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**Relationships Among Arm Strength, Wrist Release, and Joint  
Torques During the Golf Downswing**

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**Relationships Among Arm Strength, Wrist Release, and Joint  
Torques During the Golf Downswing**

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## **Dedication**

To my parents, Chu-Ching Tang and Li-Ying Lu Tang,  
and sisters, Wen-Wei Tang and Wen-Hsin Tang

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# **Relationships Among Arm Strength, Wrist Release, and Joint Torques During the Golf Downswing**

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Understanding the effects of body segment dynamics and interactions with strength boundaries on the coordination of the golf swing is crucial for improving swing performance and the design of effective training plans. We quantified kinematics of eight elite golfers performing normal golf swings, and simulated optimal solutions of the swing task with a series of mathematical models, each of which was based on a different number of body segments. We then compared these analytical solutions with the experimental data to determine the effect of segment number on the modeling and analysis of golf swings. Finally, we performed a series of optimizations involving modification of joint strength boundaries. We studied the effects of shoulder, elbow, and wrist strength boundaries on overall performance, wrist release timing, and segmental coordination.

Empirical results showed that the elbow joint should not be excluded from models of the golf swing because elbow movements often become substantial. Analysis of experimental data and optimal model results revealed that wrist strength plays a major role in golf swing performance. Simulation of golf swings indicated that increased wrist strength, yielding a delay in the wrist release, is more important for improving clubhead speed at impact than are shoulder and elbow strengths. Also, delay in wrist release timing alone is not the only available means for improving performance, as the overall joint strength profile is also an important determinant of clubhead speed. This study thus reveals that individual kinematic and dynamic characteristics of the swing must be evaluated to determine productive or counter-productive actions and to improve overall golfing performance.

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## **CHAPTER 1: INTRODUCTION**

The biomechanics of the golf swing has been studied for many years in an effort to optimize performance of golfers. Many kinematic and kinetic examinations have been conducted to analyze specific components of the swing movement and to compare groups that differ in swing performance. In particular, wrist action during the swing has been analyzed by a number of researchers. The cocking and uncocking action of the wrist during the downswing is believed to be critical to achieving maximum clubhead speed at ball impact, although other factors may also be important.

By comparing kinematic and kinetic data between low and high performing groups, most researchers have concluded that delay in wrist uncocking can increase clubhead speed during the swing (Jorgensen, 1970; McLaughlin & Best, 1994; Milburn, 1982; Nagao & Sawada, 1977; Williams, 1983). Also, Kaneko et al. (1993) used an optimal control model to reveal that club design is related to swing optimization, and to show that the dynamic characteristics of the club are related to the optimal release time of wrist uncocking. Campbell and Reid (1985) used a two-segment optimal control model to investigate several characteristics of the golf swing before impact, and found that the shoulder torque first achieved a constraint boundary, followed by the wrist, suggesting that wrist action should be



delayed to improve performance. By contrast, other researchers who employed a dynamic model suggested that delayed wrist uncocking may not be necessary for a powerful swing (Budey & Below, 1979), and that the timing of wrist uncocking does not influence club speed (Mason et. al., 1996).

In addition to wrist timing issues, the peak torques generated by the linked body-arm system may be important. Lampsa (1975), using an optimal control model, showed that the torso torque did not limit the swing, but also that the shoulder and wrist torques did reach their constraints and so may be limiting factors. Furthermore, electromyographic (EMG) data indicated that the relative activation of the trunk musculature is much less than that of the shoulder muscles during the swing (Pink et. al., 1990; 1993; Watkins et. al., 1996). As torque and EMG activity are positively correlated, these data suggest that shoulder torque dominates the swing.

However, the relationships among wrist uncocking release time, arm strength, and torque during the swing have not yet been characterized. The purpose of this study was to obtain kinematic and kinetic data necessary to determine the relationships among optimal wrist uncocking release time, arm strength, and swing performance. Optimal control modeling was used to test hypothetical predictions based on experimental data. This research can also be

used to predict optimal kinematics and strategies for improving swing performance.

## **1.1 Purposes**

To date, the relationships among release time, swing performance, and arm strength have not been resolved. Therefore, the purposes of this study are to obtain the experimental kinematic and kinetic data necessary to verify the relationship between wrist uncocking release timing and clubhead speed at ball impact, and to determine the relationship between wrist uncocking release time and the arm strength necessary to produce maximum clubhead speed. Optimal control modeling was used to test hypothetical predictions, and observed data was used to verify the model's parameters. Due to the differences between the optimal torque patterns obtained from two-segment modeling and from an actual swing, this study was use a three-segment model and a four-segment model to determine the most appropriate segment number of model.

This study also examined the relationship between optimal release time and arm strength. Based on these results, strategies for improving swing performance can be derived. The results also contributed to the understanding of the importance of joint torque limits, particularly in the context of golf swing dynamics and implications for reduction in potential injuries.

## 1.2 Hypotheses

In this study, two hypotheses about the optimal wrist release time were tested.

1. The optimal time for the initiation of the wrist release is positively associated with clubhead speed at impact.
2. The optimal time for initiation of wrist release is
  - a) positively associated with wrist strength,
  - b) inversely associated with shoulder joint strength, and
  - c) inversely associated with elbow joint strength.

## 1.3 Definitions

Clubhead speed: the magnitude of the linear velocity of the clubhead, equal to the magnitude of the product of angular velocity and the distance of the clubhead from the center of rotation plus the velocity of the center of rotation.

Double pendulum: two pendula or levers hinged in the middle which swing around a fixed center of rotation (Cochran & Stobbs, 1968).

Triple pendulum: three pendula or levers with two hinges which swing in a single plane of motion (Cochran & Stobbs, 1968).

Planar: motion performed in a single plane.

Non-planar: motion performed in more than one plane.

Wrist release: wrist ulnar deviation from a radial deviated position during downswing, similar to 'wrist uncocking'.

Wrist cocking: wrist abduction from a neutral position during back swing.

Wrist uncocking: wrist adduction (ulnar deviation) from an abducted (radially deviated) position during the downswing (Cochran & Stobbs, 1968).

Performance index: the index to be maximized or minimized in an optimal control program.

## **1.4 Delimitations and Limitations**

### **1.4.1 LIMITATIONS**

1. For experimental results, the precision of the data is limited by the accuracy and reliability of the data acquisition system. The derivation of force and torque functions comes from the recorded data of participants. It is important that measured displacements be precise and that the swings be typical for the participants. The accuracy of the data acquisition system used here is 2 mm. The precision of the data is also limited by the consistency of each golfer. Therefore, a number of swings are needed to ensure accuracy of the data.
2. The participants were drawn mainly from two elite groups: professionals from the Barton Creek Country Club, and the Women's Golf Team of the University of Texas at Austin. Participants from the same group may swing with a similar pattern because of sharing the same knowledge resources, ability, or coaching. There are probably other factors that contribute to swing pattern and overall performance as well. The investigation of these factors is outside the scope of the present study.
3. The modeling simplifications in this study are as follows:
  - The segments are modeled as rigid masses.
  - The joints are simple frictionless hinge joints.

- Joint torques are generated and applied instantaneously.
- The trunk is modeled as one segment only.
- Anthropometric parameters are obtained from statistical tables based on cadaver studies as well as measurements of the subjects.

These simplifying assumptions are made for the implementation of the dynamical model in this study, and these assumptions are necessary so that the model remains feasible. Models with such assumptions are widely accepted in the literature. Furthermore, the focus of this study is to examine the patterns of joint torque among golfers with different physical limits, but not to determine the exact joint torques necessary for an optimal swing.

4. The swing motion in the model is limited to planar motion. From literature reports (Cochran & Stobbs, 1968; Soriano, 1997), the planar motion of the golf swing has been verified.
5. The number of subjects is limited, but constraints in this model can be adjusted to include the physical limits of the participants recruited for this study.

#### **1.4.2 DELIMITATION**

Study findings are restricted to healthy, medium-sized, elite golf athletes.

## **1.5 Significance of the Findings**

One of the important goals in golf research is to provide knowledge about the proper and efficient swing. The dynamic analysis of an actual swing and the optimal control model used in this study can lead to an understanding of the relationship between strength and the optimal swing pattern, and can enable the reader to understand the physical demands placed on golfers to prevent injuries. In addition, due to individual differences in body shape and strength, having the best golf game performance may not mean that every golfer has to have the same swing. Individual characteristics may then need to be evaluated to determine productive or counter-productive actions and to improve performance. The information from this study can help golfers in designing a suitable personal plan to increase distance in an efficient way.

Ultimately, the employment of the model in computer simulation studies allows one to determine the contributions of various elements to the overall swing in golf. This model provides insight into the proper biomechanics of the golf swing for each individual, especially the relationship between movement dynamics and strength.

## **CHAPTER 2: LITERATURE REVIEW**

This section is arranged in three main parts. The first consists of a brief review of related fundamental mechanics of the golf swing to help unfamiliar readers. In part two, the related literature on wrist cocking and uncocking motion is presented and conflicting results are described. In the third part, the literature relevant to optimal control theory and golf is presented.

### **2.1 Fundamental Mechanics of the Golf Swing**

A large number of books, instructional manuals, scientific journals, and magazine articles have been written about the golf swing. However, due to the variety of sources contributing to golf swing knowledge, and since many reports are largely based on personal experience, a certain amount of controversy exists among coaches and players regarding the relative importance of fundamentals in the golf swing. Therefore, advanced biomechanics information is needed to resolve this controversy.

The first examination of dynamic impact effects on the ball and club was provided by Cochran and Stobbs (1968). They developed a simple double pendulum model based on photographic analysis of various subjects. From this analysis, they identified certain fundamental concepts: the swinging of the hands around a central hub or axis of rotation, and the swing traveling in an inclined



plane around a central hub. These two characteristics of the golf swing have been used extensively in other studies.

An obvious way for an aspiring golfer to increase clubhead speed is to increase the torque he or she is able to apply to the rotating system. However, by using a double pendulum model, Jorgensen (1999) showed that even if the shoulder torque increases by 5% in the swing, the clubhead speed at impact increases by only 1.7%. Also, a 5% decrease in the torque produces only 1.8% decrease in clubhead speed at impact with the ball. The clubhead speed at impact also is reduced by 8.5% without the influence of gravity. Based on the principles of Momentum and Energy Conservation, if the club head speed increases, then the ball flight speed increases. Of course, clubhead speed or ball flight speed is not the only determinant in golf. The loft (angle of the striking surface) of the club will also change the flight distance of the ball. If the loft of the club is higher, the ball will fly higher and shorter with the same clubhead speed at impact. On the other hand, when the hands of the golfer are ahead of the ball at impact, the ball flight angle will be decreased and the ball will fly lower. To simplify the problem, dynamic analyses consider only the clubhead speed to predict effects on subsequent ball flight.

## **2.2 Wrist Action**

### **2.2.1 WRIST ACTION KINEMATICS**

Wrist action during the golf swing has been analyzed by a number of researchers. Many kinematic and kinetic analyses have been performed to compare groups that differ in swing performance on specific components of the swing movement. From these analyses, late wrist uncocking is generally found to be characteristic of low handicap golfers (Jorgensen, 1970; McLaughlin & Best, 1994; Milburn, 1982; Nagao & Sawada, 1977; Williams, 1983). The cocking and uncocking action of the wrist during the downswing is therefore thought to be related to achieving maximum clubhead speed.

Jorgensen (1970) used a double pendulum model simulation and showed that the clubhead velocity achieved during the downswing increased with an imposed delay in the uncocking of the wrists. This simulation led to the conclusion that an exclusively free hinge at the wrist joint may not result in optimal downswing action, and that a negative torque about the wrist may be necessary during the swing to counteract the centrifugal force associated with arm rotation, thus preventing early uncocking of the wrist.

Lamps (1975) used a double pendulum model to describe the kinematic characteristics associated with the achievement of maximal club head velocity. Results from this investigation indicated that the uncocking of the wrist must be delayed in order to maximize clubhead velocity. This research supported the findings of Jorgenson (1970).

McLaughlin and Best (1994) used a larger sample size and additional statistical techniques to analyze kinematic parameters of the golf swing, and generally agreed with earlier literature which emphasized the delay of wrist uncocking. It has also been recognized that uncocking the wrists too early decreases the ability to produce large clubhead speeds (Williams, 1983).

Some researchers argued that a delay of wrist uncocking may not be necessary or appropriate to maximize clubhead speed. Williams (1967) studied torque and power generation of golfers during their swing motion. A double pendulum model was used and the equations of motion were derived and solved with joint torques as the input. The results showed that forceful extension of the wrist at impact does not significantly increase the clubhead velocity. Budey and Below (1979) used a double pendulum model with a kinematic modification, namely shoulder rotation briefly before the impact, instead of delayed wrist motion applying maximum effort for only a short time. The mathematical model showed impressive improvements, but the equivalent physical modifications were impractical.

Mason et. al. (1996) analyzed the swings of 64 Professional Golf Association (PGA) third year apprentices. A statistical analysis showed that a delay in uncocking of the wrist was not related to increased clubhead speed. However, the subjects may not have displayed enough variability in this factor to draw firm conclusions.

### **2.2.1 WRIST ACTION DYNAMICS**

Information about the strength of arm joints related to the swing has been obtained in several studies. Using a double pendulum optimal control model, Campbell and Reid (1985) showed that the shoulder torque achieved the constraint boundary first, followed by the wrist torque, prior to impact. A similar double pendulum optimal control model showed that the torso torque did not limit the swing, but that the shoulder and wrist torques did impose such a limit (Lampsa, 1975). EMG data indicate that the activation of the trunk musculature is much less than that of the shoulder muscles during the swing (Pink et. al., 1990; 1993; Watkins et. al., 1996). From these results, it appears that arm and wrist strengths are related to the clubhead speed at impact.

The most important joint strength for a golf swing may also be inferred from injury reports. McCarroll and Gore (1982) conducted a survey of professional golfers regarding injury, and found that 190 out of 226 respondents had been injured as a direct result of their profession. The left wrist was most commonly injured (27%), followed by the low back (24%) and left hand (7%). Another study examined the injury rates of amateur golfers, as self-reported in a survey, and found similar results (Duda, 1987). The highest incidence of injury was to the lower back (24%), followed by the elbow (23%), the hand and wrist (14%), the shoulder (8 percent) and the knee (6.5%). From these studies, it appears that professional golfers may rely on their wrist strength and approach a wrist torque limit much more than amateur golfers. Conversely, amateur golfers appear to use elbow power more than professional golfers. In summary, strength

is a factor for power development in the golf swing, but all other sources, such as the torque profile of each joint, must also be well coordinated to enhance performance.

### **2.3 Optimal Control**

The application of optimal control theory to the study of golf has been attempted by only a few researchers. Most optimal control studies were conducted to investigate how the characteristics of swinging or of the structure of the club are related to swing performance. Lampsas (1975) derived optimal control torques about the shoulder and wrist with a performance index based on the clubhead speed for a planar double pendulum model. Comparing the derived optimal torques and computed actual torques from film data revealed that the uncocking of the wrists should be delayed as long as possible to maximize clubhead speed. However, it is not clear neither how the actual torques were calculated from the film data, nor what simplifying assumptions were used. The results suggested that no subject, including elite golfers, actually swings a golf club with the derived optimal torques. This study also suggested that increased strength is not necessarily required for longer drives.

