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On the mechanics of the golf swing

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A perspective of the golf swing is defined and current knowledge of swing mechanics is reviewed. The implications of previous research are collected. A conventional arm–club simulation model is set up and used to show that an apparently important result from the literature is incorrect. The swing model is fitted, through a parametric optimization process, to data relating to expert golfers. The kinematic deficiencies of the arm–club model are pointed out and a shoulder–arm–club simulation model with shaft flexibility is developed. Parameters of the shoulder–arm–club model for best matching the expert-golfer data are found by optimization. The torques generated by the golfers are deduced. The shoulder–arm–club model is then applied for finding whether or not improvements to the observed torque histories are possible, with a positive result. A pattern for the optimal use of available driving torques is established. Scaling of the problem to help the understanding of the relationship between large and small players is studied through dimensional analysis. Several contributory conclusions are drawn.

Keywords: mechanics; golf; swing; optimization

1. Introduction

The focus of this paper is the review, consolidation and extension of understanding of the mechanics of the golf swing. In particular, it is desired to establish how a golfer may employ his assets to maximize the club-head speed of a driver at the point of impact with the ball. We take the view here that the upswing is simply a means whereby the starting state for the downswing can be established and is not of interest itself. We presume that the golfer is stationary at the commencement of the downswing, although it is not uncommon for the club to reverse its motion slightly after the arms and body. This viewpoint is implicit in the subsequent discussion.

Scientific treatment of the golf swing started with Williams (1967) who made deductions from detailed observations of a photographic sequence of the downswing of Bobby Jones with a driver. Williams observed that the swing is essentially planar, the plane being inclined to the vertical by perhaps 35°. He suggested that two rigid bodies are involved, the arms rotating about a fixed hub,

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located at the base of the neck and the club shaft and head, rotating about the wrist joint relative to the arms. Williams divided the downswing into two phases. In the first phase, the arms and the club rotate around the fixed hub as a rigid assembly while, in the second, the wrist action occurs. The club uncocks and the club head 'catches-up' with the hands, contacting the ball with arms and club in roughly a straight line. In this second phase, starting with the 'release' and commonly called the 'hitting area', the angular velocity of the arms is nearly constant. Williams noted the gentle start to Bobby Jones' downswing and linked it to the commonly mentioned fault 'hitting from the top' (unfortunately misprinted tap) without much explanation. He also suggested a simple torque scaling rule, which if the driving torques are multiplied by a factor r^2, the club-head speed at impact will be multiplied by r, with geometry unchanged.

Soon afterwards, the influential book of Cochran & Stobbs (1968) was published. High-speed photography was used to establish that contact between club head and ball lasts approximately 0.5 ms, confirming that the club head acts as a free projectile in its interaction with the ball. The movement of the club head just prior to impact determines the outcome of the stroke totally, to all intents and purposes. Observations of the head movements of 31 professional golfers revealed that these movements were downwards and backwards but were typically very small. The fixed-hub notion seems to be sound. Moving one's head significantly during the golf swing seems sure to complicate the mental task involved in returning the club head to the ball with precision, and so is not likely to be advantageous.

Furthermore, Cochran & Stobbs (1968) used high-speed photography to record successive positions of arm and club, with camera located on the swing axis normal to the swing plane, of the British Ryder Cup golfers, Bernard Hunt, Geoffrey Hunt and Guy Wolstenholme. They simulated the arm–club downswing digitally, using a constant driving torque for the arms and a free hinge for the wrists, with a limit stop preventing the wrist-cock angle from exceeding a set limit. It was shown that such a simulated swing looks very much as a real one and they discussed the efficiency with which the golfer can cause motion in the club head deriving from Williams' two phases. In the first phase, the arm–club system is folded up, minimizing the inertia with respect to rotation about the fixed hub and allowing maximum angular velocity to be developed for a given arm-torque capacity. In the second phase following the release, much of the momentum in the system at the end of the first phase is transferred from the arms to the club head, accounting for the arms continuing to rotate at constant angular velocity, even though the arm torque is continuous. The possibility of using muscular action at the wrists to gain advantage relative to the free-wrist-hinge swing was mentioned. Analysis of the energies involved in the 0.2–0.3 s duration of the downswing showed that a good male professional golfer of the time needed to fully use approximately 15 kg of muscle, making leg and torso muscles essential participants.

Lampsa (1975) used optimal control theory, in conjunction with the arm–club downswing model discussed already, to establish how available arm and wrist torques should be employed to maximize the club-head speed of a driver at impact. Lampsa's method is relatively sophisticated and his findings are considerably out of step with others. He found that both arm and wrist torques should build approximately linearly with time, the arm torque starting from nearly zero, the wrist torque starting from approximately 20 per cent of the full capacity. The supposed optimal swing aligns with the gentle start of the Bobby Jones swing but is problematic from an energy viewpoint, since the average muscle usage over the downswing duration is quite low. This ‘optimal’ golfer appears not to be trying very hard.

Q7 Jorgensen (1999) used stroboscopic photography to record the sequence of positions that a professional golfer passes through in his downswing. The position and time data were processed to yield club-head speed as a function of time and four very similar swings were averaged to increase the reliability of the results. An arm–club swing of the type described earlier was fitted to these averaged velocity results. It was apparently found necessary to introduce a hub motion, involving an imposed acceleration first forward and then backward, to bring theoretical and experimental results into agreement. The simulated swing was considered validated by the agreement achieved and was used with variations to make deductions about swing fundamentals. The efficacy of this procedure must, however, be questioned. The hub motion postulated is contrary to experimental evidence and the reduction of a great deal of position data to club-head speed is only a considerable negative step. It is obvious that the arm–club swing model does not represent the kinematics of a real swing at all precisely, so that a very close parallel between arm–club simulation results and observations should not be expected. Jorgensen’s attempt to match simulation with experiment seems also to have been influenced by a conviction that the golfer’s arm torque is constant, which must be regarded as an open issue.

The swing mechanics literature expanded with the organization of conferences on *Science and golf* in 1990, 1994, 1999, 2002 and 2008. The proceedings to 1999 and other literature to 2001 have been nicely reviewed by Penner (2003), whose account will be largely relied upon. However, the shoulder–arm–club swing model set up by Turner & Hills (1999), see below, is of special note, since it demonstrated the capability of such a model to replicate reasonable swings, provided that the driving torques are suitably chosen. It is clear from the account, which only involved constant torques, that many unreasonable swings can be constructed if the driving torques are not suitably chosen. Sprigings & Mackenzie (2002) also conducted simulations based on a shoulder–arm–club swing model, paying particular attention to the influences of wrist torque and the sources of power in the swing: they used optimization to show that club-head speed at impact can be increased by delaying the release and included limited results on shoulder, arm and wrist torques. More recent studies of the wrist-torque function include those of Chen *et al.* (2007). Tutorial material at arm–club model level can be found in White (2006).

The basic kinematic difficulty with the arm–club model is illustrated in figure 1, where it is clear that an arm–club representation does not allow a sufficiently long backswing. As soon as the right arm of a right-handed golfer bends to allow a realistic arm-swing length, the swing geometry is much more than that of the three-link model than of the two-link one, as shown in figure 2. As evident in the figure, the shoulder–arm–club swing presumes the shoulders to rotate around a fixed hub, the left arm (of the right-handed player) to rotate around the left shoulder joint restrained by a limit stop and the club to rotate around the wrist joint, again restrained by a limit stop.

