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3D Dynamic Modelling and Simulation of a Golf Drive

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Abstract

A validated dynamic model of a golf driver can be an invaluable asset in designing a well performing club. We present a model for simulating the golf drive that includes experimentally-measured inputs from actual golfers, and a three-dimensional (3D) flexible shaft model based on Rayleigh beam theory. The result is a computationally-efficient dynamic model that includes the "signature" of an individual's swing, and the bending and torsion that is so important to the performance of the club. Good agreement was obtained between the simulated and experimental results.

Keywords: Golf drive simulation; Computer simulation; Dynamic modelling; Experimental validation; Flexible beam; 3D model.

1. Introduction

A validated model of driver which can accurately represent its motion can be an invaluable asset in designing a well performing club. In the literature, there are many mathematical models and computer simulations of the drive of a golf ball [1]. Unfortunately, most of these are based on overly-simplified models of either the golfer or the driver. In many cases, the golfer is replaced by a pivot point that is fixed or translating, resulting in a simple double- or triple-pendulum model of the swing [2-4]. In other cases, the club itself is overly simplified, either by neglecting its three-dimensional characteristics [5-6] or the flexibility of the shaft [7-8]. The three-dimensional nature of the golf swing has been demonstrated to be important in [9] while neglecting the flexibility of the shaft is particularly difficult to justify for a modern driver. MacKenzie's 3D golf drive model uses a simple representation for the shaft flexibility [9]. In the very few references that include both a detailed golfer and club model [10], the authors have used finite element methods that require extremely long computer simulation times. Faster simulation times allow the club designer to try "what-if" scenarios, not to mention design optimization and sensitivity analyses.

In our work, we developed a model for the golf drive that includes experimentally-measured inputs from actual golfers, and a three-dimensional (3D) flexible shaft model based on Rayleigh beam theory. The result is a computationally-efficient dynamic model that includes the "signature" of an individual's swing, and the bending and torsion of the shaft that is so important to the performance of the club. The golf team at the University of Waterloo (UW) was recruited to provide swing input data for our model. Reflective markers were placed on the golf club's grip, shaft, and head and tracked by a camera-based motion capture system during testing sessions. The data was

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filtered and post-processed to obtain the translational and rotational motion of the grip used to drive our computer model. In addition, data was collected for the motion of the shaft and clubhead for experimental model validation.

CGSim, a computer model for simulating the swing, was created by connecting a 3-D clubhead, with known geometry and inertial properties, to a Rayleigh beam model of a flexible shaft. Optimized simulation code was created using the commercial Maple/DynaFlexPro [11] software (now part of the MapleSim computer simulation environment). A second computer model was created for validating CGSim using Ansys [12] to model the flexible shaft, with this finite element model imported into an MSC.Adams [13] multibody dynamic simulation of the swing. The Ansys/Adams results were found to be in close agreement to the results of our beam-based model, with the latter computer simulations being significantly faster (approximately 3 times faster for the representative case). In a second stage of CGSim validation, we compared our simulation results against the experimental data from the motion capture system. Very good agreement was obtained between the simulated and experimental clubhead speed, with reasonable agreement for clubhead dynamic loft[†] and droop[‡] at the instant of ball contact.

2. Methods

In this section we present first the experiments conducted in this study and then present the simulation methods used to develop the computer model of a golf drive.

2.1. Experimental Methods

Four members—referred to here as S1, S2, S3 and S4—of the UW golf team were recruited for this study to provide swing input data for the computer simulation of a golf drive. Each athlete was asked to swing four drivers, each of which had a shaft with different geometric and material properties. These clubs are referred to by the color of their shafts: red, green, orange and yellow. Two good swings (good being defined by the athlete) of each athlete was recorded using each club. The main objective of this experiment is to record the motion of the grip which, after filtering and postprocessing, results in the 6 dof (degree of freedom) motion required to drive the simulation model. The 6 dof motion consists of the translational (X, Y and Z components) and rotational (3-1-3 Euler angles) position of the grip and its time derivatives (velocity and acceleration).

The club kinematics were recorded using an 8-camera Vicon MX-F20 motion capture studio at the UW Kinesiology department. The infrared cameras were used to track the positions of 10 mm diameter reflective spheres attached to the golf club grip, shaft and head at 500 frames per second (fps). The grip's 6 dof motion, which serves as the main input to the simulation model, was measured using an array of 3 markers attached to the grip of the golf club (Fig. 1). Markers were also attached to the shaft and clubhead to record their motion for validation purposes.

