta$, the following equations are derived :

$$C_N = 2\,\kappa_N^2\,\lambda\cos(\theta-\phi) + \beta^2 \tag{4}$$

$$C_k = 2\,\kappa_k^2\,\lambda\{\cos(\theta_k-\phi)/\kappa_{k+1} - \cos(\theta_k-\phi)/\kappa_k\} + (\kappa_{k+1}/\kappa_k)^2\,C_{k+1} \tag{5}$$

The non-dimensional arc length at arbitrary position on the elastic curve is obtained by integrating equation (3).

$$\zeta_B = -\int_{\theta_A}^{\theta_1} d\zeta_1 - \sum_{k=1}^{N-2}\left(\int_{\theta_k}^{\theta_{k+1}} d\zeta_{i+1}\right) - \int_{\theta_{N-1}}^{\theta_B} d\zeta_N \tag{6}$$

Furthermore, the non-dimensional horizontal and vertical position ξ_B, η_B at the end of the pole is given by:

$$\xi_B = -\int_{\theta_A}^{\theta_1} d\xi_1 - \sum_{k=1}^{N-2}\left(\int_{\theta_k}^{\theta_{k+1}} d\xi_{i+1}\right) - \int_{\theta_{N-1}}^{\theta_B} d\xi_N \tag{7}$$

$$\eta_B = -\int_{\theta_A}^{\theta_1} d\eta_1 - \sum_{k=1}^{N-2}\left(\int_{\theta_k}^{\theta_{k+1}} d\eta_{i+1}\right) - \int_{\theta_{N-1}}^{\theta_B} d\eta_N \tag{8}$$

2.3. Three-dimensional Vaulter model

The mathematical model of the vaulter shown in Figure 1 is represented by eight rigid links, namely, a forearm, an upper arm, torso, waist, right thigh, right shank, left thigh and left shank. The joint moments acted on each link to either side of each joint, and horizontal and vertical reaction forces acted at each joint. The moment (M_B) and force (magnitude P, direction ϕ) act simultaneously on the forearm top (point B) because we have modeled both arms on one arm from the pole. The equations that described the motion of the model are derived using Newtonian mechanics and we can simulate the model by using an Open Dynamics Engine (ODE).

3. Optimization by using Genetic Algorithm

Genetic algorithms are search algorithms inspired by the mechanics of genetics and natural selection. These search algorithms combine survival of the fittest among chromosome-like string structures with a structured yet randomized information exchange. It evolves candidate solutions to problems that have large solution spaces and are not amenable to exhaustive search or traditional optimization techniques. Genetic algorithms have been applied to a broad range of optimization problems since their inception by Holland (1975)[5]. In this paper, a genetic algorithm is used to solve the optimization problem of the joint torque of pole vaulting and pole stiffness.

3.1. Objective function

The pole vault competes for height, so in this simulation, we have chosen the maximum height of the lower trajectory to be the objective function of an optimization problem. According to the rules, the crossbar may be set anywhere in the range ($-0.9\text{m} < x < -0.1\text{m}$), so the height is the maximum height that can be attained for the trajectory

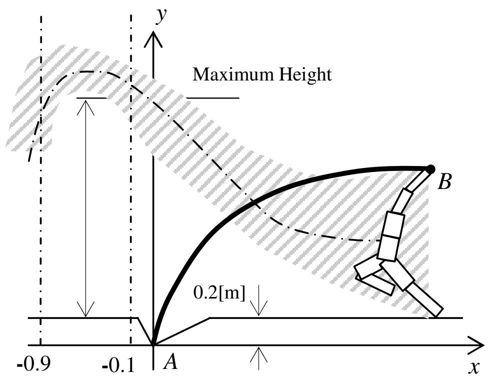

Figure 2. Vault trajectories showing the maximum attainable vault height within the allowable range.

within the crossbar setting range (Figure 2). In other words, the position of the crossbar for maximum height can be readily determined by just setting the crossbar at the maximum height.