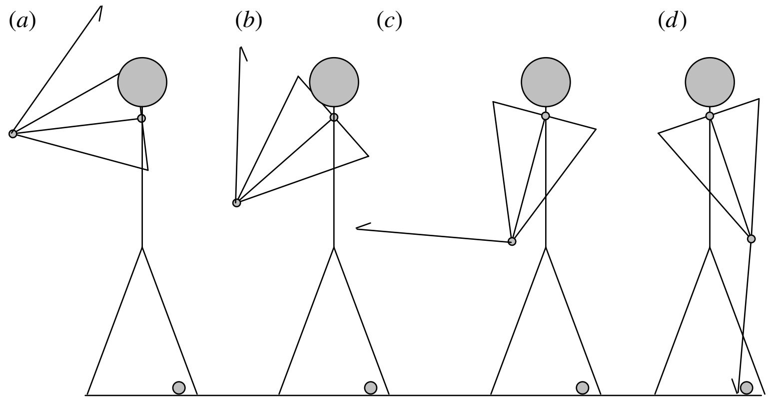


Figure 1. (*a*–*d*) Illustration of the restricted backswing implied by a strictly two-link model.

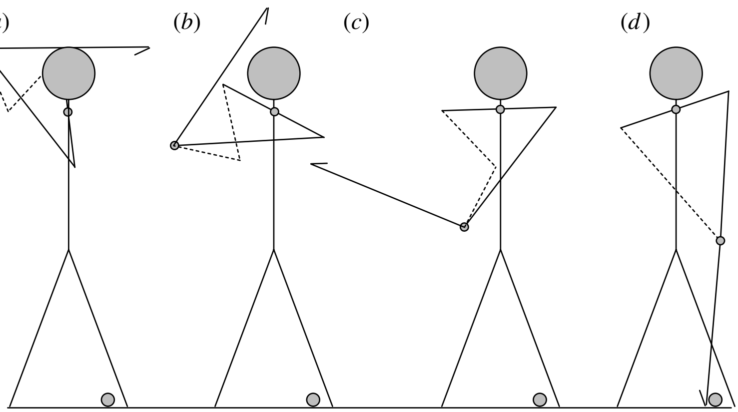


Figure 2. (*a*–*d*) Illustration of the full-length backswing possible with a three-link swing model.

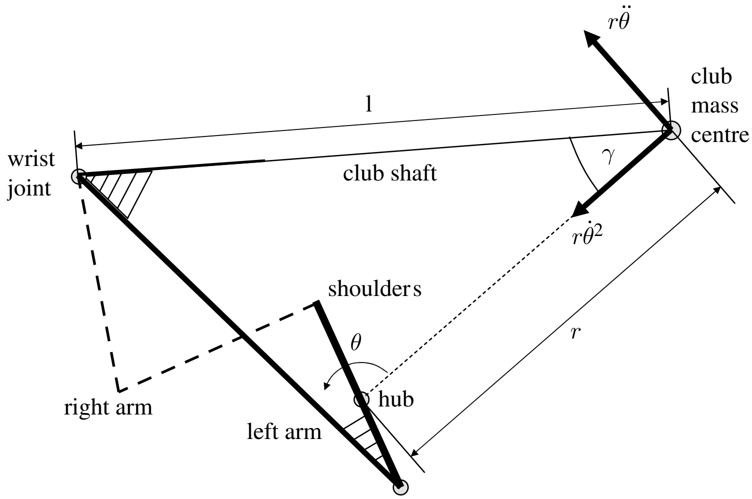
Important observations by Penner (2003) includes the following.

- (i) Delaying the release is advantageous and requires that the club be held back from the hit at the point in the swing where natural (wrist-torque-free) release would otherwise occur.
- (ii) Golfing skill is strongly associated with the delayed release and the late hit.
- (iii) Measurements of wrist torques applied by golfers demonstrate driving torque until late into the downswing. Around release, golfers use hold-back torque to delay the release, some of them maintaining that torque up to impact, while others use driving torque again for the last 30 ms before impact. (It should be noted that initial driving torque is an inevitable consequence of the wrist-motion limit stop, and does not necessarily derive from muscle action.)
- (iv) Effective distribution through time of the delivery of the various driving torques depends on the lengths of shoulder, arm and wrist swings.
- (v) The contribution of shaft flexibility to the swing is minor.
- (vi) Constant-torque models may be unrepresentative of what skilled players do.
- (vii) Matching of good quality simulations to measurements is desirable to further the understanding of optimal strategies.

The picture for the optimal swing that emerges from an arm–club treatment of the problem is given below.

(i) The fixed-hub and planar motion ideas substantially accord with observations but the kinematics of the two-body swing misrepresent to some extent the swings of real golfers.
(ii) From rest at the top of the backswing position, driving (positive) torque should be applied to the arms, while hold-back (negative) torque should be applied to the wrists. Near the start, the wrist torque is unimportant, since the wrist limit stop will be engaged (unless a positive wrist torque is applied before the arm torque, giving a motion such as the cast of a fisherman), but the negative wrist torque is needed later to delay the release.
(iii) It should not be presumed that the golfer should apply his maximum arm torque throughout the swing, since this may cause a too-early release. Such a release gives maximum club-head speed before impact and what may well be a substantial loss of speed at impact.
(iv) When release occurs, despite the negative wrist torque applied, the wrist torque should reverse to increase the transfer of momentum from arms to club head before impact occurs. The more negative the wrist torque can be made before release, the later will release occur for a given arm-torque history. The late release provides opportunity to apply more arm torque earlier in the downswing, without spoiling the efficiency by early hitting. The later the release is, the less time there is for the club to catch up with the hands, necessary for efficiency, and the more need there is for positive wrist torque after release.
(v) Long arm swings are potentially advantageous, since the work done on the motions before impact, assuming that the muscle forces available are unaffected by the arm-swing length, can be increased. For the same reason, the ball should be located as far forward in the golfer's stance as is comfortable. However, long arm swings are more likely to be spoiled by early hitting if the gentle-start discipline is not followed.
(vi) Large wrist-cock angles are potentially advantageous in allowing more efficient use of muscle forces to create club-head speed at impact, but the arm-torque distribution must be appropriate to the geometry employed.
(vii) Within the normal range of club-shaft flexibilities in use, these ideas are unlikely to depend on shaft details.

In terms of the shoulder–arm–club swing model, it can be imagined that the shoulders and arms replicate, to some extent, the arms and club of the simpler model. According to this view, the downswing should start with positive shoulder torque and negative arm and wrist torques. The initial motion is effectively a rigid-body rotation around the hub. At some point, the arm torque should switch from negative to positive and the arms should start to rotate relative to the shoulders. Some efficiency in the transfer of momentum from shoulders to arms should derive from the movement out from the hub of the mass of the arms, increasing the effective inertia of the system about the hub. However, the effect can be expected to be much smaller than the corresponding one for the arms and club, since the arm mass does not move out from the hub by much. Once the



Q1 Figure 3. Depiction of the rigid-body rotation phase at the start of a good-quality downswing.

arms are in relative motion, the mechanical actions can be expected to be similar to those of the arm–club swing. There should be a second switch time at which the wrist torque changes from negative to positive, the release occurring as late as possible, consistent with the club head being able to catch up with the arms at impact.

To make the crucial issue of the release and its implications for the application of shoulder and arm torques as clear as possible, an elementary analysis of the rigid-body-rotation phase of the swing, Williams' phase 1, is carried out. Referring to figure 3, let us imagine that the driving torque at the hub is given by $(A+Bt)$ with the effective inertia of the system about the hub being I_e and the distance from the fixed hub to the club mass centre being r. Let the rotation angle of the composite system about the hub be θ and let θ and $\dot{\theta}$ be zero when $t=0$. Let m_c be the club mass and l the distance from the wrist joint to the club mass centre.