All of the calculations needed to extract the position and orientation of the club from the grip array assume that the relative positions of the array markers are fixed. Extracting the 6 dof grip motion to drive the simulation and the club face kinematics for validation required significant effort (Fig. 2). A summary of each step can be found below.

1. *Correct Vicon Mislabelling Errors*: Vicon tracks a series of unlabeled passive markers, and labels them by fitting a labelled template to the observed data. This process was error-prone and often left many markers mislabelled. These errors were corrected by hand[§].
2. *Patch Small Data Gaps with Splines*: This is a standard practice in processing optical motion capture data [14]. Often cameras are unable to track markers for brief periods of time (25 ms) due to occlusion. These small gaps are interpolated with a spline in order to recover the data.
3. *Velocity Dependent Low Pass Filter*: We had to devise a filter that changed its bandwidth as a function of the swing velocity in order to get plausible velocity and acceleration estimates. While Vicon is excellent at tracking the positions of markers to less than 1 mm error, velocity and acceleration estimates using these systems are challenging to obtain due to the highly nonlinear noise that is present. The position noise is

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[†] The loft is the angle the clubface makes with the vertical axis when its sole is at rest on the ground. The dynamic loft is the change in this angle during swing due to bending of the club in the swing plane.

[‡] Droop is the angle that measures the bending of the club in a plane perpendicular to the swing plane.

[§] The Vicon system that was used in this study is typically used for tracking motions slower than golf swing. The high frame rate (500 fps) employed for tracking the golf club might have lead to a high rate of dropout seen in this study. In addition we were not able to change the camera locations. It is possible that changing the camera locations would have improved the quality of the data.

particularly difficult to remove because it has a bandwidth of 5-250 Hz—overlapping significantly with the bandwidth of the grip motion (estimated to be 0-32 Hz). This nonlinear noise is amplified significantly when numerical derivatives of the position data are taken to estimate velocity and acceleration required to drive the simulation. The bounded position noise corrupts the grip kinematics less when the grip position is changing faster than the noise. A velocity-dependant, low-pass 4th order, Butterworth filter with a velocity dependent upper cutoff frequency, 5-30 Hz, helped reduce the position error aggressively when the grip was moving slowly and very noise-corrupted, yet increased the bandwidth of the filter when the grip was moving quickly and less corrupted by noise. This velocity-dependent filter greatly improves the signal.

4. *Trim Waggle Data*: The low-velocity movements of the waggle were corrupted with position noise. This position noise created erroneous velocity and acceleration estimates and generated unrealistic vibrations in the simulated shaft. Fortunately the higher velocity data (clubhead speed > 75 mm/s) beginning with the back swing was more reliable. The waggle movements were trimmed from the input data until the first inflection point of the speed of the grip prior to the backswing.
5. *Extract 6 dof grip input motion*: The grip's translational and rotational position, velocity and acceleration was extracted from the movement of the processed filtered triad kinematics using script in Matlab.
6. *Reconstruct clubface kinematics for validation*: The experimental clubface kinematics were reconstructed using 3 markers placed on top of the clubhead for validating the simulated clubface.

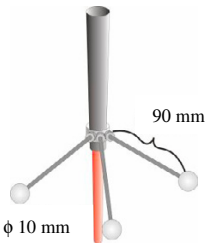


Figure 1: The grip triad was constructed using a pipe clamp and 3 slender 3.5" bolts and 3 reflective markers.

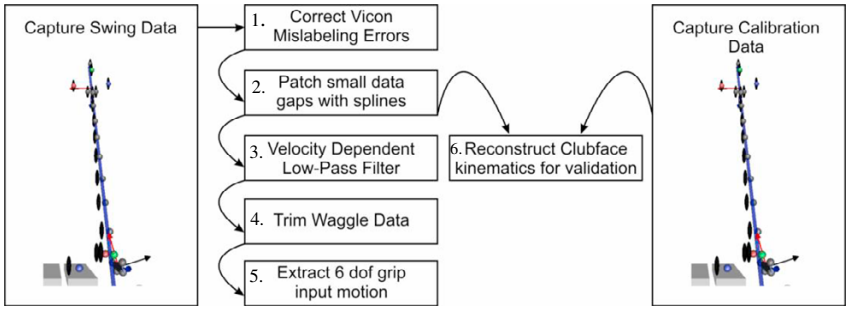


Figure 2: A block diagram of the processing done to extract 6 dof grip and club face kinematics from raw marker data.