Kaneko et. al. (1993) used a three-segment optimal control model to derive an individual optimal golf swing that included the effect of physical characteristics of the club. The performance index used was the clubhead velocity at impact divided by the work done at the upper limb joints during

downswing. This performance index was defined as the efficiency of a golf swing. The results showed that dynamic characteristics of the club were related to swing optimization and to the timing of the wrist uncocking motion. For a heavy club, the delayed uncocking of the wrist was found to be an advantage for an efficient swing. However, that study did not characterize the relationship between arm strength and the timing of the wrist uncocking motion.

From the above literature review, it appears that wrist release time should be delayed to increase the clubhead speed at impact, that shoulder strength influences clubhead speed up to a limit, and that wrist strength may play an important role in swing motion. The optimal wrist release time is increased if the club mass increases. Therefore, it is plausible to conclude that swinging a golf club may require a different optimal release time according to the particular arm strength profile. In order to arrive at a comprehensive description of swing patterns and their effects on performance, a dynamic study of the golf swing is clearly needed.

## **CHAPTER 3: METHODOLOGY**

### **3.1 Instrumentation**

The two main components of this study are the quantification of the golf swings and mathematical modeling. Golf swings were performed, recorded, and quantified within a 3m wide x 6 m long x 5 m high enclosed volume located in a large biomechanics laboratory. All modeling computations were performed on a personal computer and a computer workstation.

Specific instruments that were used in this study include:

1. A state-of-the-art three-dimensional video motion analysis system, Locus MA-8000 (Anima Corp., Tokyo) with six high-speed CCD cameras and reflective markers. Sampling rate was at 240 Hz.
2. A driver for the cameras and a desktop personal computer equipped with a video board and software (MA-8000 package from Anima Corp., Tokyo) to collect video image files and process the files into three-dimensional data.
3. Autolev, a dynamical simulation software package, for the formulation of the equations of motion and forward simulation routines.
4. One desktop computer with software (Autolev, Matlab, Fortran compiler) to analyze video data, to run forward simulations, to run inverse dynamic

simulations, to solve minimization problems, and to serve as an instructional display during data collection.

5. A workstation Indy 5000 (SGI Corp.) with a Fortran compiler package to solve minimization problems.
6. Reflective markers attached to the subject and to the golf club, providing the spatial data to be captured by CCD cameras.

### **3.2 Design**

To answer the study questions, the following procedure was followed. First, a motion analysis system was arranged to collect swing data accurately from the subjects. Anthropometrics were obtained to derive the participants' inertia and mass property values. Next, a dynamic system model of a human swinging a golf club was developed, including the governing differential equations. Then, using the anatomical anthropometric measurements and calculated kinematic data, an inverse dynamics analysis was performed to obtain the torque history for the model. The torque history was used as an initial input for the optimal control modeling. (If the model did not match the actual swing, then the complexity of the model was enhanced until the model provided a good representation of the experimental data.) Complexity was added to the model in the form of additional degrees of freedom, either additional segments or actions at selected joints.



### **3.3 First Study: Experimental Analysis**

#### **3.3.1 SUBJECTS**

Four elite women golfers from the varsity golf team of The University of Texas at Austin, and four male professional golfers from Barton Creek Club participated in this study as volunteers after signing the consent forms. The consent form is given in Appendix A. The subjects ranged in age from 21 to 25 years old and were highly skilled golfers (a handicap index less than 5). The handicap system developed by the United States Golf Association (USGA) was used to classify the test subjects. This system gives a ranking to participants based upon their golfing skill. A description of the USGA handicap system is given in Appendix B.

#### **3.3.2 TEST ADMINISTRATION**

##### **3.3.2.1 Task Preparation and Definition**

A standard 5 iron was selected for use in data collection. This club is at intermediate length, is stiff enough to be modeled as a rigid segment, and is similar to clubs analyzed in previous studies. Each participant was outfitted with 16 reflective markers on his/her joints, and the test golf club was equipped with 3 reflective markers. The markers were attached with adhesive tape to the skin at the wrists, elbows, shoulders, trunk, hips, knees, and ankles. The details of marker placement are described in Appendix C. Two semi-spherical markers were

attached to the clubhead, and associated data were used to evaluate the linear hitting direction. Participants were tested standing on a mat and hitting the ball, which was suspended by a short cable from a low bar so that it would not travel far when struck (Fig 3.1). Each participant was allowed as many practice swings and practice hits as necessary to gain confidence with the testing environment. After standard warm-up and some drills involving wrist release, their subsequent motion was recorded. The task for participants was to produce the best possible swings in terms of accuracy and speed.

### **3.3.2.2 Set-Up Position**

The initial condition started from a relaxed quiet stance with slightly bent knee and straight arm. The order of the swing movement was as follows: ball address, backswing, downswing, ball contact, and follow-through. A pause usually occurred at the top of the swing between the backswing and downswing, indicating that the angular velocities of club and body were zero at that time (Geisler, 1996).

### **3.3.2.3 Data Collection Procedures**

Six high speed CCD cameras with high resolution capture were used to measure high speed movements with adequate resolution. Video data were also collected at a rate of 240 Hz for 3 s per trial. Video data for ten successful (or best) and consistent trials were collected. The positions of the cameras were

determined by a pilot study to confirm that each marker could be tracked, and also that each camera could irradiate sufficient infrared light to illuminate the reflective ball markers. Image data were saved on the computer hard disk through a video capture board, and the markers on the image were automatically tracked and digitized. The entire process was controlled by a desktop computer. The digital data were transferred to 3D positional data using the software provided with the system run by the computer.

The golf swing motion was defined relative to an inertially fixed Cartesian coordinate system, where the positive  $x$ -axis was aligned horizontally, pointing in the direction that the participant faces; the positive  $y$ -axis was also horizontal, pointing along the direction of flight of the ball; and the positive  $z$ -axis was upward, perpendicular to the ground. With an anterior view of the participant, the projected body motion onto the  $y$ - $z$  (coronal) plane can be seen. The motion of the reflectors was in 3D, and from this setup accurate measurements of the wrist extension angle in 2D could be obtained. All previous golf studies have assumed that the wrist is a hinge joint with 2D motion (Cochran & Stobbs, 1968; Lamps, 1975; Soriano, 1997).

#### **3.3.2.4 Isokinetic Measurements**

Isokinetic wrist, elbow, and shoulder torques were measured for participants as a gross indicator of participant strength. Measurements were performed using a Biodex Dynamometer owned and operated by the Division of Women's Athletics at The University of Texas at Austin. The participants'

maximum peak isokinetic wrist, elbow, and shoulder torque through five trials, defined as strength, were compared to the peak torque derived from the actual swing motions in order to examine how closely the participants reached a strength boundary. On the other hand, peak isokinetic torques were determined to establish constraints on the control variables as functions of the states of the system in the optimal control model, and also were compared to optimal peak torque values in the model.

### **3.3.3 DATA PROCESSING AND ANALYSIS**

#### **3.3.3.1 Kinematic Data**

The first step was to process the 3D data transferred through the package software from the system. Then, the trajectories of each marker were smoothed using a zero phase-lag Butterworth second-order digital filter. The cutoff frequency was decided from residual analysis of all markers to obtain the best cutoff frequency, and varied across trial and markers from 7 to 14 Hz. In some cases, trials were discarded at this point because one or more markers could not be tracked accurately. The decision of the number of missing frames was set to 8. This was most common near impact when markers were moving fastest.

The following formula was used to calculate joint angular positions:

$$\vec{A} \bullet \vec{B} = |\vec{A}| |\vec{B}| \cos \mathbf{q} \quad (3.1)$$

where  $A$  and  $B$  are the vectors from the joint to the ends of the two segments, and

? is the joint angle between the two segments. For the trunk joint angle, however, the definition was the angle between the upper thorax and the horizontal axis projected in the swing plane.

From marker position data, joint angle histories can be computed. The time at the start of the downswing was known, as was the time of impact. Thus, a plot of the joint angle during the downswing through impact was generated in a straightforward manner. Angular velocity and acceleration curves were calculated using a central finite difference method for each trial. Average plots for all trials of each subject and all subjects of the above plots then were obtained.

Definition of and an example of the wrist release angular position during downswing are shown in Figure 3.2. The other joint angles are shown in the same manner in chapter 4. The wrist release point is defined as the beginning point of the wrist ulnar deviation movement (Soriano, 1997). Wrist release initiation was judged to occur during a sharp change in wrist position, and was identified as the time when wrist angular velocity increased from zero. This instant is graphically exhibited by the rapid change in the slope of the curve of wrist angle versus time. The full extension point of the wrist as it relates to the impact point is seen as the maximum value or the peak of the curve.

### **3.3.3.2 Kinetic Data Derived from Inverse Dynamics**

The resultant force and torque for each joint were derived through the Inverse Dynamics method. The human body is a very complex mechanical entity.

The combination of our knowledge and our available computing power limited us to constructing simple mathematical models.

The dynamic model simplifications used in this study were:

-The segments were modeled as rigid masses.

-The joints were simple frictionless hinge joints.

-The inertia parameters of the model were derived based upon anthropometric measurements and equations developed by Dempster via Winter (1990).

Newton's third law of Action-Reaction was applied on interactions between segments. D'Alemberts principle ( $\mathbf{SF} - m\mathbf{a} = 0$ ,  $\mathbf{ST} - I\mathbf{a} = 0$ ) was used to calculate net joint forces and torques. The net joint torque induced by the muscles crossing each joint as well as by ligaments and by all other soft tissues was modeled as the net joint moment. The bone-on-bone forces were modeled with a pair of perpendicular vectors acting at the center of each joint onto each segment. With the derived function and with position, velocity, and acceleration information, the inverse dynamics problem was then solved and the net joint forces and torques were obtained (Winter, 1990).

Joint torque patterns for all participants were compared. The participants' potential ability was evaluated from the net joint torques and the strength test results. The peak torque and strength of each joint were exhibited as a scatter plot and a correlation test was performed among strength, applied torque, wrist release initiation, and performance (clubhead speed at impact).

## **3.4 Second Study: Optimal Control Model**

### **3.4.1 MATHEMATICAL MODEL**

A full model of the golf swing is complicated, as the swing requires the coordinated effort of several body parts. Consequently, understanding and modeling this motion is difficult. Some attempts have been made previously to analyze the problem dynamically. These models determined the forces and torques at the club handle necessary to produce a golf shot. Moreover, some endeavors tried to approach the problem simply by optimal control theory. Specifically, optimal control problems have been solved to determine the optimal torques needed to maximize the distance of a golf drive (Lampsas, 1975; Campbell, 1985; Kaneko & Sato, 1993). Modeling entire body is too complex for detailed solutions, so a simpler model is necessary. As other researchers have done, this study simplified the model of the golf swing to include a (left) single arm and only the upper torso. This type of simplification has been successful in previous efforts. Lampsas (1975), using a double pendulum model, and Campbell (1985) and Kaneko and Sato (1993), using three-segment pendula, reported differences in joint dynamic patterns, with the latter model being more similar to experimental data. To be most comparable to the experimental data, this study compared three-segment and four-segment pendulum models in their dynamic pattern. Moreover, in these earlier studies, only the shoulder and wrist were included, but the elbow was excluded. In order to understand whether elbow joint contributions can play an important role, the elbow joint was added in the third

model for this study as well. The linked system of segments in the model (Figure 3.3) are identified as:

- (a) the club hinged at the wrist,
- (b) the left arm hinged at the shoulder (3-segment), or the left arm hinged at the elbow and shoulder (4-segment),
- (c) the upper torso fixed to rotate at the midpoint between the left and the right shoulders.

As discussed in the above literature review, kinematic data have shown that the swing occurs along a single inclined plane (Cochran & Stobbs, 1968; Soriano, 1997). Therefore, all segment motion was restrained to an inclined plane with the degree of inclination determined by average values from experimental data.

### 3.4.2 FORWARD DYNAMICS

Kane's method (Kane & Levinson, 1985) of describing dynamical systems was used to derive the equations of motion (EOM) for each model used. The software package Autolev was used to process each model's description and task-specific constraints to derive its EOM and to produce FORTRAN77 code for the forward simulation. The simulation code was modified to accept joint torque values in the form of ten control nodes for each joint.

The equation of motion for this model is:

$$\{A(q)\}\ddot{q} + \underline{C}(q, \dot{q}) + \underline{G}(q) + \underline{T} = 0, \quad (3.2)$$

where :



$\{A(q)\}$  is the mass matrix,

$\underline{C}$  is the Coriolis and centrifugal forces vector,

$\underline{G}$  is the gravitational force vector,

$\underline{T}$  is the joint torque vector, and

$\underline{q}$  is the vector of segment angular position.

The mass matrix  $\{A(q)\}$  is invertible. Therefore, from equation 3.2, the forward dynamics equation of motion is determined as follows:

$$\ddot{\underline{q}} = \{A(q)\}^{-1} [\underline{C}(q, \dot{q}) + \underline{G}(q) + \underline{T}]. \quad (3.3)$$

### 3.4.3 GENERAL OPTIMIZATION PROBLEMS

The performance index  $J$  is the quantity to be minimized or maximized in an optimal control problem. In general the performance index is of the form

$$J = \mathbf{f}(\underline{y}_{t_f}) + \int_{t_i}^{t_f} L(t, \underline{y}, \underline{x}) dt, \quad (3.4)$$

where  $t_i$  is the initial time,  $t_f$  is the final time,  $\underline{y}(t)$  is the system state vector,  $\underline{x}(t)$  is the control vector, and  $\phi$  and  $L$  represent scalar functions. In order to include these constraints, the solution may be required to satisfy a given set of final states (equality constraints) expressed as:

$$\underline{\mathbf{y}}(t_f, \underline{y}_{t_i}) = 0 \quad (3.5)$$

Furthermore, the states  $\underline{y}$  and the controls  $\underline{x}$  may be required to satisfy certain boundary conditions (inequality constraints) expressed as:

$$\underline{C}(\underline{y}, \underline{x}, t) \leq 0 \quad (3.6)$$

However, solving optimal control problems for a system described by

highly coupled, nonlinear differential equations poses a considerable problem computationally. In fact, the computations required to arrive at a solution may be prohibitive. Converting the optimal control problem into a parameter optimization problem greatly reduces the problem size and allows for the use of any standard nonlinear programming algorithm which is able to handle constraints on the system controls and states. Pandy et al. (1992) demonstrated how to transform optimum control musculoskeletal problems into parameter optimization problems. Parameter optimization was therefore employed in this study.

In order to transform an optimum control problem into a parameter optimization problem, the performance criterion  $J$  and inequality constraints  $C$  from time-dependent to point conditions and the control histories  $\underline{x}$  were parameterized. After transformation to parameter optimization, the performance criterion, performance equality constraint, and performance inequality constraints become:

$$J = \phi(t_f, \underline{Y}_f), \quad (3.7)$$

$$\underline{Y}(t_f, \underline{Y}_f) = 0, \quad (3.8)$$

$$\underline{C}(t_f, \underline{Y}_f) \leq 0, \quad (3.9)$$

where  $\underline{Y}$  consists of the old states  $\underline{x}$  combined with the new states.

#### 3.4.4 GOLF SWING OPTIMIZATION

In this study, the general problem was for 3-segment and 4-segment models to create a coordinated swing in a set time, ending in ball impact. The performance criterion to be maximized was the horizontal clubhead velocity at impact:

$$Ja=V(t_f) \tag{3.10}$$

Before application of the general conditions, it was also necessary to impose constraints on the states of the system to guarantee a golf swing that is physically possible. The initial conditions were the same beginning position, angular velocity, and angular acceleration as the experimental data. The physical constraints on the system were identified as:

- (a) the final position of the segments and clubhead, and
- (b) the maximum control torque.