The force components to sustain the assumed motion of the club mass centre are $m_c r\ddot{\theta}$ tangentially and $m_c r\dot{\theta}^2$ radially and the angular acceleration of the assembly about the hub is given by $\ddot{\theta}=(A+Bt)/I_e$. Integrating with respect to time, we have $\dot{\theta}=(2At+Bt^2)/2I_e$ and $\theta=(3At^2+Bt^3)/6I_e$. The wrist joint torque necessary to sustain the presumed club motion is

$$\frac{I_c}{I_e}(A+Bt)+\frac{m_c rl}{I_e}(A+Bt)\cos\gamma-\frac{m_c rl}{I_e^2}\left(At+\frac{Bt^2}{2}\right)^2\sin\gamma,$$

where I_c is the club moment of inertia about its mass centre. Since all the parameters involved are positive, it can be seen that the wrist torque is positive for small t, going to zero as the system accelerates. When the moment becomes zero, natural release occurs. The positive wrist torque at the start does not demand muscular action since it is provided by the wrist limit stop.

If the driving torque at the hub is constant, $B=0$ and release occurs when $t^2=(I_e/m_c rlA\sin\gamma)(I_c+m_c rl\cos\gamma)$. Numerical evaluation of this expression for representative parameters $I_e=I_a+m_c z_1^2+I_c+m_c r^2=1.525\ \mathrm{kg\ m^2}$,

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Table 1. Arm–club model parameters in S. I. according to Lampsa.

parameter	symbol	value
arm mass	M_a	7.312
club mass	M_c	0.394
arm principal inertia	I_a	0.373
club principal inertia	I_c	0.077
n0 to arm mass centre, nominal	z_1	0.326
n0 to wrist hinge, nominal	z_2	0.615
n0 to driver mass centre, nominal	z_4	1.624
n0 to driver face centre, nominal	z_5	1.72
inclination of swing plane to vertical	ϕ	0.785
arm swing angle	θ_a	−2.792
wrist-cock limit angle	θ_w	−2.242
limit-stop stiffness	C_w	1000
limit-stop damper coefficient	D_w	5
arm driving torque	T_a	$9.5+811t$
wrist driving torque	T_w	$9.5+63.52t+73.81t^2$

$I_a=0.373\ \text{kg m}^2$, $m_c=0.394\ \text{kg}$, $z_1=0.326\ \text{m}$, $r=0.8692\ \text{m}$, $l=1.106\ \text{m}$ and $\gamma=0.5861\ \text{rad}$ (see §2*a* below and table 1) yields a release angle $\theta_{\text{rel}}=0.937\ \text{rad}$, irrespective of the value of A. If the driving torque at the hub rises in proportion to time, $A=0$, release occurs at $t^3=(4I_e/m_c rlB\sin\gamma)(I_c+m_c rl\cos\gamma)$ and $\theta_{\text{rel}}=1.25\ \text{rad}$, irrespective of the value of B. These values are close to those given by the corresponding simulations (see §2). Release times do, of course, depend on the values chosen for A or B, as can be seen in the equations. In each case, small values of the angle γ contribute to increasing the angle traversed by the arms before the natural release, θ_{rel}. The ramp driving torque is significantly better than the step in terms of avoidance of a too-early hit. Simulated swings illustrating the consequences of a constant driving torque and a time-proportional driving torque are included in the electronic supplementary material, appendix A.

In §2, the more or less standard arm–club swing model is established, with a view first to checking Lampsa's result. A generalization of the arm–club model is then used to confirm the conventional view that gravitational effects and aerodynamic drag on the club head are unimportant and to determine how well the generalized arm–club swing is able to match the real swings of presumed high quality.

2. Mechanics of the arm–club swing

To demonstrate that Lampsa's solution is incorrect, it is necessary to set up a simulation model that mimics his conditions. We then run the model and compare the results against the original and show that a much better result than that found originally to be optimal is, in fact, available. The opportunity is also taken to see how much influence gravity exerts and to include in the model the aerodynamic drag on the club head, a feature normally neglected. It becomes clear that the gravity effect is small and that the drag influence is very small. These features are included in the subsequent simulations but they are optional.

(a) The basic arm–club model

The simulation models used here are automatically built in C++ by the symbolic multibody modeller, VehicleSim (formerly called AutoSim, see www.carsim.com; Sharp *et al.* 2005). The system of interest is described in a special description language, consisting of statements such as add-body, add-point, add-line-force, add-moment and add-speed-constraint. The language includes functions such as sin, cos, atan, max, min, sign and ifthen, and it also allows the use of vector operations such as dot, cross and dir. The modeller 'knows' a form of Kane's equations, embodying the virtual power principle (Schiehlen 1997) and it will output the symbolic equations of motion in addition to writing the problem-dependent part of a simulation programme. The problem-independent part of a simulation code is contained in library files that are linked together with the dependent file and compiled to give an executable programme. The extreme algebraic complexity typical of multibody models of any size mitigates against the usefulness of examining the symbolic equations of motion, so that, normally, the analyst will not engage with the equations, or indeed the C++ code, themselves. In the case of the arm–club golf swing model, the basic behaviour is well-enough known to reveal whether or not a model has been built correctly, from a careful examination of simulation results.

Our arm–club model starts with a fixed revolute joint with its axis normal to the swing plane, inclined at angle ϕ to the vertical. It is convenient to call the reference point, where the axis intersects the plane, n0. The arms rotate around this joint, driven by a torque from the inertial base. At the outer end of the arm body, there is another revolute joint, with its axis also normal to the swing plane, around which the club rotates relative to the arms. This wrist joint includes an elastic, damped limit stop, which comes into play when the wrist-cock angle exceeds a specified magnitude. The club is acted on by a driving torque, which is reacted on the arms, representing the wrist action. A standard gravitational field is included. To check Lampsa's results, we adopt his parameters (table 1). The simple polynomial descriptions of Lampsa's supposed-optimal driving torques are good representations of his numerical results. Limit-stop parameters were not used by Lampsa, since he employed constraints to control the relevant club motions. The values chosen make the wrist limit an order of magnitude stiffer than a typical driver, with the damping being sufficient to prevent the club motion from showing unrealistic oscillations.

Club-head speed and wrist-cock angles from the base simulation model are shown in figure 4, in the same form as given in the original paper. Simulation results for wrist limit-stop stiffnesses of 200, 1000 and 5000 $\mathrm{N\,m\,rad^{-1}}$ with the standard damping coefficient, 5 $\mathrm{N\,m\,s\,rad^{-1}}$, and with damping coefficients of 1 and 25 $\mathrm{N\,m\,s\,rad^{-1}}$ and the standard stiffness, 1000 $\mathrm{N\,m\,rad^{-1}}$, are shown superimposed on each other, indicating that the wrist-stop properties are not at all influential. The results have similar shapes to the original ones but there are clear differences in timing, too great for both solutions to be in any sense correct.