2.2. Simulation Methods

A 3-D simulation model of a golf drive, CGSim, is created by using commercially available multibody modelling software, DynaFlexPro, is used to generate the symbolic equations of motion of the driver. The model consists of a flexible shaft and a rigid clubhead. A correct representation of the bending and torsional deformation of the shaft is critical in determining the driver's performance and the kinematics of the club face at ball contact. The bending, torsional and axial flexibility of the shaft were modeled using Rayleigh beam theory. The motion of the driver was represented using the following set of system coordinates:

1. Position of clubhead center of gravity (C.G.) x_c, y_c, z_c.
2. The 3-1-3 Euler angles of the clubhead ψ_c, θ_c, ϕ_c.
3. Flexible coordinates of the deformable shaft (Rayleigh beam): (a) three bending coordinates along each of the two transverse directions of the shaft, (b) one torsional coordinate, and (c) one axial coordinate.

Many physical parameters of the clubhead and the shaft properties were required to construct the model. Material (Young's modulus, shear modulus, and density) and geometric (shaft length, cross-sectional area, and second moment of inertia) properties of the shaft were required for the Rayleigh beam model. It should be noted that the shaft is modelled to incorporate variation of second moment of area ($I(x)$), crossectional area ($A(x)$), Young's modulus ($E(x)$), and shear modulus ($G(x)$), along the length of shaft (x). This is done by representing these quantities as sixth-order polynomials. Geometric (location of the center of gravity) and inertia properties (mass and inertia matrix) were required for the rigid body model of the clubhead.

The development of CGSim is summarized in Fig. 3. DynaFlexPro used Maple to automatically generate the symbolic equations of motion of the golf club, which were exported to an optimized C procedure. The kinematic variables of interest in the model were obtained in terms of the system coordinates using DynaFlexPro's built-in function, GetFrameMotion, and this code was exported to Matlab procedures. Finally, a script was implemented in Matlab to integrate the equations of motion and post-process the results to obtain the variables of interest, animations, and plots. This script is compiled using Matlab to obtain the standalone software for golf club motion simulation, CGSim. Once the grip's 6 dof** motion is provided to the model the simulation of the driver can be conducted using CGSim and its motion can solved for together with other interesting physical quantities like the dynamic loft, droop, and the velocity of the clubhead.

The Ansys/Adams model of golf club simulation was created using a two-step procedure involving first the general-purpose finite element (FE) software Ansys and then a general-purpose multibody software MSC.Adams. Crossectional area ($A(x)$), bending stiffness ($EI(x)$), and torsional stiffness ($GJ(x)$) are modeled as a function of length along shaft (x). The finite element (FE) model of a flexible shaft is created in Ansys using 3-D two node elastic beam elements (Beam188) based on Timoshenko beam theory. This element is suitable for analyzing slender to moderately stubby/thick beam structures and thus suffices for the purpose of modelling a flexible golf shaft. The model contains 100 elements and 101 nodes.

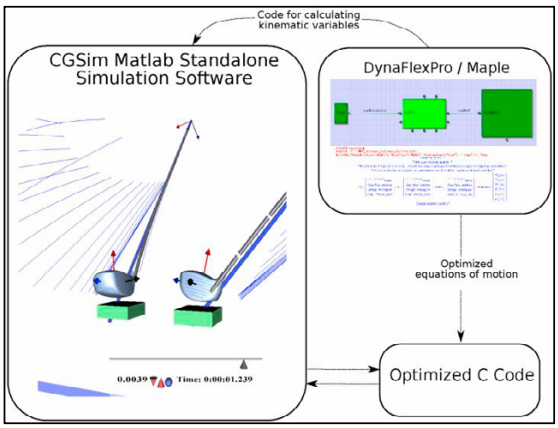


Figure 3: CGSim simulation model.

The Ansys model was used to create a modal neutral format (MNF) file of the shaft, which contains the modal mass, stiffness, and deflection characteristics, and which facilitates the representation of the shaft as a flexible body in MSC.Adams. After generating the MNF file, it was imported into MSC.Adams as a flexible shaft. The clubhead was represented as a rigid body. The clubhead and shaft hosel are welded together in the model. To simulate the motion of the golf club, a 6 dof motion driver is applied at the grip end of the shaft and the motion of the golf club numerically simulated using MSC.Adams time integrator.