The time to complete the downswing was fixed to match the experimental data of subject M4.

For the solution of the parameter optimization problem, the subroutine VF02DP from the Howell library was used. It is based on a sequential programming algorithm developed by Powell (1978) and requires the user to provide evaluations of performance and constraints, as well as the first derivatives of performance and constraints with respect to each torque control node for each iteration.

A set of ten, evenly spaced, joint torque control nodes was used for each joint included in the model. The maximum torque value allowed for each joint was based on the maximum isokinetic contractions and applied torque for the golf swings obtained in Study One. Inequality constraints were used to convert the limits to the optimum control algorithm. The optimization process followed the following steps:

- (a) An initial guess of control torque node values from Study One was entered.

- (b) The constraint evaluation was obtained with a forward simulation.
- (c) Numerical differentiation using central differences was applied to performance and the constraints for each of the torque nodes.
- (d) All the information was provided to VF02DP, which returned a new set of control node values.

The process was repeated from (a) to (b) until VF02DP failed to provide substantial improvement to the performance criterion.

#### **3.4.4 RESULTS**

The optimal torques for each joint were used to examine the optimal dynamic pattern. Kinematic results were also displayed for joint angles, joint angular velocities, and joint angular accelerations to compare with experimental data.

In order to obtain the best fit between the experimental and simulated results, Pearson correlation tests were performed between the experimental results of the most complex model and each of the simulated solutions.

### **3.5 Study Three: Simulation Analysis**

To understand how strength is related to the dynamic pattern and optimal wrist release time, joint torque constraints for the wrist and shoulder joints in the model were set up in different ways. This study employed the model which offered the best similarity and realistic representation of experimental data from

study two. The boundary of joint torque strength was treated as an independent variable. Three joints (wrist, elbow, and shoulder) were compared with increased and reduced torque boundaries starting from study two. The optimal joint torque histories and the angular displacements and velocities were obtained, and the optimal wrist release time was derived.

Then, the optimal wrist release time was calculated for (a) high wrist strength down to low wrist strength, (b) high elbow strength down to low elbow strength, and (c) high shoulder strength down to low shoulder strength.

## **CHAPTER 4 RESULTS**

### **4.1 Study One: Experimental Analysis**

The first study was performed on eight elite golfers (four females and four males) recruited from the University of Texas at Austin, Barton Creek Country Club, and the Golfsmith Club. Subject age ranged from 21 – 25 years (mean age: 22 years). Each subject participated in two measurement sessions for three-dimensional swing kinematics and for isokinetic strength testing; anthropometric parameters were also obtained on all subjects. Experimental kinematic and kinetic time histories of swings were analyzed using a four-segment inverse dynamic model. The goal of the first study was to describe general kinematic and torque patterns of the swing, and to examine the relationships between the parameters of strength, release initiation, clubhead speed, and applied torque in each joint.

#### **4.1.1 STRENGTH PROFILE**

Isokinetic strength testing was carried out using a Biodex dynamometer (Biodex Medical System, Shirley, NY). Testing included measurements on wrist-radial/ulnar deviation, elbow flexion/extension, shoulder abduction/adduction, shoulder motion in the rotational plane of the swing, and forearm supination/pronation. All tests were repeated at angular velocities of 30°/s,

120°/s, and 180°/s with the left arm, and with the hand only. Except for the case of left shoulder abduction/adduction, shoulder rotation testing also included mimicking the golf swing under two conditions: using the left arm only, and using both arms. Wrist testing similarly included the two cases of left wrist only and the two wrists together. Peak torque was obtained from five repeated trials. Figure 4.1 and Table 4.1 show mean and peak torques for all subjects.

In the data presentation, Direction 1 refers to the predominant joint rotational direction in the downswing, which includes wrist ulnar deviation, elbow extension, forearm supination, and the shoulder downswing. Direction 2 is the analogous opposite joint rotational direction during the upswing.

Males exhibited an average strength 66% higher than that of females in Direction 1 (downswing), and 71% higher than that of females in Direction 2 (upswing). The strength of wrist/ulnar deviation was 90 - 110% greater for males relative to females in Direction 1, but was only 50 - 70% higher in males for Direction 2 (Table 4.1). This result implies that males may use more and/or train more for wrist strength in the golf downswing direction.

The participants' average peak isokinetic wrist, elbow, and shoulder torques, defined here as strength, were measured and compared to peak torques derived from actual swing motions to examine how closely participants reached their strength boundary in 4.1.4 session.

#### **4.1.2 3D KINEMATIC ANALYSIS AND TYPICAL PROFILE FOR A GOLF SWING**

The 3D kinematic data were collected at 240 Hz using 18 markers located on study subjects. Detailed positions of markers are described in Appendix C. Figure 4.2 and Figure 4.3 show examples of male (subject M4) and female (subject F2) kinematic data, including  $(x, y, z)$  directional displacements, derived velocities, and derived accelerations for shoulder, elbow, wrist, and club locations. The  $x$  direction is the direction the body faces, the  $y$  direction is the ball flight direction, and the  $z$  direction is upwards vertically. Data collection was set to begin ten frames ( $t=0.0416$  s) prior to the downswing onset. Joint angles for the wrist, elbow, shoulder, and trunk were derived from marker positions, from which angular velocity and angular acceleration were then derived. The wrist, elbow, and shoulder angles were defined as the angle between the two relevant connected segments. Trunk angle was defined as the angle between the upper trunk and the vertical gravitational axis.

##### **4.1.2.1 Angular Position, Velocity, and Acceleration**

Figures 4.4 - 4.9 are examples of male (subject M4) and female (subject F2) subjects' angular position, angular velocity, and angular acceleration over five trials and the respective mean values of these variables. However, due to missing markers, only five usable trials were obtained from subject M4 and subject F2 also had only five usable trials; subject F3 and subject F4 had only three usable trials each; subjects M3 and F1 had two usable trials each; and subject M1 and



subject M2 only had one usable trial. The remarkable kinematic consistency for all subjects with multiple trials was the basis for different deciding to represent subjects' data with different numbers of trials. The data were synchronized by impact time ( $t=0$ ) to obtain homologous timing values. The beginning of the downswing started at the tenth video frame. Wrist release initiation was judged to occur during a sharp change in wrist position, and was identified as the time when wrist angular velocity increased from zero.

The trial-averaged angular position, velocity and acceleration for each subject, as well as the overall mean for all individuals, are shown in Figure 4.10 to Figure 4.12. In general, wrist release occurred 0.10 - 0.15 s before impact, and the wrist fully extended within 0.013 s before or after impact (Table 4.2). Peak angular velocities were attained first by the shoulder, then the trunk, the elbow, and finally the wrist. In order of decreasing magnitude, peak angular velocity of the wrist ranged from 10 - 20 rad/s, that of the trunk from 8 - 13 rad/s, that of the shoulder from 2 - 5 rad/s, and that of the elbow from 2 - 5 rad/s. In order of reaching the time of peak angular velocity, the trunk was first, followed by the shoulder, elbow, and then wrist.

The wrist-projected angle in two dimensions was derived to compare with the two-dimensional and three-dimensional true wrist angles (see Figure 4.13 and explanation in Methodology). Wrist motions from the start of the downswing to the time of wrist release were very similar in projected and non-projected systems, so that the times to reach wrist peak acceleration and about 0.05 s prior to peak were equivalent.

#### **4.1.2.2 Clubhead Speed**

Representative clubhead velocities of males (subject M4) and females (subject F2) are shown in Figure 4.14 and Figure 4.15. The average clubhead speed for all subjects is shown in Figure 4.16. Most subjects' maximum clubhead speed did not occur immediately at impact, but in some cases occurred right before or right after impact (Table 4.3).

#### **4.1.2.3 Rotation Plane**

Tilt of the longitudinal body axis from vertical was variable; average tilt angles for each subject are shown on Table 4.4. These values were used in the inverse dynamic model.

### **4.1.3 KINETIC DATA ANALYSIS AND TYPICAL PROFILE IN A GOLF SWING**

#### **4.1.3.1 Anthropometric Data**

Anthropometric data were determined using Plagenhoef's (1983) model for use in the inverse dynamic model (Table 4.5). In Plagenhoef's model, the data included both men and women college-age athletes, and used a segmented trunk with pelvis, abdomen, and thorax.

#### **4.1.3.2 Inverse Dynamic Model**

The inverse dynamic model considers planar motion with four segments: club and hand, forearm, upper arm and upper thorax. Joints are the wrist, elbow, shoulder, and the rotation center of the upper thorax (sternum).

Torque histories were derived from the kinematic histories (Figures 4.10-4.13) and anthropometric data (Table 4.5). Figure 4.17 and Figure 4.18 are examples of male and female joint torques. Positive torque indicates a rotational direction along the downswing; negative torque indicates an upswing rotational direction. Figure 4.19 shows derived torques for the upper trunk, shoulder, elbow, and wrist as calculated from the inverse dynamic model in the projected plane. Torque patterns were similar within subjects until about 0.025 s before impact. The averaged kinetic data from all subjects are shown in Figure 4.20. The averaged torque histories from all subjects showed sequential motion with the occurrence order of peak joint torque as follows: trunk, shoulder, elbow, then finally wrist.

#### **4.1.4 COMPARISONS BETWEEN STRENGTH, APPLIED TORQUE, WRIST RELEASE TIMING, AND PERFORMANCE (CLUBHEAD SPEED AT IMPACT)**

##### **4.1.4.1 Performance Evaluation: Comparison between Strength and Applied Torque**

Strength and peak torques are shown in Figure 4.21 and Table 4.6. The peak torque in isokinetic strength testing was less than the peak torque calculated

during a swing for the elbow, and especially for the wrist joint. Overall, the applied torque of the wrist exceeded the isokinetic testing strength, but the shoulder did not reach maximum torque during the swing. This result implies that wrist strength may be a limiting factor for the swing in both males and females. Females generally had a higher percentage of applied joint torque relative to their strength in the wrist joint, but a lower percentage in the shoulder joint when compared to males (Table 4.6).

#### **4.1.4.2 Relationship between Performance (Clubhead Speed) and Strength**

Clubhead speed and strength were positively but not significantly correlated in all joints (Table 4.7 (i)).

#### **4.1.4.3 Relationship between Performance (Clubhead Speed) and Peak Applied Torque**

The relationship between peak torque in various joints and clubhead speed at impact is shown in Table 4.7. Performance (clubhead speed at impact) was significantly positively correlated with peak applied torques in the wrist especially and elbow joints but not in the shoulder and upper trunk, implying that applied torques in the wrist joints are more critical in determining clubhead speed. Clubhead speed at impact was not significantly related to the timing of peak applied torque, except at the wrist joint. The earlier the wrist peak torque was reached, the higher was clubhead speed at impact.

#### **4.1.4.4 Relationship between Performance (Clubhead Speed) and Wrist Release Timing**

Wrist release initiation and clubhead speed at the peak exhibited no significant relationship (see Table 4.8).

#### **4.1.4.5 Relationship between Wrist Release Timing and Strength**

Wrist release timing exhibited no significant relationship with strengths of the wrist, elbow, and shoulder (Table 4.9).

#### **4.1.4.6 Relationship between Wrist Release Timing and Peak Applied Torque**

Peak applied torque and wrist release timing were not significantly related (Table 4.10).

#### **4.1.4.7 Relationship between Wrist Release Timing and Angular Position**

Correlations between joint angular positions at the onset of the wrist release motion with (i) release timing and (ii) clubhead speed at impact are shown in Table 4.11. For wrist release motions, a later release initiation was high and significantly correlated ( $p < 0.05$ ) with a greater angular position of the wrist, a shorter angular displacement to impact, and a greater angular rotation of the trunk at the initiation of wrist release.

The relationship between clubhead speed and wrist location at release initiation in this study showed no significant correlation (Table 4.11, wrist location).

#### **4.1.4.8 Peak Torque Timing Pattern**

Peak timing for joint torques in each subject is shown in Figure 4.22. Most subjects reached peak torque first with the trunk, then the wrist, then the elbow, and finally the shoulder. Some subjects (M3, M4, and F3) started with peak torques in the trunk, then the shoulder, the wrist, and finally the elbow.

## **4.2 Study Two: Optimal Control Model**

The goal of this study was to compare a three-segment model and a four-segment model, to determine the effects of the multiple segments and to select the modeling configuration to be used in subsequent simulations in study three. Two models were compared with the experimental results. One model contained three segments; the other model contained four segments. Kinematic results were calculated for joint angles, joint angular velocities, and joint angular accelerations to compare with experimental data.

In order to evaluate the fit between the experimental and simulated results, time histories of kinematic and dynamic variables were plotted for visual

comparison and Pearson correlation tests were performed between the experimental results and each of the simulated optimal solutions.

The three-segment model contained upper torso, arm and hand with club. The four-segment model contained upper torso, upper arm, forearm, and hand with club. The physical configuration and time period of the simulation model was based on one subject, subject M4 in this study (Table 4.5). All the joint torques were normalized by the maximum torque of the subject in a golf swing for optimal control modeling, and then returned to an absolute scale in the end. The torque boundary was set by the subject's peak torque in the experiment. Figures 4.23 through 4.25 contain the animation of an experimental performance and two modeling optimal solutions. The kinematic analysis results of these performances are shown in Figures 4.26 to 4.28. In the four-segment model simulation, the trunk angular position and shoulder angular position were similar to experimental results, and the angular position of the wrist was more similar to experimental results when close to impact. The torque and clubhead speed results are shown in Figure 4.29 and 4.30. Table 4.12 shows the correlation values of the various angular kinematics, torques, and clubhead speed between optimal solutions and the experimental results. In the four-segment model simulation, the results showed that clubhead speed and angular position, velocity, and acceleration through time were highly correlated for all four joints except for the elbow angular acceleration. The torques showed low correlation with the experimental results. However, the pattern was similar between experiment and model from the beginning of the swing to the point the subject reached and held the peak

torque. From the beginning of the swing to this time, the correlations between experimental torque and simulation torque were high, 0.814 – 0.997. Overall, the trunk joint displayed the best fit to the experimental results, then the shoulder and wrist, then the elbow. In the three-segment model, the trunk and wrist fit the best, then the shoulder, but there was no elbow to be compared.

Generally, both models had high and significant correlations with the experimental data. However the four segment model was better able to represent the actual experimental data because it successfully included the elbow joint. On the basis of these results, the four segment model was selected for use in further analysis of golf swing dynamics.

### **4.3 Study Three: Simulation Analysis**

The goal of this study was to gain understanding of the relationship between the optimal wrist release time and the joint strengths of the wrist, elbow, and shoulder. Toward this end, joint torque constraints were varied for the wrist, elbow, and shoulder joints in the model. This study employed the four-segment model, which offered the greatest similarity with experimental data in Study Two. The boundary of joint torque strength was treated as an independent variable. Three joints (wrist, elbow, and shoulder) were compared with increased and reduced torque boundaries starting from study two. Optimal joint torque histories and angular displacements and velocities were obtained, and the optimal wrist release time was derived.



The optimal wrist release time was simulated from (a) high wrist strength going down to low wrist strength, (b) high elbow strength going down to low elbow strength, and (c) high shoulder strength going down to low shoulder strength.