3.2. Control parameters

The height of the pole vault is influenced with various parameters, i.e., vaulter's physical constitution, takeoff velocity and direction, pole length and stiffness, joint torques and so on. Except the joint torques, these parameters are independent of time. The parameter independent of time can be expressed as one control parameter for optimization. However, since it is dependent on time, the joint torque cannot be directly expressed as a control parameter for optimization. Then, in this study, we proposed expressing a joint torque with the spline functions. The expression for the torque was given by the control points $\mu_{i,0}$, $\mu_{i,1}$, … ,$\mu_{i,11}$.

$$M_{i,j}(t)=a_{i,j}+b_{i,j}(t-\tau_{i,j})+c_{i,j}(t-\tau_{i,j})^2+d_{i,j}(t-\tau_{i,j})^3 \tag{9}$$

Thus, one torque was expressed by eleven control parameters as a function of time. Therefore there are 88 control parameters expressing the torque of eight joints. Moreover, we chose the stiffness of the pole as control parameters, i.e. $(EI)_1$, κ_k(k=2…N), so it becomes the optimization problem for a large number combination.

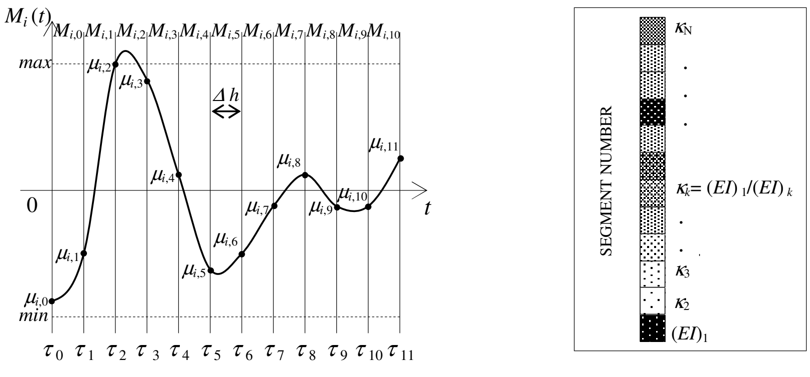

Figure 3. Control parameters of optimization. (a) Spline function expressing the joint torque. (b) Parameters expressing pole stiffness.

4. Simulation Results

In our simulation, pole length of L=4.5m, vaulter's height of 1.8m and weight of 77 kg where used as the primary input. The initial conditions, control parameters and range of joint angles were selected by considering realistic ranges that can be achieved.

The result of the optimize simulation, for joint torque and stiffness are presented in different cases. The first case, shown in Figure 4(a) and Figure 5(a), is calculated based on the pole model with uniform stiffness(N=1). The second case, shown in Figure 4(b) and Figure 5(b), takes account of multi-segment stiffness(N=11).

The overall result of the simulation show that the pole with high stiffness was selected by the Genetic Algorithm in proportion to an increase in the horizontal initial velocity. Furthermore, as can be seen in Figure 4 and 5, the model that considered multi-segment can obtain higher height.

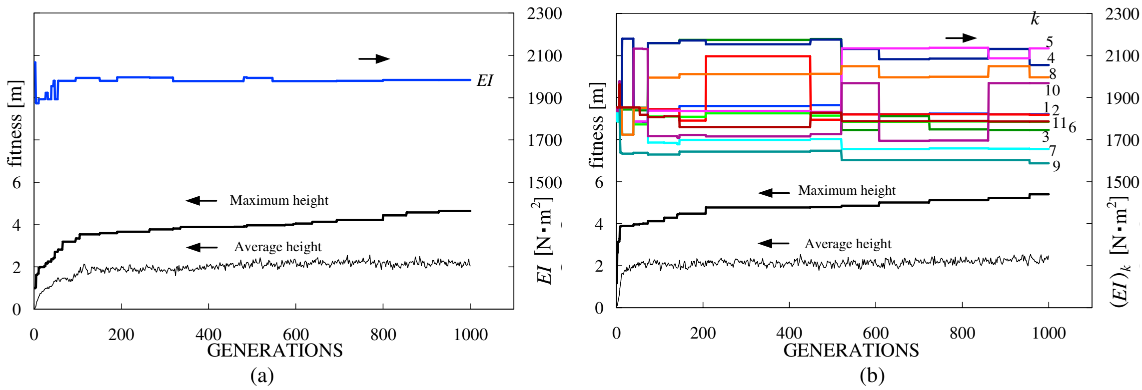

Figure 4. Transition of fitness value and control parameter(s) $(EI)_k$ by optimization. (a) Pole modeled with uniform stiffness (N=1). (b) Pole modeled with multi-segment (N=11). Horizontal initial velocity of center of gravity $v_{c.g.}$= -7.5[m/s].