(b) Improving on Lampsa's results

From the discussion in §1, the use of the wrist action in the Lampsa swing appears to be far from optimal, so it might be expected that the result can be improved upon. To examine this possibility, the terms of the simulation model are changed, such

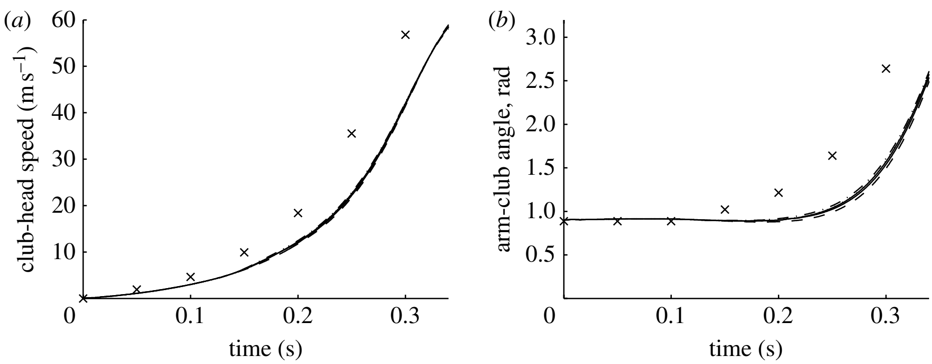

Figure 4. (*a*) Club-head speed and (*b*) arm–club angle results from supposed-optimal Lampsa swing. Lampsa's results are shown by crosses (solid curve, present; dashed curve, Lampsa).

that (i) the arm driving torque is defined as a simple function of time ($\tau_{a0}+\tau_{a1}t$), with maximum value set to Lampsa's maximum, 265 N m, and (ii) the wrist torque is given by $36\tanh(\lambda\times(t-t_w))$, 36 N m being Lampsa's maximum value. This torque is negative at $t=0$ and becomes positive after a time, t_w, to be chosen. The switching from negative to positive occurs at a finite rate, controllable by the parameter λ in the switching function $\tanh(\lambda\times(t-t_w))$, which is asymptotic to -1 for $(t-t_w)\rightarrow-\infty$ and to $+1$ for $(t-t_w)\rightarrow+\infty$. Typically $\lambda=100\ \mathrm{s}^{-1}$.

The revised simulation model is now run repeatedly under the control of the MATLAB parameter optimization routine 'fminsearch' with free parameters τ_{a0}, τ_{a1} and t_w, the cost function to be minimized being the negative of the club-head speed at impact. Impact occurs at that point in the swing when the longitudinal position of the club head reaches that of the ball, here 0.15 m in front of the base point, n0. Best parameters are $\tau_{a0}=0$, $\tau_{a1}=16\,236\ \mathrm{N\,m\,s^{-1}}$ and $t_w=0.1400$ s and the club-head speed at impact is $65.69\ \mathrm{m\,s^{-1}}$, improving greatly on Lampsa's result. If the influence of gravity is omitted by setting the swing inclination angle to $\pi/2$, the results are altered only slightly, and when aerodynamic drag on the club head is modelled, in the form of a force of magnitude $(1/2)C_d\rho A V^2$ (table 2) opposing the velocity in direction, the changes are even less. This result confirms the conclusion of Budney & Bellow (1979), based on a simple analysis of the forces acting on a club with a known motion. Results shown in figure 5 indicate a rigid-body-rotation phase to each swing, lasting approximately 0.13 s.

The near-optimal driving torques shown in figure 5 do contain a gentle start for the arms but the arm torque quickly builds to the maximum allowed and remains there. The wrist torque arises from compression of the limit stop initially and contains a very short holding-back phase. We have seen already that the limit-stop properties can vary widely without influencing the swing very much and also it should be recognized that the golfer can choose the limit-stop character to some extent by gripping tightly or loosely at the start of the downswing.

(*c*) *Fitting the arm–club model to real swings*

As indicated in §1, some expert swings have been documented in the literature, notably those of Bernard Hunt, Geoffrey Hunt, Guy Wolstenholme (Cochran & Stobbs 1968) and Jorgensen's (1994) subject. The best-known data

Table 2. Club-head drag parameters in S. I.

parameter	symbol	value
drag coefficient	C_d	0.4
club-head cross-sectional area	A	0.0036
air density	ρ	1.227
club-head speed	V	variable

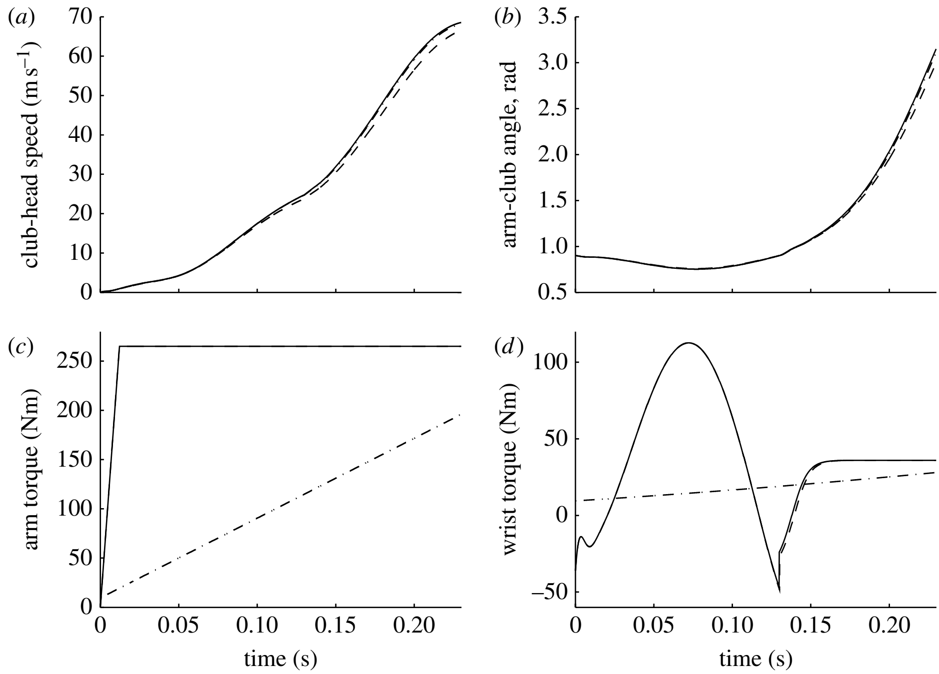

Figure 5. (*a*) Club-head speed, (*b*) arm–club angle and (*c,d*) driving torque results ((*c*) arm torque (*d*) wrist torque) from variations of the Lampsa swing. The influence of aerodynamic drag on the club head can hardly be seen. (*a,b*) Solid curve, optimal; dashed curve, no gravity; dot-dashed line, with drag; (*c,d*) solid curve, optimal; dashed curve, with drag; dot-dashed line, Lampsa.

are probably those of Bobby Jones (Williams 1967), but the camera employed for the photographic sequence was off-axis by an unknown amount, so their value is not so high. Scaled diagrams in Cochran and Stobbs allow the successive angles at 10 ms intervals of arms and club to be determined from wrist-joint and club-head positions, while Jorgensen's data are for the speed of a marked point near to the club head of a driver, averaged over four swings by one player.

To find how well the arm–club model will fit these data, the model is first developed a little. The parametric description of the arm driving torque applied by the golfer is extended to $(\tau_{a0}+\tau_{a1}t+\tau_{a2}t^2)$, so that the coefficients can be chosen by an optimizer to minimize the differences between model-swing and actual-swing angles. Also, for greater realism, the club bending flexibility is incorporated in the form of a revolute joint at the base of the grip, with a joint stiffness designed to give the model club a realistic lowest natural frequency of

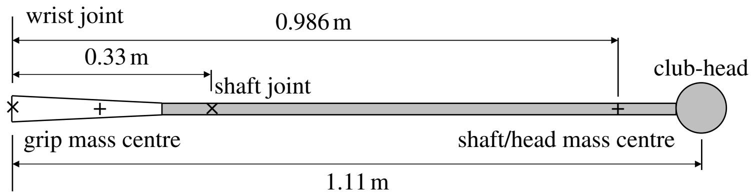

Figure 6. Dimensions of driver with compliant shaft.