#### **4.3.1 KINETIC AND KINEMATIC DATA**

##### **4.3.1.1 Low and High Wrist Strength Boundary**

Optimal simulations were calculated with the wrist strength boundary varied from -50% to +100 % of the subject M4's peak wrist torque. The torque, angular displacement and velocity are shown in Figures 4.31 to 4.33. Trunk, shoulder, and elbow torque did not vary much but wrist torque did when wrist torque constraints increased. With the higher wrist strength limit, the wrist torque constraint was achieved later (Figure 4.31). Angular displacements showed that the elbow had more flexion and the wrist release timing was later when the wrist strength boundary became higher. Differences in angular velocity between high wrist strength and low wrist strength became greater in the wrist but was not much in the trunk.

##### **4.3.1.2 Low and High Elbow Strength Boundary**

Simulation results were obtained for solutions with the elbow strength boundary varying from -50% to +100 % of the subject's peak elbow torque. The torque and angular displacement and velocity are shown in Figures 4.34 to 4.36. The trunk, shoulder, and wrist torques did not vary when the elbow torque

constraints increased, but they varied some when the constraints were reduced. With higher elbow strength limits, the elbow torque constraint was achieved later but dropped more (Figure 4.34). Angular displacement at all joints varied less than 0.1 rad when the elbow strength boundary became greater, but varied up to 1.1 rad when the elbow strength boundary was reduced. The angular velocity analysis showed similar results.

#### **4.3.1.3 Low and High Shoulder Strength Boundary**

Simulation results were obtained for solutions with the shoulder strength boundary varying from -50% to +100 % of the subject's peak shoulder torque. The torque and angular displacement and velocity are shown in Figures 4.37 to 4.39. The shoulder torque achieved the constraint earlier and it lasted longer when the shoulder strength boundary was reduced. The trunk, shoulder, and wrist torque varied less than 10% when the shoulder torque constraints were increased by 50% (Figure 4.37). The angular displacement of the shoulder, elbow and wrist changed up to 75% when the constraint was reduced 50%.

#### **4.3.2 WRIST RELEASE TIMING**

The wrist release timing resulting from varied wrist, elbow, shoulder strength boundaries is shown in Figures 4.40 to 4.42. The wrist release timing was delayed more when the wrist strength boundary was increased from 0%

through 40%, then did not change after the wrist strength boundary increased to 80%, but was prolonged when the wrist strength boundary was reduced from 0% to -40% (Figure 4.40).

The wrist release timing delay was 0.025 s when the elbow strength boundary was increased from 0% through 20%, did not change much after a 20% increase or a reduction from 0%, but increased again when the wrist strength boundary was reduced from -20% to -40% (Figure 4.41).

Wrist release timing was a little earlier when the shoulder strength boundary was increased from 0% through 60%, was delayed further after wrist strength boundary increased to 100%, but did not change substantially when shoulder strength boundary was reduced from (0% through -40%) except at -50% (Figure 4.42).

### **4.3.3 PERFORMANCE (CLUBHEAD SPEED AT IMPACT)**

Results for clubhead speed at impact as derived from the optimal control model for the variable wrist, elbow, and shoulder strength boundaries are shown in Figure 4.43. Clubhead speed at impact increased up to 9.0% when the wrist strength boundary increased through 100%, but only improved up to 1.8% for elbow strength boundary increases and up to 0.9% for shoulder strength boundary increases. The clubhead speed at impact did not change after an increase of 20% of elbow strength. The clubhead speed at impact also dropped much faster with a reduction of the wrist strength than with reduction in elbow and shoulder strengths. This result shows that wrist strength is the most crucial to performance in terms of clubhead speed at impact.

## **CHAPTER 5 DISCUSSION**

### **5.1 Strength Profile**

A principal finding of these experiments is that joint torque of the wrist during the golf downswing is much higher than the wrist strength as determined isokinetically. This may be due in part to several factors: (a) constraints on isokinetic testing, (b) error in inverse dynamics calculations, (c) dynamic physiological processes not represented in the model, and (d) kinematic differences between testing and experimental conditions. Isokinetic testing may constrain the wrist to rotate differently than in the swing, so the motions in the two cases may not be strictly comparable. When teaching golf, it is typically required that the wrist remains flat and that ulnar deviation remains planar during the downswing. However, the wrist may still exhibit some flexion and supination throughout the swing. In addition, wrist action will still include some flexion and a range of motion and angular velocity which is different from that used in isokinetic testing. Nevertheless, the wrist joint is probably closer to a strength threshold than the other joints given the much higher ratios of torque to isokinetic strength seen in the wrist (Table 4.6). At times, the angular velocity and acceleration in the experimental data were outside the range of kinematic tested isokinetically.

Wrist and elbow strength varied substantially among female subjects relatively to male subjects, perhaps according to training strategy. However, females generally tended to maximize wrist variables. Also, isokinetic strength in

wrist/ulnar deviation was 90 - 110% greater for males relative to females in Direction 1 (downswing), but was only 50 - 70% higher in males for Direction 2 (upswing) (Table 4.1).

Males may thus build up greater wrist strength than females for use during the golf downswing. Comparisons between isokinetic strength testing and applied torque during the swing suggest that wrist strength training at high angular velocities should be reinforced for maximum performance.

Threshold joint strength in a golf swing may also be inferred from injury reports. A survey of professional golfers regarding injury was conducted by McCarroll and Gore (1982) , who found that the left wrist was most commonly injured (27%) in professional golfers. By contrast, the most commonly injured region in amateur golfers is the lower back (24%; see Duda, 1987). The present study using elite golfers shows that wrist strength likely imposes a limiting factor on the swing, a finding that is consistent with injury reports among professionals.

## **5.2 Clubhead Speed and Wrist Release Timing**

Subjects in this study initiated wrist extension at 0.10 - 0.15 s before impact, a timing very close to values for the expert subjects (0.07 - 0.12 s) studied by Soriano (1997). McLaughlin and Best (1994) showed that late release of the wrist was a characteristic of low handicap golfers, supporting the claim of

Cochran and Stobbs (1968) that delayed wrist release yields an effective swing. McLaughlin and Best (1994) compared players with a broad range of handicaps, and found a significant relationship between late release motion and clubhead speed. This study tested the hypothesis that the optimal time for the initiation of the wrist release is positively associated with the clubhead speed at impact (Hypothesis 1). The present data show that late wrist release is not significantly correlated with clubhead speed, as was also found by Mason et al. (1996). Both this study and that of Mason et al. (1996), however, used trained individuals (all with low handicaps and with fairly low kinematic variance) who may not show such a relationship. The relationship between late wrist release and high clubhead speed may not be demonstrable among elite golfers if they all have relatively late wrist release compared to high handicap golfers.

Some of the differences between studies may also derive from different definitions of wrist release. Mason et al. (1995) defined the release motion as a wrist angle beyond  $90^{\circ}$ . Soriano (1997) defined the beginning point of wrist extension as a rapid change in the slope of the curve of wrist angle versus time. Cochran and Stobbs (1968) describe wrist uncocking (release) using anatomical definitions, i.e., wrist adduction (ulnar deviation) from an abducted (radially deviated) position (see also McLaughlin and Best, 1994). In the present study, five of eight subjects had already exceeded a wrist angle of  $90^{\circ}$  at the beginning of the downswing. Therefore, the wrist release timing was defined as the time that the wrist's angular velocity became positive, instead of being defined relative to a particular angular position.

### **5.3 Joint Angular Position and Wrist Release Timing**

The correlation of joint angular position at the initiation of wrist release and the timing of wrist release revealed significant correlations for the trunk and wrist angles (Table 4.11). This result shows that the later the release motion, the greater the trunk rotational angle, which is to be expected. However, later wrist release is also associated with more 'leaking' (i.e., reduced uncocking) of the wrist. The correlation between clubhead speed and joint angular position showed a moderately negative correlation such that the lower the wrist angular positions (less leaking) at the initiation of wrist release, the greater the clubhead speed. In this study, six out of eight subjects tended to extend their wrists just before impact, and all subjects tended to strike the ball with their wrists slightly flexed.

### **5.4 Swing Pattern**

Motions of the segments in the golf swing can generally be sequenced in a proximal-to-distal fashion (Putnam, 1993). The way segments move in sequence, and particularly the way their motions are linked, can vary considerably according to task. At the initiation of the sequence, the joint moment at the proximal end of the next segment in the linked system is usually in the opposite direction and assists in backward acceleration of this more distal segment. However, if the proximal segment is at right angles to the distal segment, then the linear

acceleration-dependent interaction is most effective. The relative angle between segments thus has a significant effect on the way segments interact (Putnam, 1993). In golf, the angle between the club and the leading arm is about  $90^\circ$  at the beginning of the downswing (in this study, range: 1.59 - 1.91 rad;  $90^\circ - 109^\circ$ ) (Figure 4.11). Therefore, the forward acceleration of the leading arm tends to accelerate the club in the direction of the downswing. Wrist angle starts to increase well before (-0.1 - 0.15 s before impact) the leading arm reaches its maximum angular velocity (-0.07 - -0.1 s before impact). These results are consistent with previous studies (see Millburn, 1982). However, shoulder and elbow angular velocity timing patterns vary among subjects, while the magnitude of these velocities is relatively low compared to those of the wrist and trunk angular velocities. In the same way, most subjects exhibited similar angular acceleration and deceleration patterns in the trunk and wrist, but the shoulder pattern was somewhat variable, and the elbow pattern varied substantially. In the beginning of the downswing, the trunk starts to accelerate forward, followed by the shoulder and elbow, but the wrist shows little initial acceleration. When trunk acceleration reaches peak values, the shoulder and elbow start increasingly to accelerate forward. Then the wrist starts to accelerate and the trunk, shoulder, and elbow begin deceleration. About 0.05 s before impact, all joints are decelerating, although joint angular velocities are still positive and the club head velocity is still slowly increasing or is constant (Figures 4.11 and 4.16).

Many other studies have recorded only 2D kinematics of the golf swing for use in a planar pendulum model. Therefore, wrist kinematic data in this study



were used to compare non-projected wrist motions (3D) with those in 2D projected onto the swing plane (Figures 4.10 - 4.13). Wrist motions from the start of downswing to the time of wrist release are very similar in projected and non-projected systems, so that the times to reach wrist peak acceleration are the same (about 0.05 s before impact). At this time, the 3D wrist angle becomes nearly constant and then decelerates, but the 2D projected wrist angle continues to increase and to accelerate. This difference is associated with forearm supination and flexion that is not apparent in a two-dimensional perspective, but does not influence kinematic and kinetic estimates until 0.05 s prior to impact.

## **5.5 Simulation and Experimental Strategy**

Campbell and Reid (1985) showed with optimization modeling that maximum impact velocity is achieved by sequential activation first of the shoulder, then of the upper torso, and finally of the wrist. The corresponding torque values are about 339 Nm, 191 Nm, and 35 Nm, whereas timing of peak torque is -0.125 s for the shoulder, and -0.05 s before impact for wrist. The shoulder and wrist follow the boundary constraint. In this study, joint torques of the four-segment model reached the peak torque of the torso at -0.171 s, of the shoulder at -0.138 s, of the elbow at -0.136 s, and of the wrist at -0.113 s. Torque values thereof are 169.0 Nm, 103.0 Nm, 60.5 Nm, and 36.5 Nm, respectively, including the elbow which the above model of Campbell and Reid (1985) did not contain. The timing of peak torques and the torque history before reaching peak torque are closer to experimental results than predictions of the

three-segment model. However, joint torques in both models tend to hold the maximum torque to the end of the swing, unlike the experimental results, except for the shoulder torque of four-segment model. To hold peak torques in trunk, elbow, and wrist appear to be the best swing strategy, but human strength is constrained and cannot attain this goal. The force-velocity relationship for contracting muscles suggests in particular that torque cannot be maintained at a high constant value.

Comparing angular position data, the trunk and shoulder joint move similarly between real swings and the four-segment model, but differ in that the elbow in the model tends to bend more, followed by elbow extension. However, some wrist release is evident, the elbow bends further, and then further release occurs (at  $-0.08$  s). In the three-segment model with no elbow, the pattern of wrist joint release is much closer to the experimental results. In other words, the whip motion is greater in the optimal control model (Figures 4.27 and 4.28). It seems that the addition of the elbow rotation and the delayed wrist release yield the best strategy for the optimal control model.

## **5.6 Joint Strength Boundary and Strategy**

From this study, it is possible to determine an efficient training plan based on variable physical strength. Training can improve strength typically up to 40%. This study tested the hypothesis that the optimal time for initiation of wrist release is (a) positively associated with wrist strength, (b) inversely associated with shoulder joint strength, and (c) inversely associated with elbow joint strength

(Hypothesis 2). The model used here showed that within the range of a 40% increase in the wrist or elbow strength boundary, a later wrist release is preferred for optimal performance, supporting Hypothesis 2(a). However, earlier or constant wrist release time is preferable within the range of a 40% increase in the shoulder strength boundary, also supporting Hypothesis 2(b). Interestingly, the relationship between the elbow strength boundary and wrist release timing is similar to that exhibited by the wrist strength boundary. This result does not support Hypothesis 2(c). If wrist strength is improved over the shoulder strength, then the wrist release timing is delayed for optimal performance.

## **5.7 Joint Strength Boundary and Clubhead Speed at Impact**

Modeling results of Study Three show that an increase in wrist strength is more effective for improving clubhead speed relative to strength changes in the shoulder and elbow (Figure 4.43). The increase of elbow strength is the most ineffective for improving clubhead speed among these three joints. An increase in shoulder strength of 10% is similar to an equivalent increase in wrist strength for improving the clubhead speed. However, for a higher percentage increase, an increase of wrist strength is more effective in improving performance. At the same time, a delay in wrist release time is preferred with an increase in wrist strength. Additionally, strength at high speeds is important in order to maintain a high peak torque during fast movements.

## **5.7 Conclusions**

These experiments showed that elite golfers extend the elbow by 0.2 - 0.5 rad though the downswing, so the elbow joint should not be excluded from modeling efforts when such movements are substantial. Comparison of experimental results to optimal model predictions revealed that wrist strength plays a major role in golf swing performance, especially the wrist strength in high-speed rotation. Simulations of Study Three indicate that wrist strength is more crucial for improving the clubhead speed at impact than are shoulder and elbow strengths. For wrist release timing, simulations showed that for certain increases in wrist and elbow strength, a later wrist release timing may benefit the clubhead speed at impact. With increased shoulder strength, later wrist release may not be necessary. It seems that substantial delay in wrist release is not the only way to improve performance, but that the joint strength profile is also important. Furthermore, the strength at high speeds is crucial to performance, but may be difficult to achieve because of limitations on muscle activation. While these observations are based on a model representing only a single swinging arm, the general principles seem to be very appropriate for application to real, two-arm golf swings.

This study has thus shown that individual dynamic characteristics must be evaluated in order to determine productive or counter-productive means of improving golfers' performance.

## **5.8 Future Work**

Previous studies of golf kinematics and dynamics have assumed planar motion. In our study, the inverse dynamic model also assumed planar motion of the swing, as well as one-dimensional movement of the wrist, in order to compare this model with prior research results. The simplicity of our optimal control model allows us to perform a series of optimizations in an acceptable amount of time. This allowed us to determine effects of various strength boundaries for different joints on golf swing performance. In future research, both supination/pronation and radial/ulnar deviation should be included to understand the influence of associated torques on performance. Additionally, the torque-velocity (or force-velocity) relationship of particular joints can be imposed in the optimal control model to investigate effects on the optimal torque solution, and to see how these influence segmental coordination and motion strategies necessary to achieve maximum clubhead speed. By systematically improving the complexity and detail of such models, we will be able to understand how varied biomechanical factors affect the optimal golf swing.