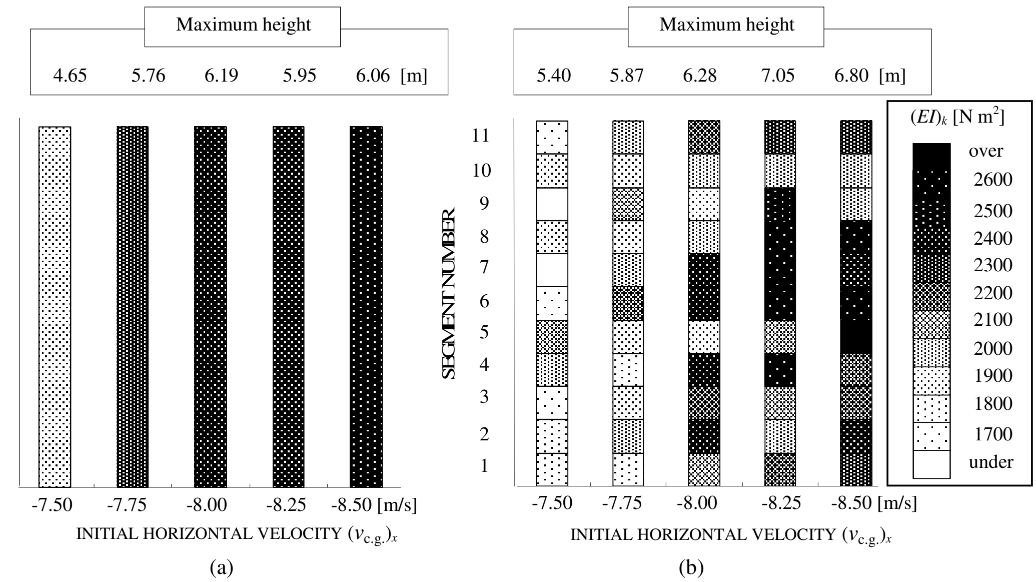

Figure 5. Distribution chart of optimized stiffness. (a) Pole modeled with uniform stiffness (N=1). (b) Pole modeled with multi-segment (N=11).

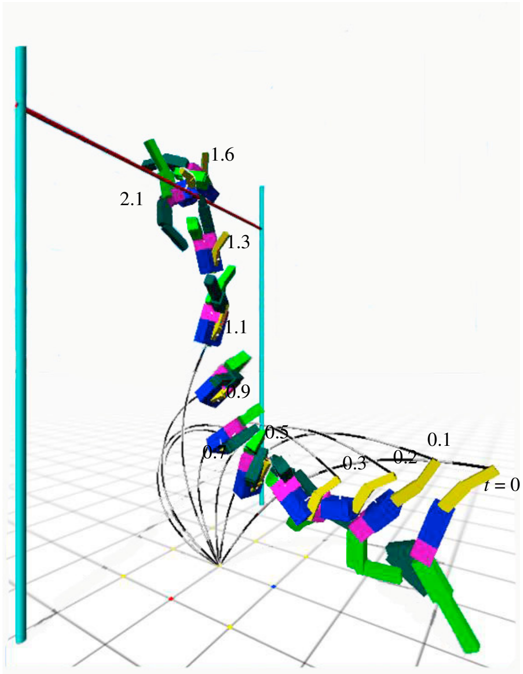

Figure 6. An example of vaulting by optimization using the multi-segment pole model. Horizontal initial velocity of center of gravity $v_{c.g.}$= -7.75[m/s].

5. Conclusions

We have shown in this paper that, a genetic algorithm can be used to solve the optimization problem of the joint torque and stiffness of pole vaulting. The model is used to study the effects of different initial conditions and the effect of time histories in the control torques during the vault. Moreover, these optimization methods might be useful for the selection of the pole, and the optimization method using the multi-segment and the result are useful for pole design corresponding to vaulter's characteristic.

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