Table 3. Parametric data for individual golfers, B. J. Hunt (BJH), G. M. Hunt (GMH) and G. Wolstenholme (GW), in S. I.

parameter	symbol	BJH	GMH	GW
club-head cross-sectional area	A	0.0036	0.0036	0.0036
drag coefficient	C_{d}	0.4	0.4	0.4
shaft-joint stiffness	C_{s}	109.2	109.2	109.2
wrist-stop stiffness	C_{w}	1000	1000	1000
shaft-joint damping coefficent	D_{s}	1	1	1
wrist-stop damping coefficient	D_{w}	5	5	5
arm principal inertia	I_{a}	0.4369	0.354	0.354
grip principal inertia	I_{g}	0.01367	0.0104	0.0104
club principal inertia	I_{c}	0.02571	0.02571	0.02571
mass of arms	M_{a}	11.526	9.8	9.8
mass of grip	M_{g}	1.8994	1.63	1.63
mass of shaft and head	M_{s}	0.2924	0.2924	0.2924
switch-rate parameter	λ	100	100	100
swing-plane angle	ϕ	0.6109	0.6109	0.6109
air density	ρ	1.227	1.227	1.227
arm mass centre to n0	z_1	0.3048	0.2888	0.2888
wrist joint to n0	z_2	0.675	0.64	0.64
shaft joint to n0	z_3	1.005	0.97	0.97
head mass centre to n0	z_4	1.661	1.626	1.626
club face centre to n0	z_5	1.785	1.750	1.750

approximately 4.3 Hz. The club mass is divided into two parts, one above the joint associated with the grip and the other below the joint, associated with the shaft and club head (figure 6). The golfer's hands, being below the wrist joint, are treated as part of the grip of the club. Furthermore, well-estimated parameter values are required to represent the golfers in a mechanical sense.

For a description of the golfer, we adopt as standard a 90 kg man with height 1.83 m. We employ biomechanical data from Webb (1964) to get the mass, length and inertia contributions from body parts. Bernard Hunt is especially tall at 1.93 m, so his data are scaled to account for this. Table 3 shows the outcome. Free parameters in the optimal fitting computations are τ_{a0}, τ_{a1}, τ_{a2}, τ_{wmx}, τ_{w}, $Q(1)$ and $Q(2)$. τ_{wmx} is the maximum wrist torque allowed, τ_{w} is the wrist-torque switch time and $Q(1)$ and $Q(2)$ are the initial arm and wrist-cock angles, respectively.

The simulated swing is embedded in the parametric optimizer as before, but the cost function to be minimized is now the sum of squares of the differences between the recorded arm and simulated arm angles and the recorded and

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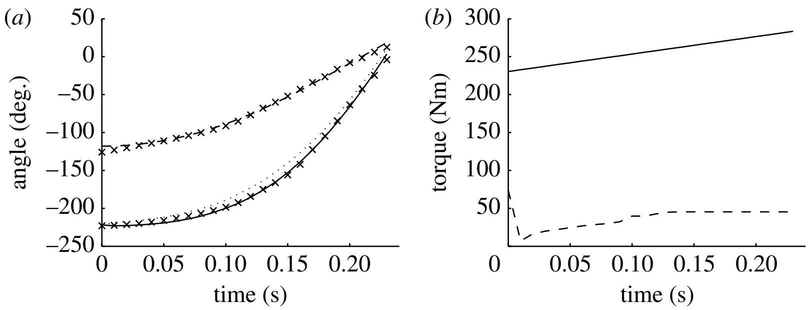

Figure 7. (*a*) Arm, grip and club angle, and (*b*) driving torque results for best matching Bernard Hunt's swing. Actual swing data are shown by crosses. (*a*) Dashed curve, arm; dotted curve, grip; solid curve, shaft; (*b*) solid line, arm; dashed curve, wrist.

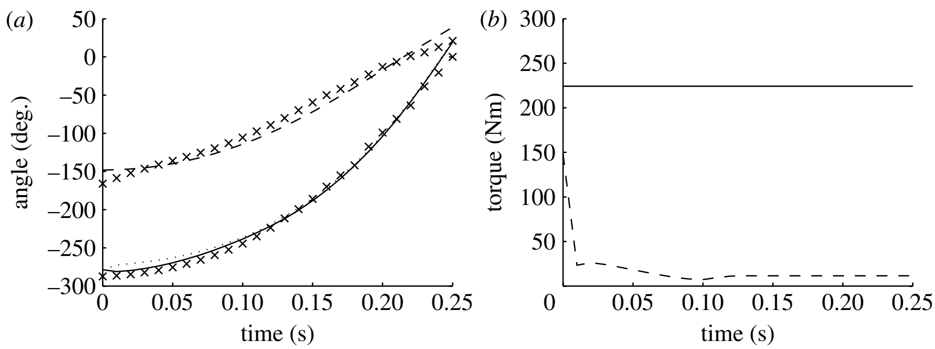

Figure 8. (*a*) Arm, grip and club angle, and (*b*) driving torque results for best matching Geoffrey Hunt's swing. Actual swing data are shown by crosses. (*a*) Dashed curve, arm; dotted curve, grip; solid curve, shaft; (*b*) solid line, arm; dashed curve, wrist.

simulated shaft angles. Results giving a best fit for Bernard Hunt are shown in figure 7, for Geoffrey Hunt in figure 8 and for Guy Wolstenholme in figure 9.

Model swings are capable of matching real swings quite well, although there are systematic differences between measured and simulated results. Bernard Hunt's swing is unusually short and his arm torque builds modestly through time. His wrist torque is perhaps unrealistically high. Geoffrey Hunt's arm torque is steady, while Guy Wolstenholme's contrasts with Bernard Hunt's in starting high and falling. The arm torques are all substantially linear with time, since the optimizer has chosen τ_{a2} to be near to zero, in each case. The model swing is also capable of yielding club-head speed results similar to Jorgensen's but the details are omitted in the interests of brevity.

3. Mechanics of the shoulder–arm–club swing

Even though the arm–club swing model, as enhanced with shaft compliance, gives a reasonable match to real swings, the kinematic mismatch between that and the real thing is troublesome, possibly leading to compensating errors in the

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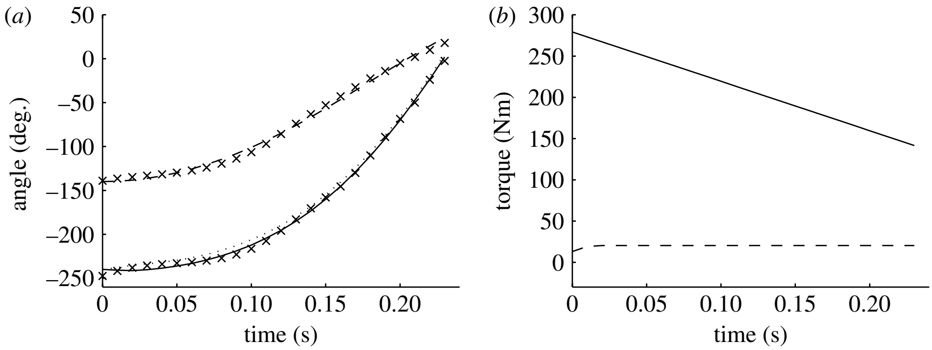

Figure 9. (*a*) Arm, grip and club angle, and (*b*) driving torque results for best matching Guy Wolstenholme's swing. Actual swing data are shown by crosses. (*a*) Dashed curve, arm; dotted curve, grip; solid curve, shaft; (*b*) solid line, arm; dashed curve, wrist.

torque computations. The arm–club model can easily become a shoulder–arm–club model by (i) including a new shoulder body as the first in the kinematic chain, free to rotate about the fixed point, n0, (ii) defining a point in that body, corresponding to the left shoulder joint location (of a right-handed player), (iii) joining the arm to the shoulder joint, including an arm to shoulder limit stop and (iv) redefining the base driving torque as acting on the shoulder body and the arm torque as being reacted on the shoulders.