## **TABLES**

Table 4.1 Isokinetic strength profile of subjects.

<b>Direction 1</b>	<b>Speed</b>	<b>Male</b>	<b>± Standard</b>	<b>Female</b>	<b>± Standard</b>	<b>Male/</b>	<b>Total</b>	<b>± Standard</b>
<b>Peak Torque (Nm)</b>	<b>(°/s)</b>	<b>Mean</b>	<b>Deviation (n=4)</b>	<b>Mean</b>	<b>Deviation (n=4)</b>	<b>Female</b>	<b>Mean</b>	<b>Deviation (n=8)</b>
Wrist	30	19.4	± 4.9	10.2	± 0.9	1.9	14.8	± 5.9
Ulnar deviation	120	13.9	± 3.9	6.5	± 2.3	2.1	10.2	± 4.9
	180	12.5	± 1.8	6.0	± 2.3	2.1	9.2	± 4.0
Wrist	30	21.7	± 4.1	18.0	± 3.6	1.2	19.9	± 4.6
2 arms	120	19.4	± 3.2	15.3	± 4.9	1.3	17.3	± 4.5
	180	15.7	± 3.2	11.6	± 4.1	1.4	13.6	± 4.4
Forearm	30	15.3	± 0.9	7.9	± 0.9	1.9	11.6	± 4.0
Supination	120	12.9	± 0.0	7.4	± 1.5	1.8	10.2	± 3.1
	180	12.5	± 0.9	7.4	± 1.5	1.7	9.9	± 3.0
Elbow	30	63.8	± 1.1	34.7	± 2.3	1.8	49.2	± 16.1
Extension	120	51.8	± 2.6	31.0	± 2.9	1.7	41.4	± 11.8
	180	44.9	± 5.5	26.8	± 3.8	1.7	35.8	± 12.1
Shoulder	30	76.3	± 4.6	57.8	± 6.2	1.3	67.0	± 11.3
Abduction	120	74.4	± 5.7	48.1	± 4.6	1.5	61.3	± 14.9
	180	62.9	± 6.6	33.3	± 11.2	1.9	48.1	± 19.0
Shoulder	30	67.0	± 6.4	48.6	± 3.8	1.4	57.8	± 14.9
Downswing	120	62.9	± 9.1	40.2	± 4.6	1.6	51.6	± 13.9
	180	42.1	± 18.1	28.2	± 6.1	1.5	35.1	± 14.6
Shoulder	30	143.8	± 37.2	89.7	± 5.6	1.6	116.8	± 38.2
2 arms	120	134.6	± 37.8	79.5	± 5.8	1.7	107.0	± 39.4
	180	122.5	± 31.0	69.4	± 7.7	1.8	95.9	± 35.5

<b>Direction 2</b>	<b>Speed</b>	<b>Male</b>	<b>± Standard</b>	<b>Female</b>	<b>± Standard</b>	<b>Male/</b>	<b>Total</b>	<b>± Standard</b>
<b>Peak Torque (Nm)</b>	<b>(/s)</b>	<b>Mean</b>	<b>Deviation (n=4)</b>	<b>Mean</b>	<b>Deviation (n=4)</b>	<b>Female</b>	<b>Mean</b>	<b>Deviation (n=4)</b>
Wrist	30	23.6	± 2.3	15.7	± 0.9	1.5	19.7	± 4.5
Radial dev.	120	18.0	± 3.5	11.6	± 1.5	1.6	14.8	± 4.3
	180	15.3	± 2.3	9.2	± 1.8	1.7	12.3	± 4.1
Wrist	30	38.4	± 2.3	25.4	± 1.5	1.5	31.9	± 7.2
2 arms	120	31.9	± 4.1	21.7	± 2.7	1.5	26.8	± 6.6
	180	26.8	± 5.3	16.2	± 2.4	1.7	21.5	± 7.0
Forearm	30	8.8	± 1.8	4.2	± 1.5	2.1	6.5	± 3.0
Pronation	120	6.5	± 1.8	3.2	± 0.9	2.0	4.9	± 2.2
	180	7.4	± 1.5	4.2	± 1.5	1.8	5.8	± 2.3
Elbow	30	62.4	± 9.6	35.6	± 2.5	1.8	49.0	± 15.9
Flexion	120	46.7	± 3.5	31.0	± 4.6	1.5	38.8	± 9.2
	180	43.0	± 3.8	25.4	± 3.5	1.7	34.2	± 10.1
Shoulder	30	71.7	± 10.7	59.2	± 9.5	1.2	65.4	± 11.7
Adduction	120	67.0	± 13.9	48.1	± 4.5	1.4	57.6	± 14.3
	180	68.0	± 14.1	41.6	± 6.9	1.6	54.8	± 17.5
Shoulder	30	80.5	± 25.5	35.6	± 5.7	2.3	58.0	± 29.5
Upswing	120	72.6	± 26.0	34.2	± 5.4	2.1	53.4	± 27.0
	180	58.3	± 23.1	30.5	± 8.0	1.9	44.4	± 22.1
Shoulder	30	152.6	± 28.9	89.2	± 10.3	1.7	120.9	± 39.4
2 arms	120	141.5	± 32.0	83.7	± 5.3	1.7	112.6	± 37.5
	180	133.6	± 40.8	75.8	± 5.5	1.8	104.7	± 41.2

Table 4.2 Wrist release duration before impact and the angular position of joints at the initiation of wrist release.

<i>Subject#</i>	<i>Total Duration (s)</i>	<i>Release</i>		<i>Trunk Angle * (rad)</i>	<i>Shoulder Angle* (rad)</i>	<i>Elbow Angle (rad)</i>	<i>* Wrist Angle * (rad)</i>
		<i>Duration before Impact (s)</i>	<i>Release Timing(%)</i>				
<i>M1</i>	0.29	0.10	0.64	-0.68	1.08	2.75	2.16
<i>M2</i>	0.30	0.15	0.48	-1.16	0.94	2.74	1.75
<i>M3</i>	0.31	0.11	0.64	-0.81	1.03	2.66	1.68
<i>M4</i>	0.24	0.14	0.40	-1.14	0.85	2.65	1.58
<i>F1</i>	0.30	0.15	0.48	-1.26	1.17	2.61	1.71
<i>F2</i>	0.30	0.12	0.59	-0.99	1.03	2.82	1.95
<i>F3</i>	0.27	0.13	0.50	-1.07	1.03	2.77	1.70
<i>F4</i>	0.30	0.15	0.47	-1.27	1.03	2.72	1.68

\* : at the time of wrist release initiation.



Table 4.3 Clubhead speed at peak and at impact.

<i>Subject#</i>	<i>Peak Time before Impact*</i> (s)	<i>Clubhead Speed at Impact (m/s)</i>	<i>Clubhead Speed at Peak (m/s)</i>
<i>M1</i>	-0.0125	31.9	32.0
<i>M2</i>	-0.0167	35.1	35.4
<i>M3</i>	0.0083	36.4	36.5
<i>M4</i>	0.0083	37.1	37.3
<i>F1</i>	0.0250	34.0	35.6
<i>F2</i>	0.0125	33.1	33.6
<i>F3</i>	-0.0083	31.3	31.4
<i>F4</i>	0.0083	31.8	31.9

\* : negative: before impact; positive: after impact

Table 4.4 Tilt angle of swing plane.

	<i>Tilt Angle of Swing Plane</i> ( <sup>o</sup> )
<i>M1</i>	27.2
<i>M2</i>	27.0
<i>M3</i>	34.7
<i>M4</i>	35.8
<i>F1</i>	31.3
<i>F2</i>	34.5
<i>F3</i>	33.9
<i>F4</i>	34.2

Table 4.5 Anthropometrics of all subjects for the inverse dynamics model.

				Length (m)				Mass (kg)				COG-proximal (m)		
Subject#	Age	Height (m)	Weight (kg)	Hand	Forearm	Upper Arm	Upper Thorax	Hand	Forearm	Upper Arm	Upper Thorax	Hand	Forearm	Upper Arm
<i>Male</i>	23	1.831	92.71	5.75 %H*	15.70%H*	17.20%H*	24.50%H*	0.65%W*	1.87%W*	3.25%W*	20.10%W*	46.80%*	43.00%*	43.60%*
<i>M1</i>	24	1.880	113.400	0.108	0.295	0.323	0.461	0.737	2.121	3.686	22.793	0.051	0.138	0.151
<i>M2</i>	21	1.890	90.700	0.109	0.297	0.325	0.463	0.590	1.696	2.948	18.231	0.051	0.139	0.152
<i>M3</i>	23	1.829	87.200	0.105	0.287	0.315	0.448	0.567	1.631	2.834	17.527	0.049	0.134	0.147
<i>M4</i>	25	1.727	79.540	0.099	0.271	0.297	0.423	0.517	1.487	2.585	15.988	0.046	0.127	0.139
<i>Female</i>	21	1.643	63.735	5.75%H*	16.00%H*	17.30%H*	20.00%H*	0.50%W*	1.57%W*	2.90%W*	17.02%W*	46.80%*	43.40%*	45.80%*
<i>F1</i>	21	1.651	74.840	0.095	0.264	0.286	0.330	0.374	1.175	2.170	12.738	0.044	0.124	0.134
<i>F2</i>	21	1.575	59.900	0.091	0.252	0.272	0.315	0.300	0.940	1.737	10.195	0.042	0.118	0.128
<i>F3</i>	22	1.727	63.500	0.099	0.276	0.299	0.345	0.318	0.997	1.842	10.808	0.046	0.129	0.140
<i>F4</i>	21	1.620	56.700	0.093	0.259	0.280	0.324	0.284	0.890	1.644	9.650	0.044	0.121	0.131

		Radius of Gyration (m)				COG-Proximal (m)	Inertia of Mass (kg·m <sup>2</sup> ) at Center of Mass				
Subject#		Hand	Forearm	Upper Arm	Upper	Hand+Club	Hand+Club	Hand	Forearm	Upper Arm	Upper Thorax
<i>Male</i>		54.90%*	52.60%*	54.20%*	62.30%*						
<i>M1</i>		0.059	0.162	0.178	0.28695	0.27416	0.16568	0.0026	0.05568	0.11615	1.87687
<i>M2</i>		0.060	0.163	0.178	0.28848	0.30302	0.15536	0.0021	0.04501	0.09389	1.51717
<i>M3</i>		0.058	0.158	0.173	0.27917	0.30751	0.15390	0.00189	0.04053	0.08453	1.36599
<i>M4</i>		0.055	0.149	0.163	0.2636	0.31921	0.15017	0.00154	0.03296	0.06875	1.1109
<i>Female</i>		54.90%*	53.00%*	56.40%*	62.30%*						
<i>F1</i>		0.052	0.145	0.157	0.20571	0.36793	0.13470	0.00102	0.02471	0.05337	0.53904
<i>F2</i>		0.050	0.138	0.150	0.19625	0.40279	0.12407	0.00074	0.018	0.03887	0.39263
<i>F3</i>		0.055	0.152	0.164	0.21518	0.39474	0.12613	0.00094	0.02294	0.04954	0.50044
<i>F4</i>		0.051	0.142	0.154	0.20185	0.41190	0.12113	0.00074	0.01803	0.03893	0.3932

\*From Plagenhoef (1983)

Table 4.6 Ratio of applied joint torque to isokinetic strength at angular speeds of 30°/s, 120°/s, and 180°/s.

Speed 30°/s					
Percentage (%)	Male1	Male2	Male3	Male4	Female Mean ± Standard Deviation
Wrist	128	103	126	210	136±47
Elbow	87	73	87	90	84±8
Shoulder	87	37	62	97	66±27
Percentage (%)	Female1	Female2	Female3	Female4	Female Mean ± Standard Deviation
Wrist	178	101	107	151	127±37
Elbow	128	85	92	77	93±23
Shoulder	56	65	51	37	52±12
Speed 120°/s					
Percentage (%)	Male1	Male2	Male3	Male4	Female Mean ± Standard Deviation
Wrist	156	134	136	189	152±26
Elbow	109	83	109	116	104±15
Shoulder	88	41	61	118	70±34
Percentage (%)	Female1	Female2	Female3	Female4	Female Mean ± Standard Deviation
Wrist	178	121	116	241	150±59
Elbow	138	94	109	85	104±23
Shoulder	73	69	56	41	58±15
Speed 180°/s					
Percentage (%)	Male1	Male2	Male3	Male4	Female Mean ± Standard Deviation
Wrist	234	148	177	210	188±38
Elbow	133	89	118	150	120±26
Shoulder	90	46	74	121	77±32
Percentage (%)	Female1	Female2	Female3	Female4	Female Mean ± Standard Deviation
Wrist	312	151	142	301	197±93
Elbow	257	94	117	95	120±78
Shoulder	73	78	63	54	67±11

Table 4.7 Pearson correlation of clubhead speed with (i) isokinetic strength, (ii) timing of peak torque, and (iii) peak applied torque.

	<i>Trunk</i>	<i>Shoulder</i>	<i>Elbow</i>	<i>Wrist</i>
<i>Clubhead Speed At Impact (m/s)</i>	<i>Isokinetic Strength (Nm)</i>			
		0.16	0.24	0.27
	<i>Timing of PK Torque (normalized)</i>			
	0.40	0.28	-0.32	-0.77*
	<i>Peak Applied Torque (Nm)</i>			
	0.30	0.37	0.62 <sup>+</sup>	0.80*

<sup>+</sup>:p<0.1, \*: p<0.05

Table 4.8 Correlation of clubhead speed and wrist release timing.

	<i>Projected plane</i>	<i>Release Timing (normalized)</i>	<i>Release Duration (s)</i>
<i>Clubhead Speed at Impact (m/s)</i>	<i>Males</i>	-0.39	0.41
	<i>Females</i>	-0.02	0.08
	<i>All Subjects</i>	-0.22	0.10

Table 4.9 Correlation of isokinetic strength and wrist release timing.

	<i>Isokinetic strength</i>		
<i>Projected Plane</i>	<i>Shoulder</i>	<i>Elbow</i>	<i>Wrist</i>
<i>Release Timing (normalized)</i>	0.13	0.26	0.30
<i>Release Duration (s)</i>	-0.37	-0.42	-0.53

Table 4.10 Correlation of wrist release timing and peak applied torque.

<i>Projected Plane</i>	<i>Trunk PK Torque</i>	<i>Shoulder PK Torque</i>	<i>Elbow PK Torque</i>	<i>Wrist PK Torque</i>
<i>Release Timing (normalized)</i>	0.25	0.06	-0.02	-0.12
<i>Release Duration (s)</i>	-0.41	-0.35	-0.24	-0.16

Table 4.11 Correlation of joint angular position at the start of wrist release with (i) release timing and (ii) clubhead speed at impact.

<i>Projected Plane</i>	<i>Trunk (rad)</i>	<i>Shoulder (rad)</i>	<i>Elbow (rad)</i>	<i>Wrist (rad)</i>	<i>Wrist location<sup>1</sup> (rad)</i>
<i>Wrist Release Timing (normalized)</i>	0.86*	-0.18	0.34	0.71*	0.49
<i>Wrist Release Duration (s)</i>	-0.97*	-0.18	-0.34	-0.66 <sup>+</sup>	-0.32
<i>Clubhead Speed at Impact (m/s)</i>	-0.03	-0.56	-0.60	-0.49	0.43

<sup>1</sup> wrist location refers to the angle that wrist moves from the top of the swing to the start of wrist release.