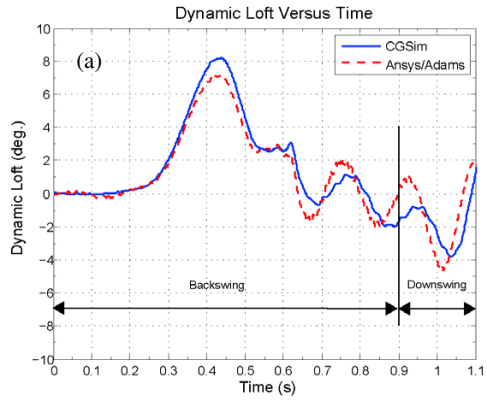
3. Results

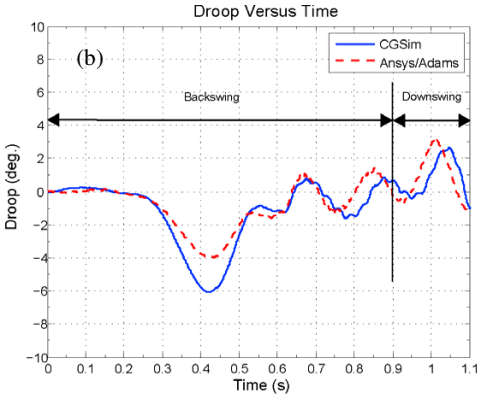
3.1. Comparison of CGSim with Ansys/Adams Model

In this section, we present the comparison of results of golf club motion simulation obtained from CGSim against those from an Ansys/Adams model. The purpose of this comparison is the validation of CGSim with another simulation approach, Ansys/Adams, and not the exhaustive comparison of the two simulation approaches. The comparison of simulation results between the CGSim and Ansys/Adams models for a representative swing (S2's first swing with red driver) is shown in Figs. 4(a)–4(c). Figure 4(a) shows the comparison of dynamic loft, Fig. 4(b) shows the comparison of droop, and Fig. 4(c) shows the comparison of clubhead speed.

Although the simulations are conducted using software packages, which use two different modelling theories for the shaft (beam theory versus finite elements), their results compare favorably with each other. The largest difference in results predicted by CGSim and Ansys/Adams occurs during the backswing, where CGSim predicts a value of 8.15 deg for dynamic loft and 6.1 deg for droop; the corresponding quantities predicted by Ansys/Adams are 7.13 deg and 4.02 deg, respectively. The predicted value of dynamic loft and droop, however, are much closer to each at the moment of contact (Fig. 4(a)–Fig. 4(b) and Table 1). Fig. 4(c) shows a good agreement between the two models for the predicted value of clubhead speed throughout the duration of simulation. Table 1 summarizes the dynamic loft, droop and clubhead speed predicted by CGSim and Ansys/Adams at the instant of ball contact. This good agreement was observed for each swing that we simulated with Ansys/Adams.

** Grip's 6 dof motion driver includes the grip's translational and rotational position, velocity and acceleration recorded during the experiments.





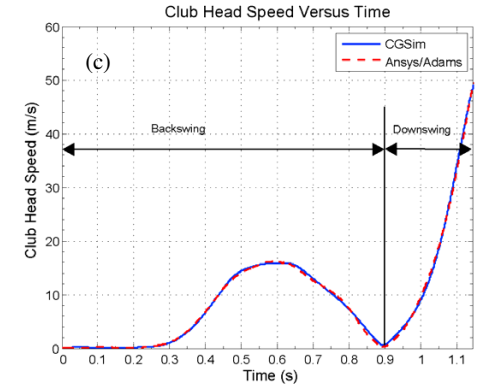
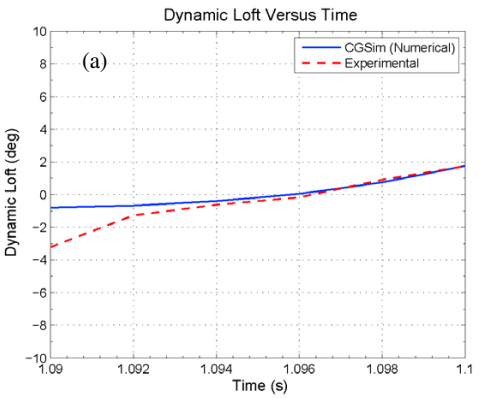
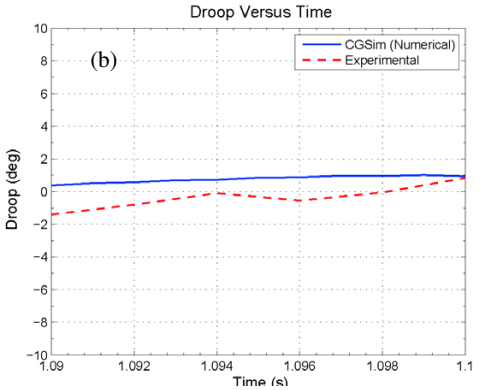