In order to obtain good matching of simulated swings to the real swings on the record, the driving torques need to be defined by parameters allowing some flexibility of form. Each torque has a maximum magnitude $\tau_{\rm smx}$, $\tau_{\rm amx}$ and $\tau_{\rm wmx}$ for shoulders, arms and wrists, respectively, and can vary within the range defined. The shoulder torque is assumed to start at its maximum positive, driving-forward, value, after a short build-up time specified by the function $\tanh(\lambda t)$, with $\lambda = 100$ typically; then it is allowed to decrease linearly with time, at a rate $\tau_{\rm s2}$, beyond a time $t_{\rm s}$. The arm torque builds linearly with time, at a rate $\tau_{\rm a1}$, to its maximum and can then decrease in the same way as the shoulder torque at rate $\tau_{\rm a2}$ after time $t_{\rm a}$. The arm limit-stop torque, when acting, reacts on the shoulders. The wrist torque is as it was in the arm–club model, fully negative to start and switching to fully positive at time $t_{\rm w}$ through the function $\tanh(\lambda(t - t_{\rm w}))$. The limit-stop torque acts on the grip and reacts on the arms.

A few parameters differ from those given in table 3, variants being specified in table 4.

(*a*) *Fitting the shoulder–arm–club model to real swings*

Since the swing data available do not show the condition of the shoulders directly, it is necessary to impose a shoulder starting angle on the simulations and to include in the cost function a term that constrains the angle at impact to have a pre-ordained value. The value chosen in each case has to be physically reasonable and to encourage a good fit between theory and experiment. The swing data from Cochran & Stobbs (1968) need to be changed from the angle form used earlier to (y, z) coordinate form for the present fitting computations,

Table 4. Variations from and additions to parametric data of table 3, in S. I.

parameter	symbol	BJH	GMH	GW
shoulder inertia	I_{s}	0.8	0.61	0.61
arm principal inertia	I_{a}	0.354	0.27	0.27
mass of arms	M_{a}	8.644	7.35	7.35
arm-stop stiffness	C_{sa}	5000	5000	5000
arm-stop damping coefficient	D_{sa}	5	5	5
arm-joint y-coordinate	y_1	0.21	0.19	0.19
arm mass centre to n0	z_1	0.343	0.322	0.322

due to the greater freedom inherent in the shoulder–arm–club model. The data in coordinate form, found by hand from the scale diagrams in the book, are given in the electronic supplementary material, appendix B.

Variables in the optimal matching problem are the torque maxima, τ_{smx}, τ_{amx} and τ_{wmx}; the rates, τ_{s2}, τ_{a1} and τ_{a2}; the times t_{s}, t_{a} and t_{w}; the arm length, z_2; the wrist joint to club-head centre distance, z_5; and the starting angles for arms and grip. As the starting angles are adjusted, so are the limit stops, such that the downswing always commences with the relevant members just touching the stops. Starting velocities are taken to be zero. Notwithstanding the large number of variables in the problem, the computations are well behaved, provided that reasonable starting values are used. Best-fit parameter vectors found are

— for Bernard Hunt [272.64 209.34 34.67 0^* 20 332 -121.67 0.298^* 0 0.019 $-$ 0.8538 -1.084 -1.056 -1.556],
— for Geoffrey Hunt [185.54 162.04 31.70 35.30 4095 0^* 0.328^* 0.0913 0 -0.7181 -1.057 -1.203 -2.354], and
— for Guy Wolstenholme [236.84 208.57 29.638 -14.696^* 5234.416.274 0.263^* 0.0638^* 0 -0.7220 -1.102 -1.101 -1.748].

with respect to the variables specified. If switch times are after impact or the rates are too small to be significant, the parameters have either no or very little influence on the swing, and they could be omitted without detriment. Such parameters are marked with an asterisk in the vectors. Note Bernard Hunt's long arms, detected by the optimizer, and his strength. In each case, once the shoulder and arm torques have built up, they are maintained.

Measured swings and best-fit simulated swings are illustrated in figures 10–12, where coordinates of shoulder joint, wrist joint and club-head centre are shown in figures 10*a*, 11*a* and 12*a* with arm and club centre lines at the start and finish of the downswing shown dashed. The driving torques used by the golfer are shown in figures 10*b*, 11*b* and 12*b*. Impact speeds are $55.3\ \mathrm{m\,s^{-1}}$, $53.7\ \mathrm{m\,s^{-1}}$ and $57.7\ \mathrm{m\,s^{-1}}$, respectively.

In agreement with conventional wisdom, all three swings start with the shoulders. For a short time, the arms are against their limit stops and the arm torque builds at a finite rate to the point where the arms start to rotate relative to the shoulders. Then the arm torque is sustained through impact. All three golfers provide arm torque that is always less than their peak shoulder torque,

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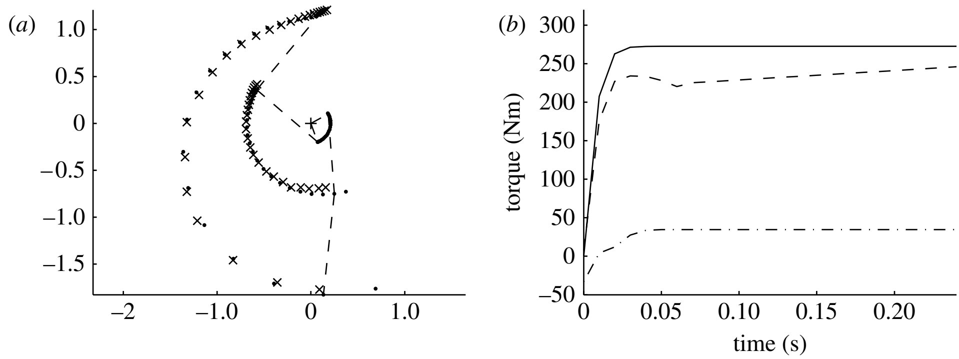
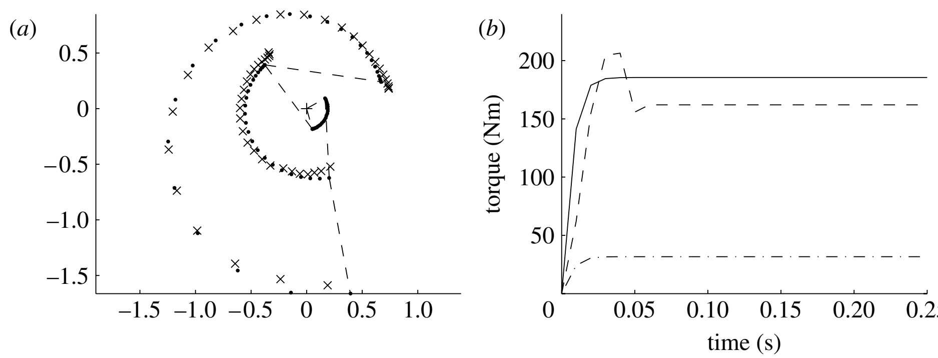
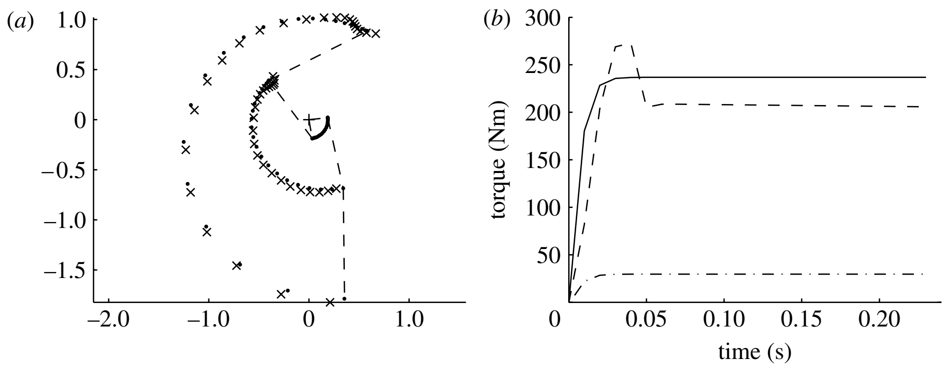


Q2 Figure 10. (*a*) Successive positions of shoulder joint, wrist joint and club head at 10 ms intervals, and (*b*) driving torque results for the best matching to Bernard Hunt's data (solid curve, shoulder; dashed curve, arms; dot-dashed curve, wrist). Actual swing points are shown by crosses.