<sup>+</sup>:p<0.1, \*: p<0.05



Table 4.12 Pearson correlation for optimal control model and experiments.

<i>Optimal vs Experiment</i>	<i>Optimal(4 segments) vs. Experimental</i>				<i>Optimal(3 segments) vs. Experimental</i>		
<i>Pearson correlation</i>	<i>Trunk</i>	<i>Shoulder</i>	<i>Elbow</i>	<i>Wrist</i>	<i>Trunk</i>	<i>Shoulder</i>	<i>Wrist</i>
<i>Angular position</i>	1.00*	0.97*	0.74*	0.87*	1.00*	0.89*	0.99*
<i>Angular velocity</i>	0.92*	0.79*	0.77*	0.77*	0.93*	0.44*	0.99*
<i>Angular Acceleration</i>	0.85*	0.56*	0.11	0.65*	0.85*	0.52*	0.89*
<i>Torque</i>	-0.20	0.05	-0.04	-0.26*	-0.17	-0.07	-0.04
<i>Clubhead Speed</i>	0.96*				0.97*		

\*:  $p < 0.05$ .

Sample size: 59.

## **FIGURES**

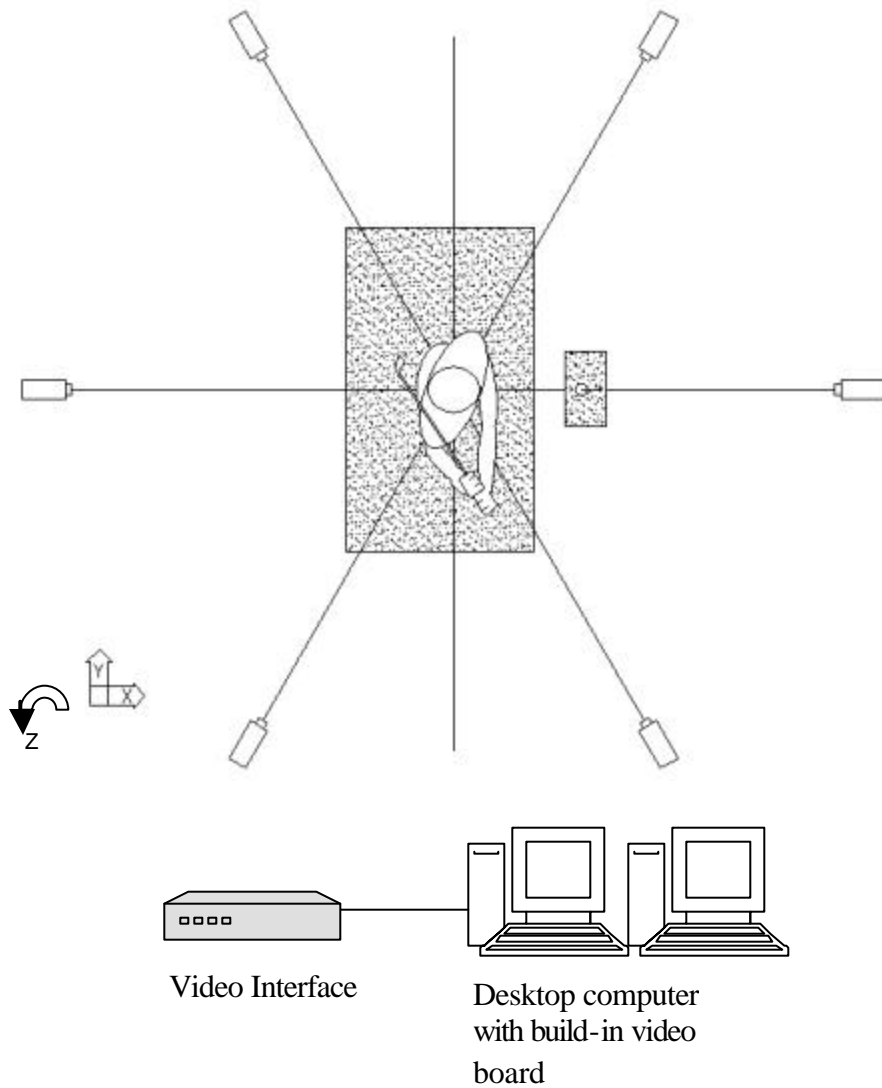


Figure 3.1 Experimental set-up showing cameras, driver, data collection system, and participant position.

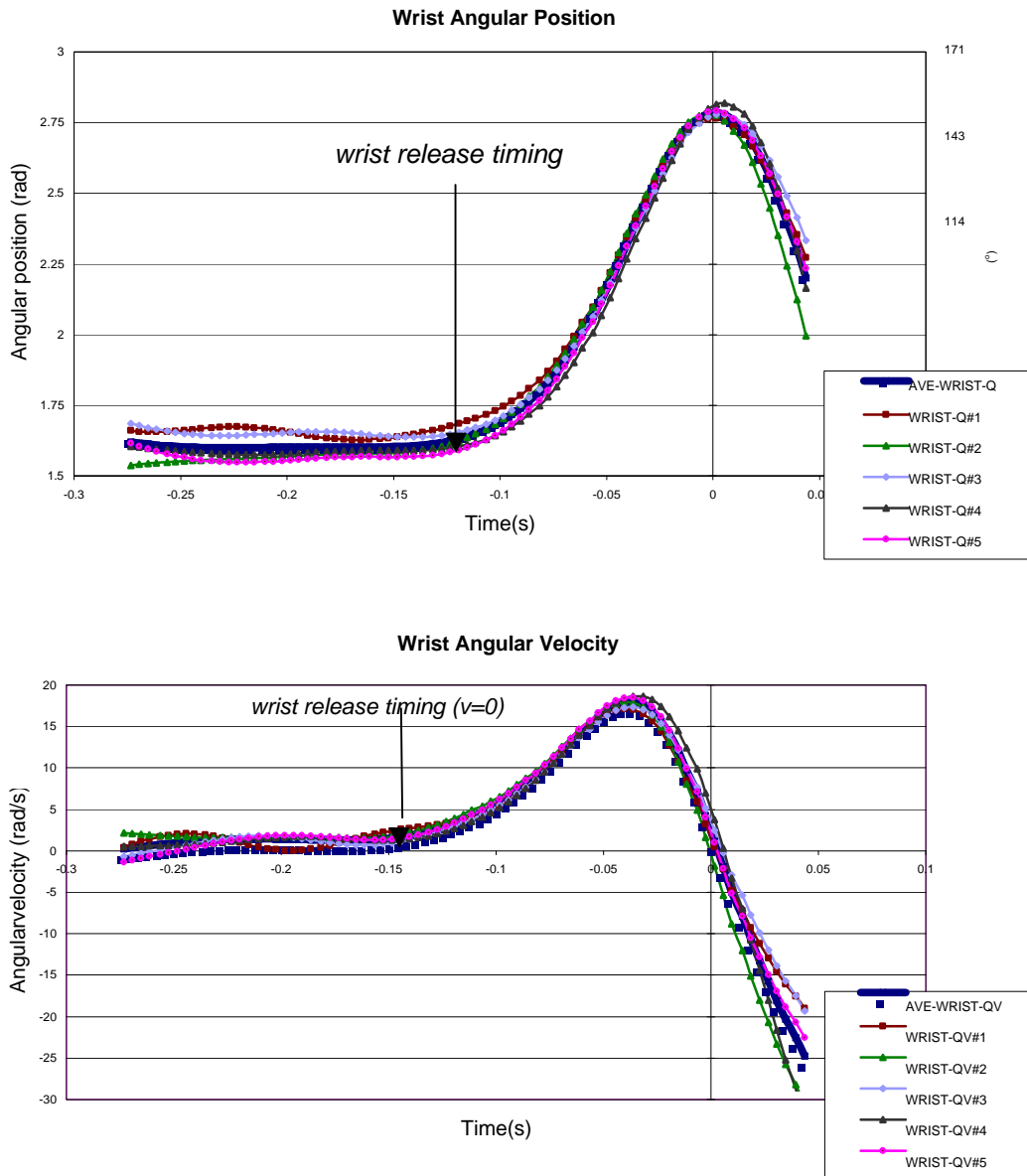
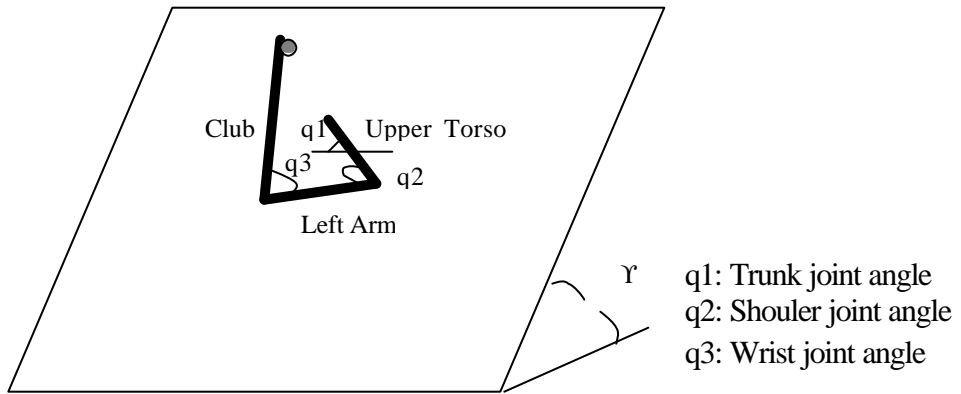
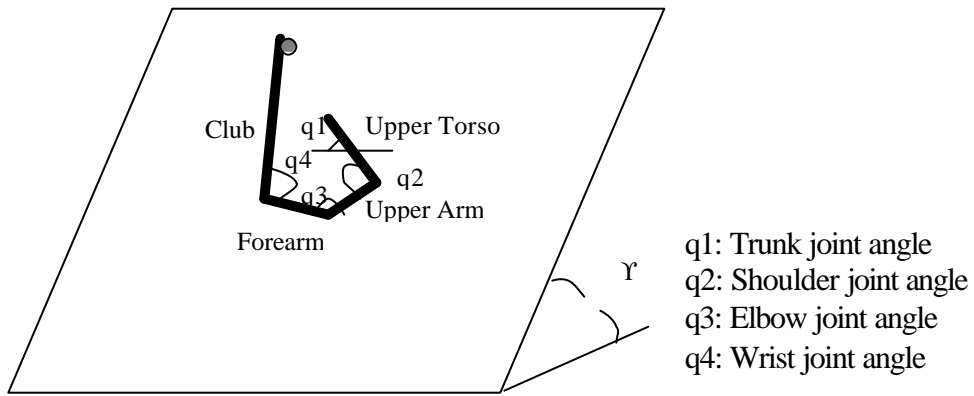


Figure 3.2 Example of wrist release timing relative to wrist angular position and velocity.



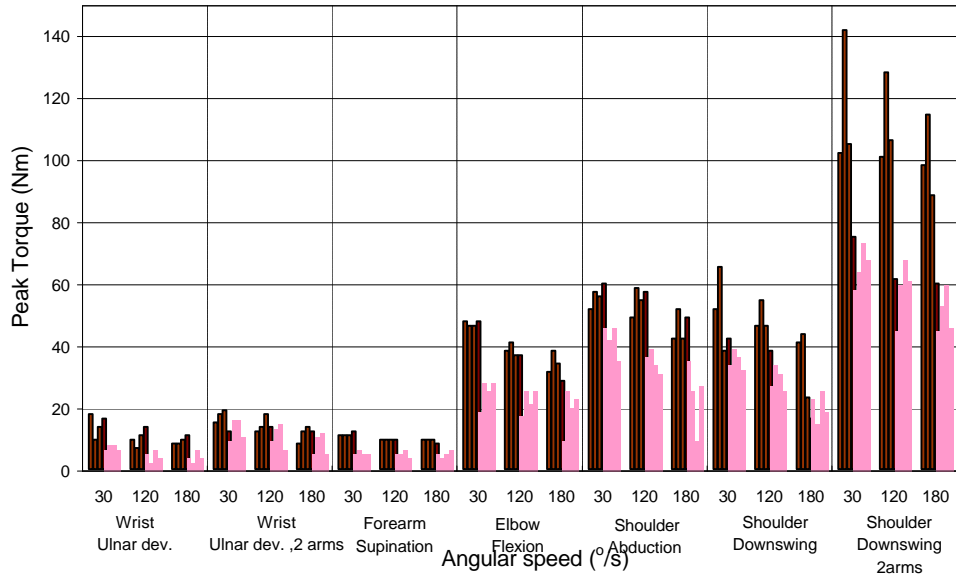
(a) Three-segment model



(b) Four-segmental model

Figure 3.3 (a) Three-segment model and (b) four-segment model of the golf swing.

### Strength Profile-Direction1 (Downswing)



### Strength Profile-Direction2 (Upswing)

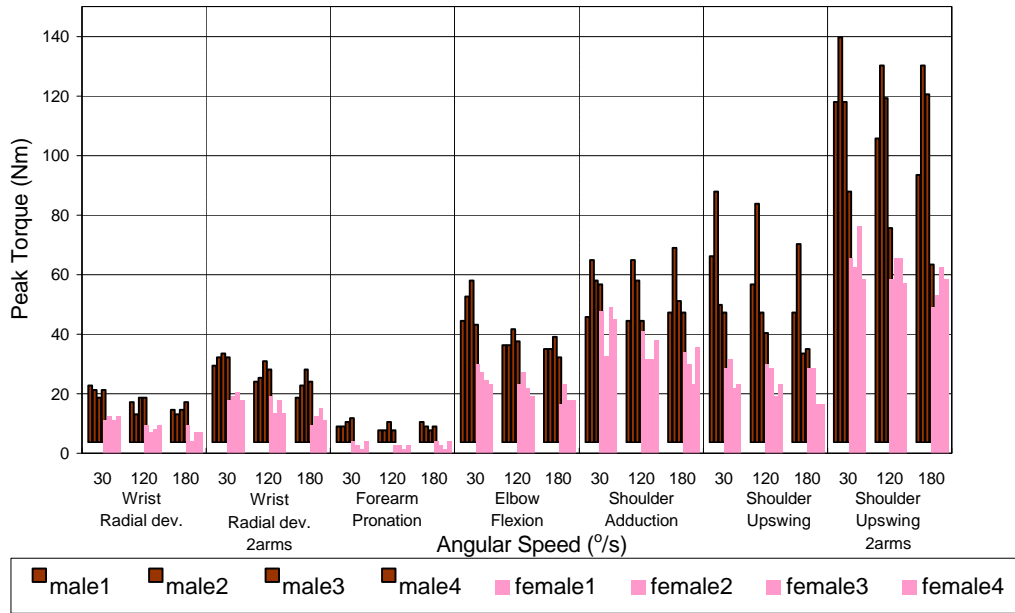


Figure 4.1 Isokinetic strength profile on wrist, elbow, forearm, and shoulder with 3 speeds (30,120, and 180 °/s).