Figure 4: Comparison of dynamic loft, droop, and clubhead speed from CGSim and Ansys/Adams for S2's first swing with red driver





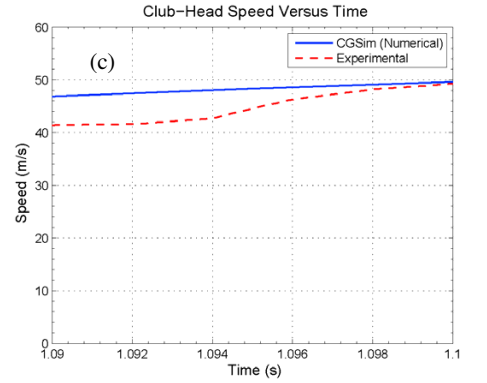


Figure 5: Comparison of dynamic loft, droop, and clubhead speed from CGSim and experiments for S2's first swing with red driver.

Table 1: Comparison of CGSim and Ansys/Adams results at the instant of ball contact for S2's first swing with red driver.

	CGSim	Ansys/Adams
Dynamic Loft (Deg.)	1.76	1.88
Droop (Deg.)	0.94	1.01
Clubhead speed (m/s)	49.6	49.2

Table 2: Comparison of CGSim and experimental results at the instant of ball contact.

S2's first swing with red driver	CGSim	Experiments
Dynamic Loft (Deg.)	1.76	1.6
Droop (Deg.)	0.94	0.86
Clubhead speed (m/s)	49.6	49.1

S3's first swing with red driver	CGSim	Experiments
Dynamic Loft (Deg.)	4.39	4.16
Droop (Deg.)	-3.84	-3.27
Clubhead speed (m/s)	46.7	45.8

3.2. Comparison of CGSim with Experiments

In this section, we present a comparison between the simulated and experimentally measured shaft kinematics. It should be noted that each simulation was conducted using swing data from individual swings and the purpose of this comparison is to validate CGSim with experiments, and not to establish a statistical characterization of the golfer's drives. The comparison of results for S2's first swing with red club are shown in Figs. 5(a) – 5(c). It is important to note that the experimental measurement of the dynamic loft and the droop are only valid near the instant of the ball impact because the markers on the club face were not picked up cleanly before this instant (whereas the slower-moving shaft markers could be tracked during the entire swing). This fact explains the discrepancy between the experimentally measured values and those predicted by CGSim before the ball contact. Thus, for our comparison, we will focus on results at the instant of impact only. Table 2 summarizes the dynamic loft, droop and clubhead speed observed by experiment and that predicted by CGSim for S2's first swing and S3's first swing (not shown here), at the instant of ball contact. The results of these two swings compare particularly well with the experiments. We found some other cases that do not compare as well due to the problem associated with accurately tracking the markers placed on the club face. Although the experiments are not always in close agreement with the simulations, the results from both do follow the same qualitative trends, as can be seen in the plots near the time of impact.

4. Conclusions

A validated dynamic model of a golf driver which includes experimentally-measured inputs from actual golfers, and a three-dimensional (3D) flexible shaft model based on Rayleigh beam theory was presented in this paper. The flexible shaft included both bending and torsion, which are vital in determining the performance of the club. For validating our model, we compared its simulation results against the experimental data from the motion capture system. Very good agreement was obtained between the simulated and experimental clubhead speed, with reasonable agreement for clubhead dynamic loft and droop at the instant of ball contact. The resulting model is also found to be significantly faster (approximately 3 times faster for the representative case) than another model that was developed using Ansys and MSC.Adams and employs finite elements.

A validated dynamic model of a golf driver can be an invaluable asset in designing a well performing club. For example our model could be used for matching the varying stiffness along the shaft to the needs of an individual golfer to achieve higher accuracy and/or greater driving distance. It could also be used to replace robot tests of golf clubs with virtual computer simulation, simply by using the golf robot data as the input motion to the model.

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