Q3 Figure 11. (*a*) Successive positions of shoulder joint, wrist joint and club head at 10 ms intervals, and (*b*) driving torque results for best matching to Geoffrey Hunt's data (solid curve, shoulder; dashed curve, arms; dot-dashed curve, wrist). Actual swing points are shown by crosses.



Q4 Figure 12. (*a*) Successive positions of shoulder joint, wrist joint and club head at 10 ms intervals, and (*b*) driving torque results for best matching to Guy Wolstenholme's data (solid curve, shoulder; dashed curve, arms; dot-dashed curve, wrist). Actual swing points are shown by crosses.

except for the arm limit-stop influence. None of the players use their wrist action to delay the release at all. Bernard Hunt's short swing is compensated by his great strength.

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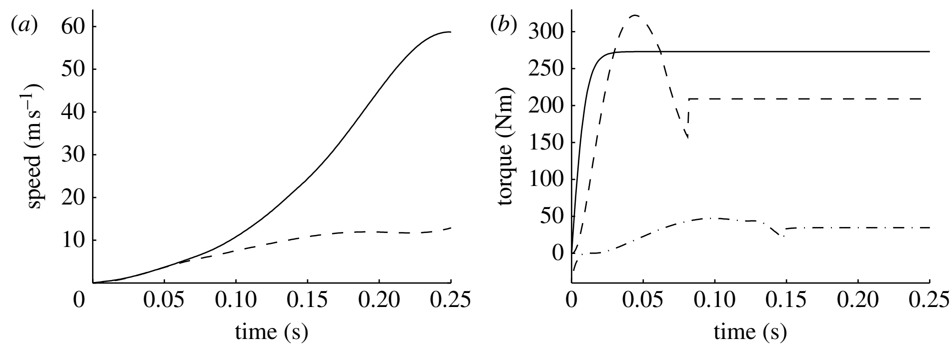

Figure 13. (*a*) Club-head centre (solid curve) and wrist (dashed curve) speeds and (*b*) driving torques for near-optimal swing for Bernard Hunt (solid curve, shoulder; dashed curve, arms; dot-dashed curve, wrist).

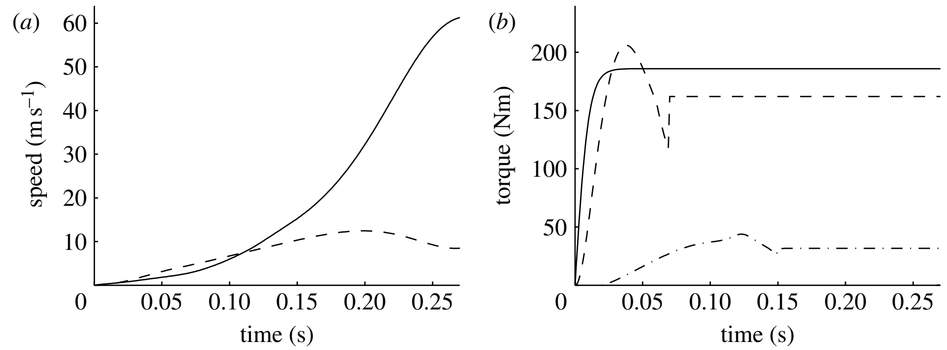

Figure 14. (*a*) Club-head centre (solid curve) and wrist (dashed curve) speeds and (*b*) driving torques for near-optimal swing for Geoffrey Hunt (solid curve, shoulder; dashed curve, arms; dot-dashed curve, wrist).

(*b*) *Near-optimal swings*

Let us regard the above results as establishing good parametric descriptions of our three golfers and let us determine whether or not they could do significantly better. To this end, the maximum torque and swing length parameters must be treated as given, while the other parameters, to do with control, can be chosen by the optimizer. In fact, due to the lack of need for diminishing either shoulder or arm torques late in the swing, the parameters τ_{s2}, τ_{a2}, t_s and t_a are unnecessary and the only variables that now need to be considered are the rate of build-up of arm torque, τ_{a1}, and the wrist torque switch time, t_w. The optimization criterion reverts to the maximization of the club-head speed at impact, with the same constraint on the finish value of the shoulder angle as used in the matching problem discussed above. The best swings are illustrated in figures 13, 14 and 15, where club-head and wrist speeds are shown 13*a*, 14*a* and 15*a* and driving torques are shown on the 13*a*,*b* and 14*b*. Further illustrations are included in the electronic supplementary material, appendix C. Best parameters for Bernard Hunt are $\tau_{a1}=3326$ and $t_w=0.1317$; for Geoffrey Hunt, $\tau_{a1}=2712$ and $t_w=0.1215$; and for Guy Wolstenholme, $\tau_{a1}=5188$ and $t_w=0.1237$. Club-head speeds at impact are now 56.28 m s^{-1}, 57.21 m s^{-1} and 59.61 m s^{-1}, better than before

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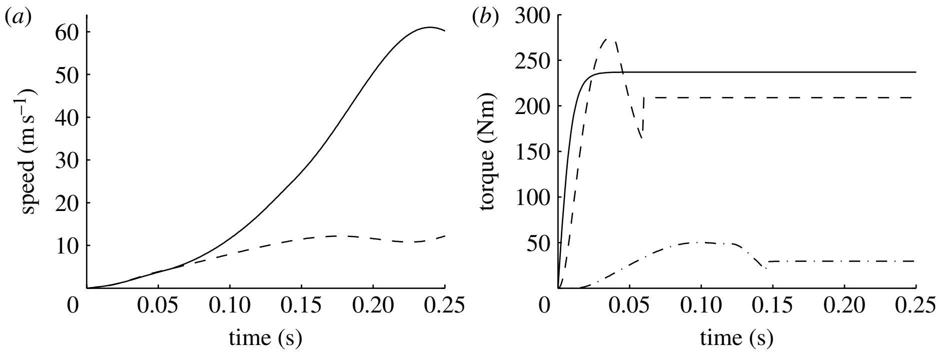

Figure 15. (*a*) Club-head centre (solid curve) and wrist (dashed curve) speeds and (*b*) driving torques for near-optimal swing for Guy Wolstenholme.