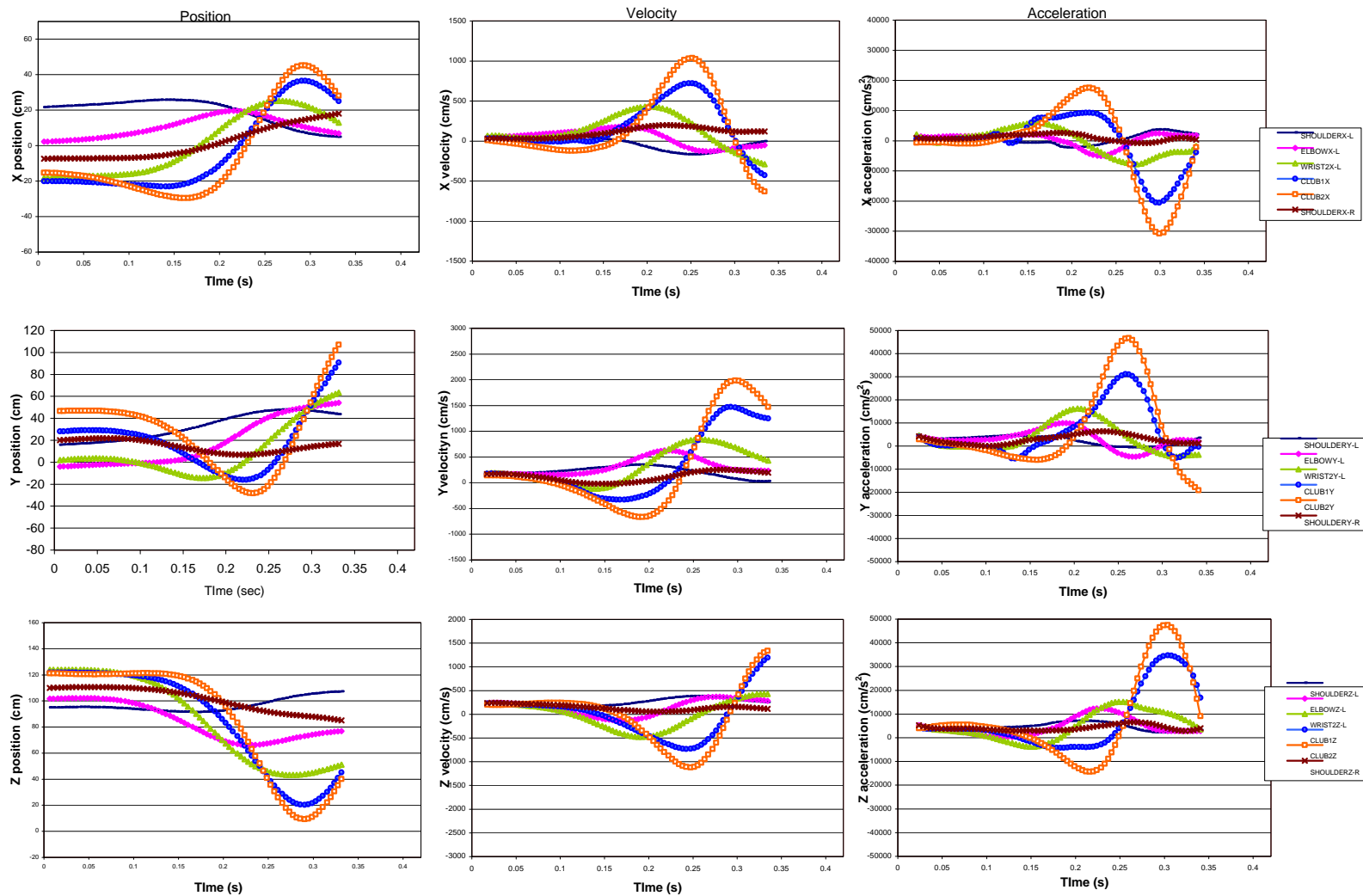


Figure 4.2 Displacement, velocity, acceleration of shoulder, wrist, elbow, club from subject M4 (male), trial 5.

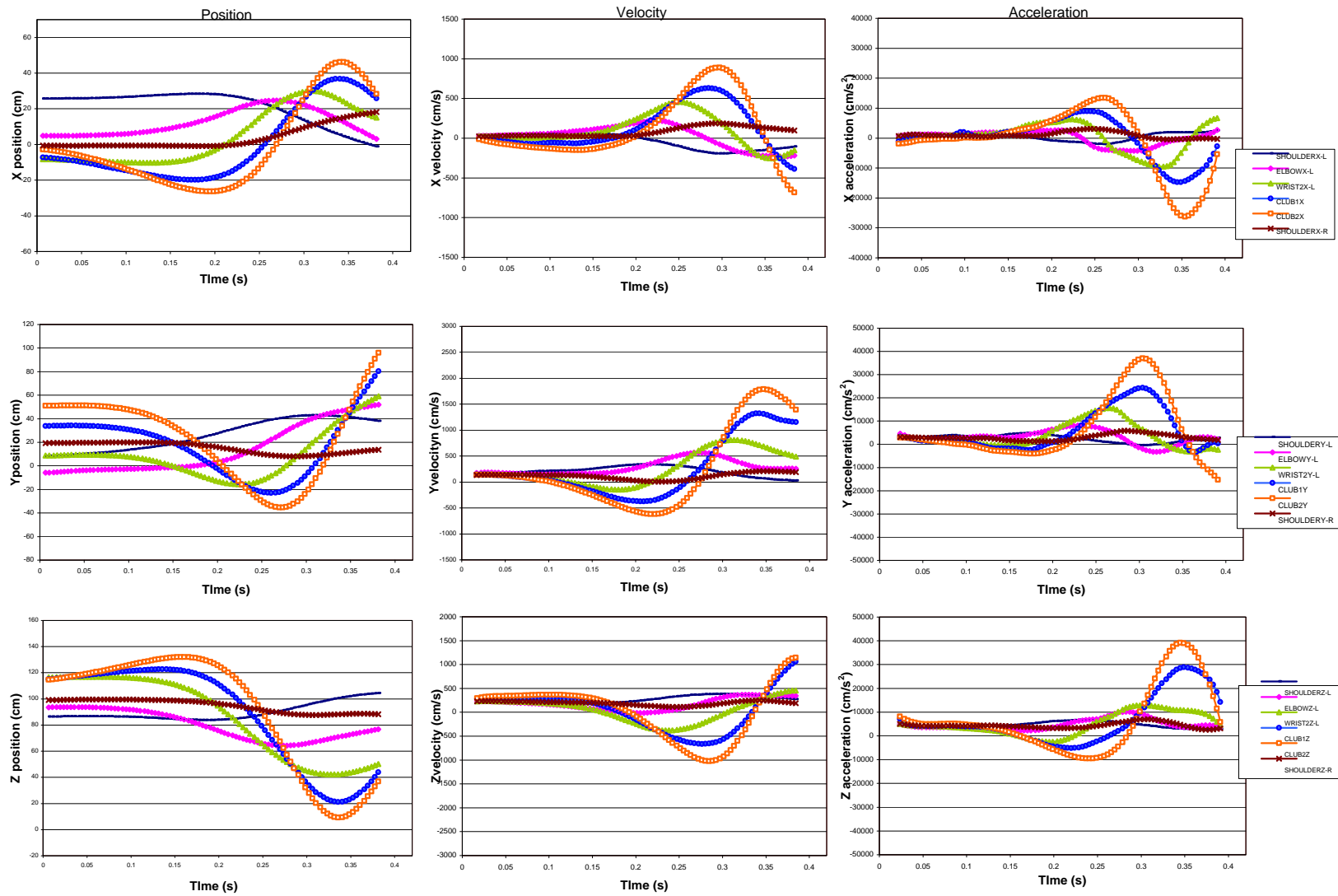


Figure 4.3 Displacement, velocity, acceleration of shoulder, wrist, elbow, club from subject F2 (female), trial 9.



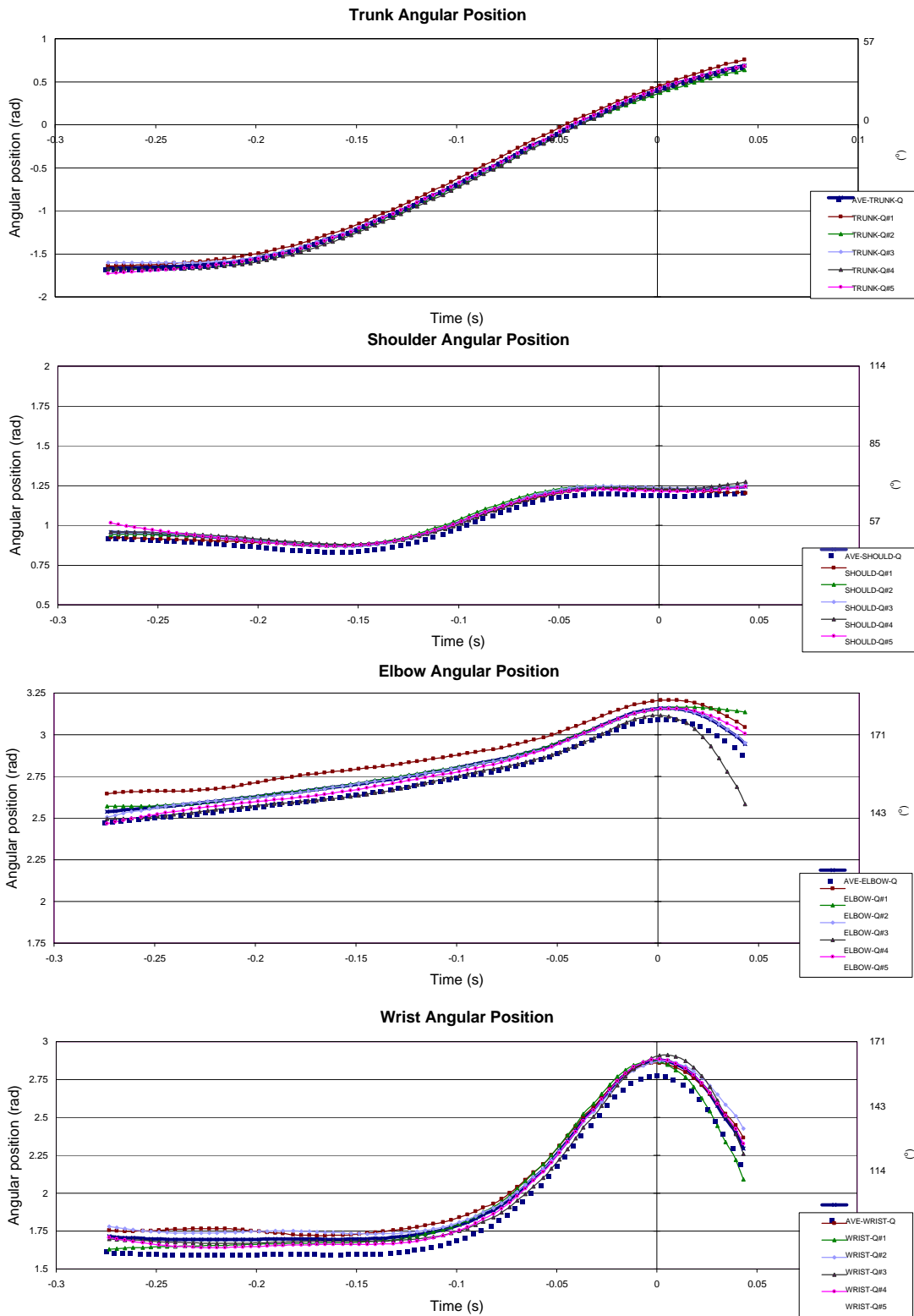


Figure 4.4 Subject M4 's angular position data of trial(s) and mean for trunk, shoulder, elbow, and wrist joints.

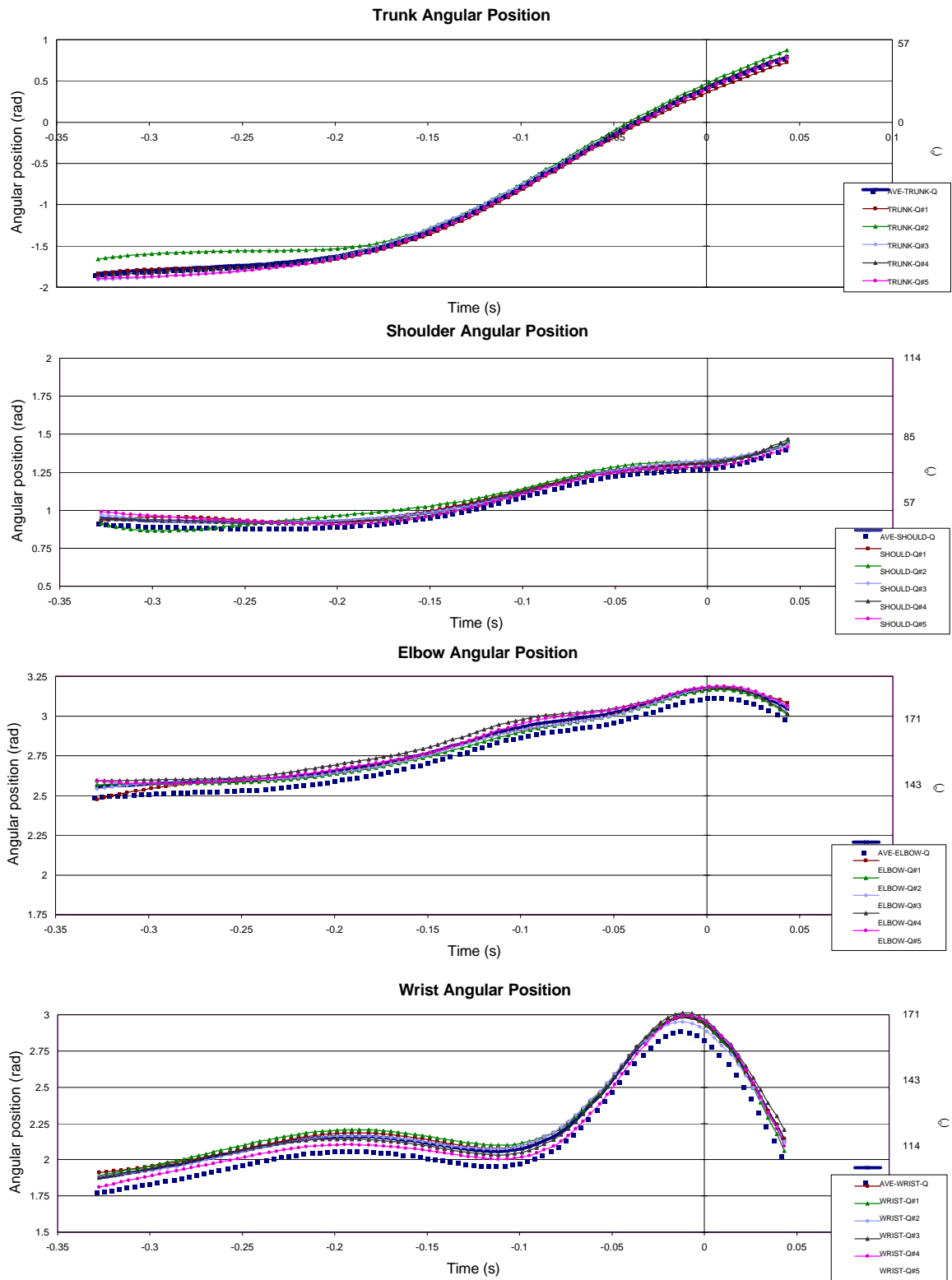


Figure 4.5 Subject F2 's angular position data of trial(s) and mean for trunk, shoulder, elbow, and wrist joints.