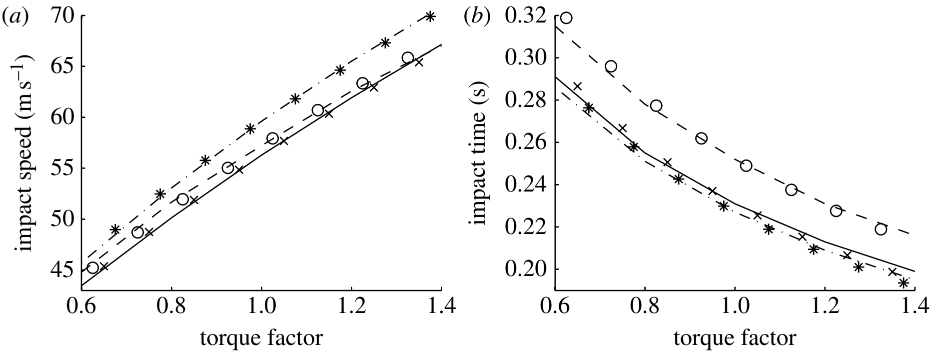

Figure 16. Influences of torque factoring on (*a*) club-head speed at impact and (*b*) downswing duration (solid curve (times), B. J. Hunt (BJH); dashed curve now only doing (unfilled circle), G. M. Hunt (GMH); dot-dashed curve (eight point star), G. Wolstenholme (GW)).

by 1.8, 6.5 and 3.3 per cent, respectively. A similar pattern for the optimal torques was given by Sprigings & Mackenzie (2002), although their shoulder and arm torques were only around half the values found here. A relevant factor is possibly their (unstated) golfer biomechanical data. The constant torques used by Turner & Hills (1999) to produce reasonable downswings were 105 N m for the shoulders, 75 N m for the arms and 20 N m for the wrists: their shoulder inertia was low, their arm mass slightly low but their arm inertia quite high in relative terms.

Shoulder and arm torques are used in a similar way to those identified in the real swings but, in these near-optimal swings, the arm torques are applied a little later. The wrist torques do not show a holding-back phase. The changes to them seem rather to result from the extra delay in applying the arm torque. All three swings show the pattern in the arm velocity pointed out by Williams (1967) for Bobby Jones that phase 1 involves the arm speed increasing uniformly with time, while phase 2 involves almost constant arm speed. In the case of Geoffrey Hunt, his arms actually slow down a little in phase 2.

When the optimizations are repeated with driving torques factored by 0.6, 0.8, 1.2 and 1.4, the swing patterns are largely preserved. With less torque, the downswing takes longer and the club-head speed at impact is lower (figure 16).

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Table 5. Dimension matrix for the golf swing.

	length, l	mass, m	inertia, I	torque, τ	gravity, g	density, ρ	speed, V
M	0	1	1	1	0	1	0
L	1	0	2	2	1	−3	1
T	0	0	0	−2	−2	0	−1

In the figure, the plot symbols represent the variations that would occur if the speed were proportional to the square root of the torque factor, as expected by Williams (1967), and the time were proportional to the inverse of the square root of the torque factor.

4. Scaling

The question of how much advantage a large golfer has over a small one is often asked. Let us apply similarity ideas from dimensional analysis, Langhaar (1951), to the issue. The club-head speed at impact is a function of length, mass, inertia, torque, shaft stiffness, gravitational acceleration and air density. For similarity, all quantities with the same dimensions have to be scaled by the same factor. Strictly, a small golfer must be thought of as wielding a shorter driver than a large golfer. Following Langhaar, we construct the dimension matrix of table 5.

The rank of the dimension matrix is 3, so that there are four dimensionless groups of parameters in the problem of expressing the club-head speed as a function of the other variables. By inspection, the following dimensionless groups can be formed: $\pi_1 = I/ml^2$; $\pi_2 = \rho/mgl$; $\pi_3 = V^2/lg$ and $\pi_4 = (\rho^2 I^3)/m^5$. The functional relationship between the club-head speed and the independent variables can be written: $V^2 = lgf(\pi_1, \pi_2, \pi_4)$, where f is a general function.

Let us suppose a particular golfer to be length scaled by a factor λ. The natural mass scale factor resulting is λ^3 and the natural inertia scale factor is λ^5. π_1 is unaltered by the scaling and π_2 is unchanged if the torques are scaled by λ^4. The muscle-mass scale factor will be λ^3, as for the other masses, and, since the leverages will be scaled by λ the torques can be expected to scale by λ^4 as required. With g and ρ remaining the same, it follows that V is scaled by $\sqrt{\lambda}$. A 21 per cent bigger player can be expected to have just a 10 per cent advantage in club-head speed capability, helping to explain why good little ones are often not so far behind good big ones.

5. Conclusions

On the basis of extensive simulations that fit well to experimental results from the literature, the common notions that gravity, club-head aerodynamic drag and club-shaft flexibility are quite small influences have been confirmed by the results obtained, although the treatment of shaft flexibility has been by no means exhaustive. Lampsa's results, which claim to show the optimal muscular strategy, at the arm–club model level, have been shown to be capable of considerable improvement.

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Although an arm–club simulation model can be made to fit recorded-swing arm- and club-angle data well, it is misleading in terms of the torques used by the golfer. The arm–club model is sufficient to show the nature and undesirability of hitting too hard, too soon in the downswing. Generating too much arm speed too soon causes an early release, with the club-head reaching its maximum speed before it arrives at the ball. The extent to which the wrist action can be employed to hold back the release is limited by the torque capacity. In the expert swings studied, control of the arms and not the wrists appears to be the priority. The arm–club model is sufficient to show that the problem of hitting from the top becomes more likely as the backswing length increases.

The shoulder–arm–club swing model fits recorded-swing arm- and club-angle data well, if suitable constraints are applied in the model to the so-far-unmeasured motions of the shoulders. Many parameter values describing the golfers have to be estimated, reducing the degree of certainty in the results. All three expert golfers are represented as employing the full available shoulder torque at the start of the downswing and applying some delay before hitting through with their arms. The wrist actions used do not show reverse action. Neither does delayed release occur directly from wrist action when the swings are optimized but later application of arm torques must act to retard the release and to improve the efficiency of the swings. The optimal strategy consists of hitting first with the shoulders while holding back with arms and wrists and after some delay, hitting through with the arms. At release, the timing of which depends on the combination of shoulder and arm actions employed, the wrists should hit through. The golfer's common experience of hitting harder with worse results is explained clearly, if the hitting harder involves hitting from the top with everything. The usually undefined idea of perfect timing comes to have a clear meaning, consisting of precisely executing the necessary changes in policy to achieve the best result possible. Animations of the best swings achieved for each of the three golfers studied are included in the electronic supplementary material, appendix C.

There is considerable scope for new experiments to find successive positions at say 10 ms intervals in the established fashion, but including shoulder data. To know the golfer parameters better, trials in which the subject golfer performs elementary swings, first with torso only, next with torso and arms and finally with torso, arms and club, would be valuable. In order to deduce both the golfer's inertial and strength properties, avoiding the usual dependence of one on the other, known inertias could be added to the golfer's shoulders, grip and club head, in turn, and peak-effort swings repeated. Data reduction of the results from such swings would contain redundancy and would allow the establishment of best-fit parameters. It would also be advantageous to employ club data specific to the tests conducted, rather than relying on generic data.

Automation of the processes developed in the work reported seems to be entirely possible. The computations necessary are rapid, lending themselves to a personal service operation, in which a golfer would have swings of various kinds recorded and software would be used to reveal his mass and inertia properties, his strengths, his muscle usage and desirable changes to obtain better results with no more effort.

Dimensional reasoning shows that dramatic differences in performance between large and small players should not be expected on the basis of size alone. Strength and inertial variations seem more likely than size to account for long and short hitting.

Based on a standard shoulder–arm–club swing with a driver, sensitivities of the maximum possible club-head speed at impact with respect to variations in muscle strength, swing length and club descriptors are revealed in the electronic supplementary material, appendix D.

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Q7 Please note that the reference citation Jorgensen (1994) has been changed to Jorgensen (1999) with respect to the reference list provided.