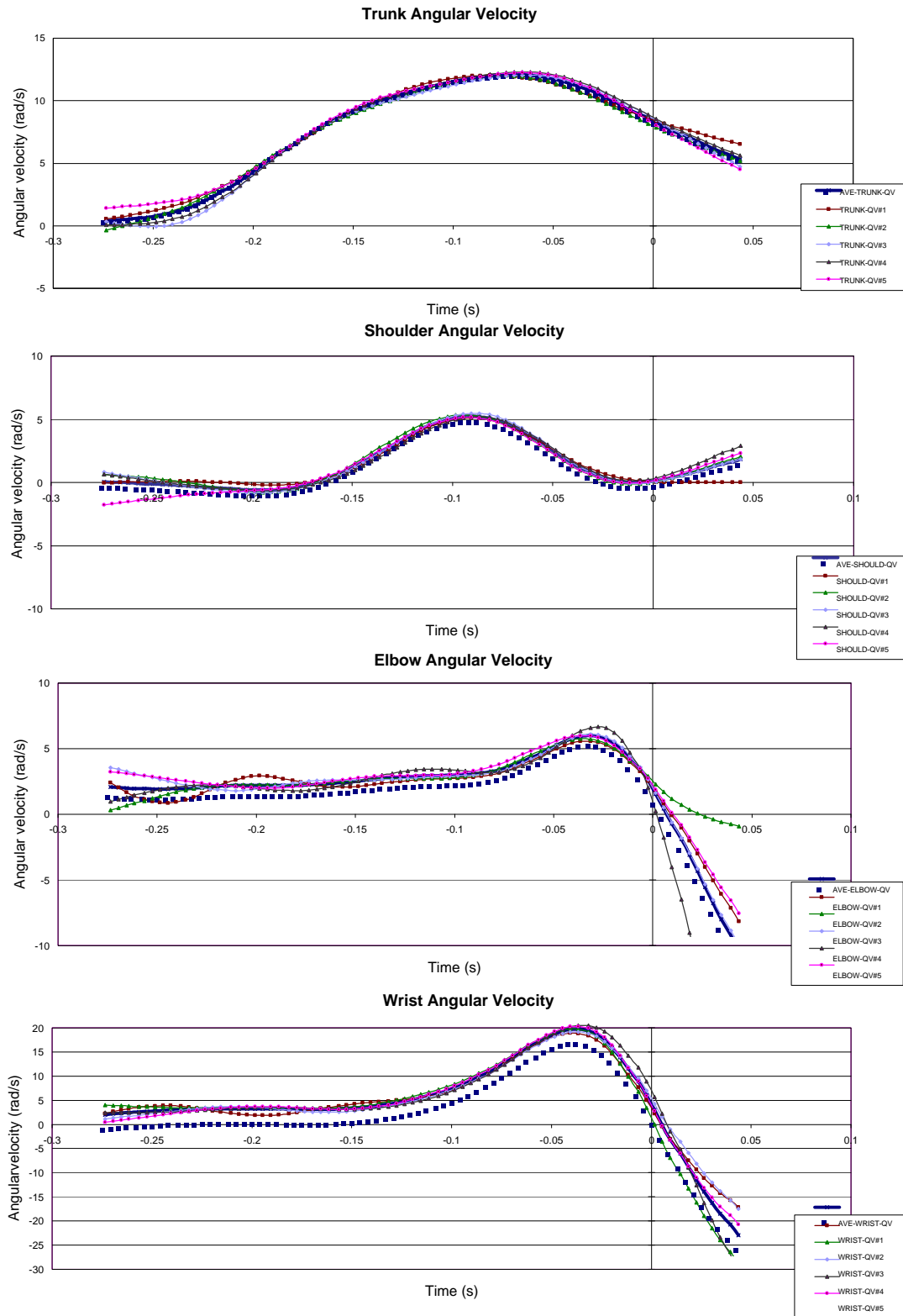


Figure 4.6 Subject M4 's angular velocity of trial(s) and mean for trunk, shoulder, elbow, and wrist joints.

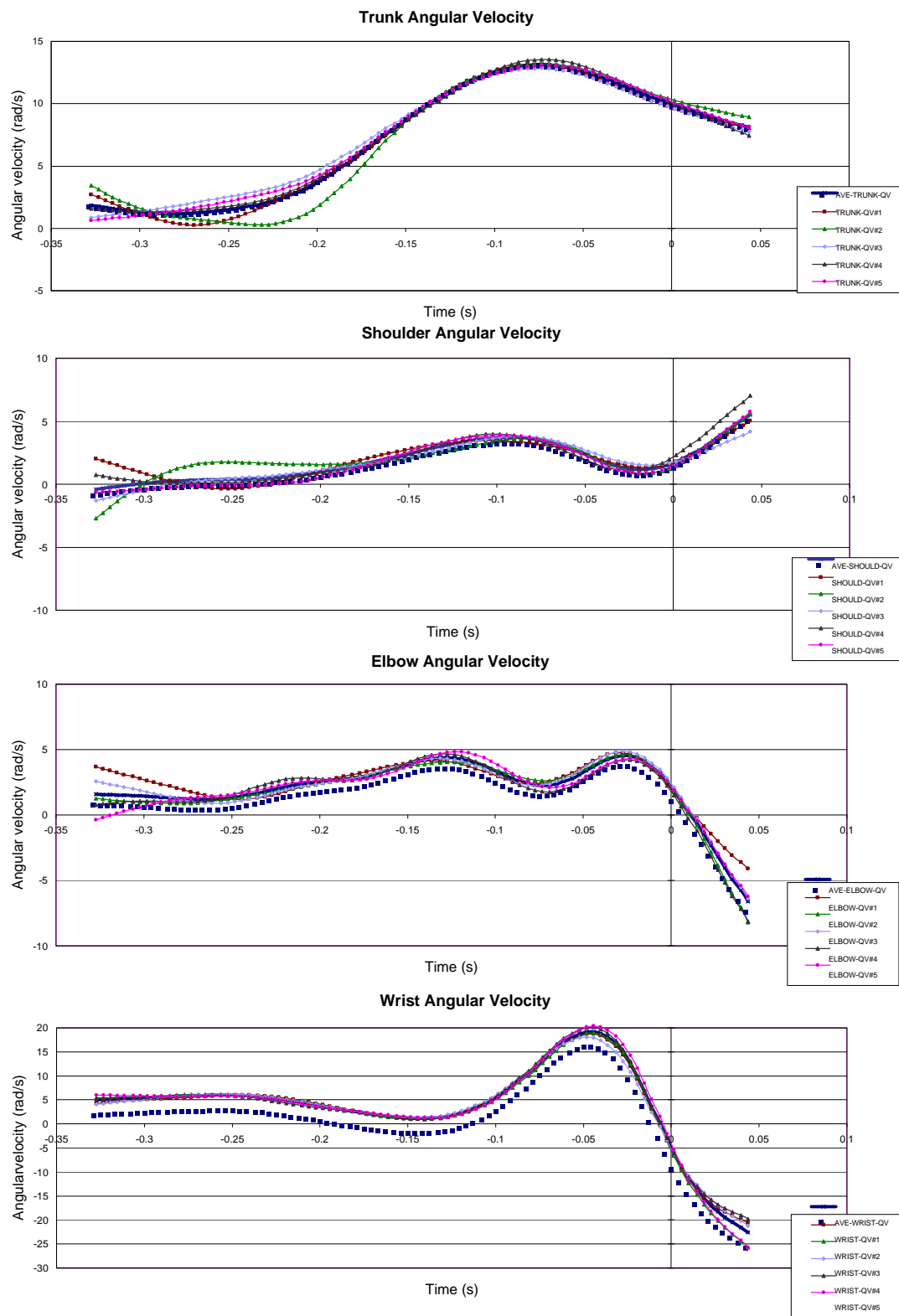


Figure 4.7 Subject F2 's angular velocity of trial(s) and mean for trunk, shoulder, elbow, and wrist joints.

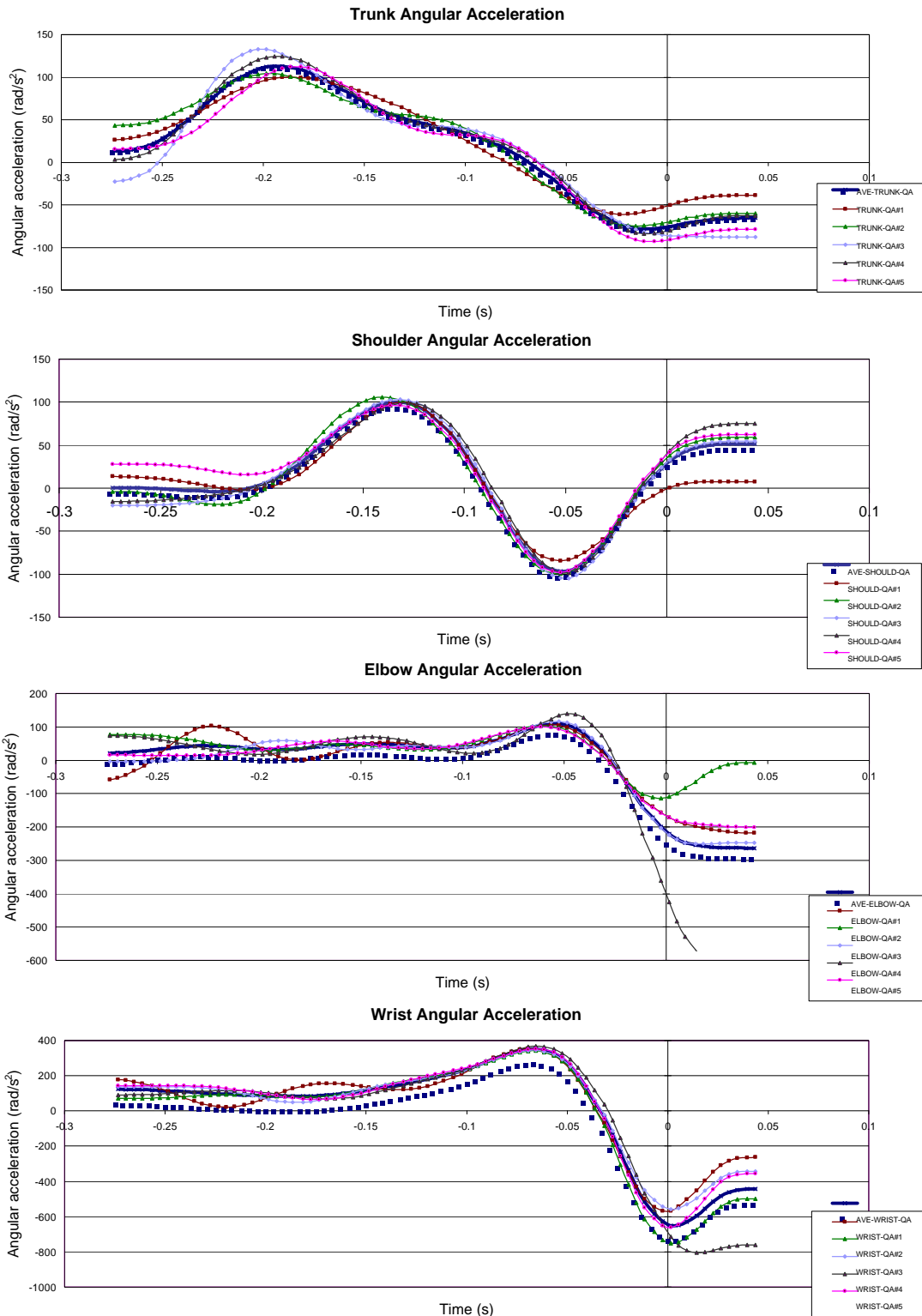


Figure 4.8 Subject M4 's angular acceleration of trial(s) and mean for trunk,shoulder,elbow, and wrist joints.

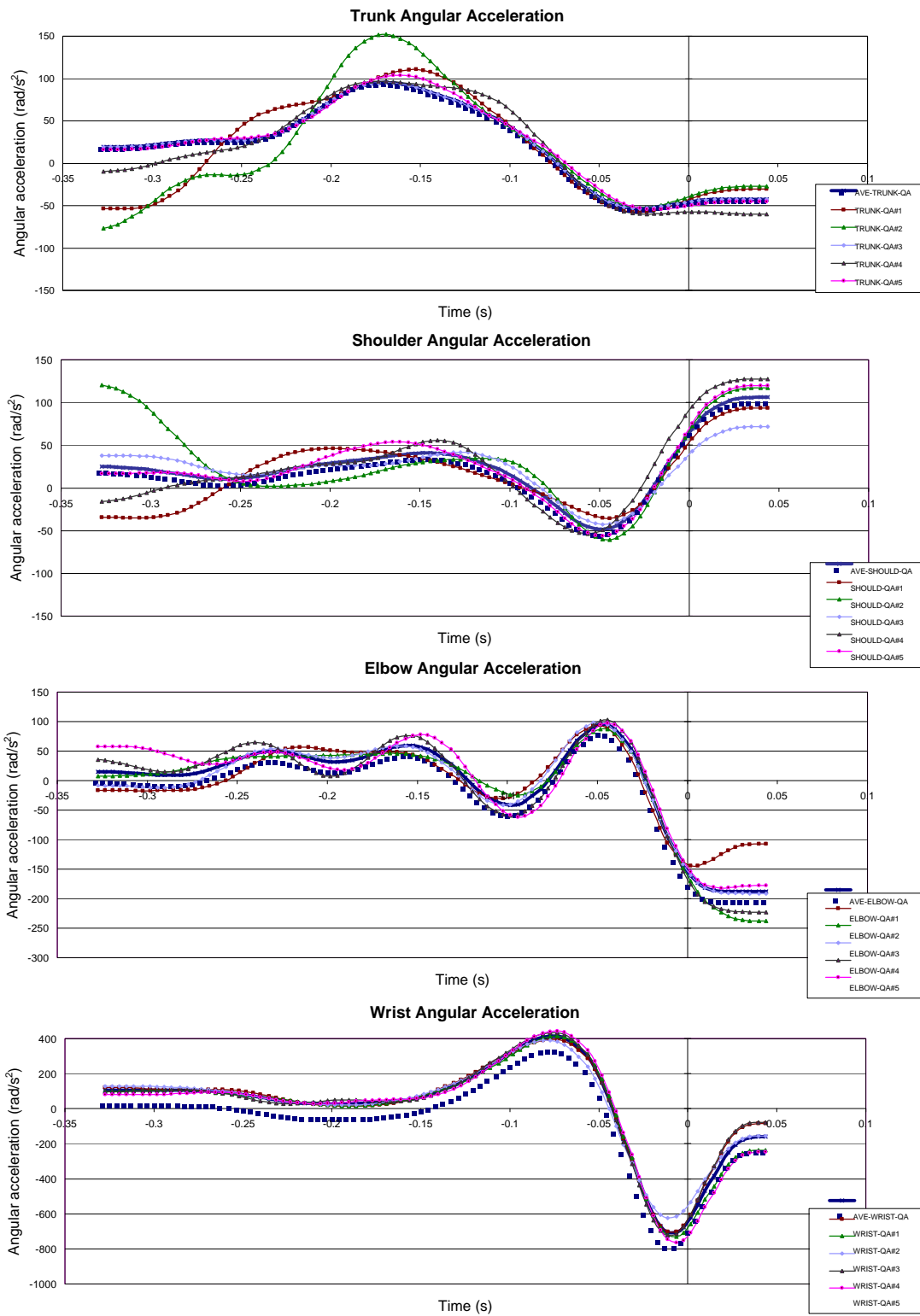


Figure 4.9 Subject F2 's angular acceleration of trial(s) and mean for trunk, shoulder, elbow, and wrist joints.

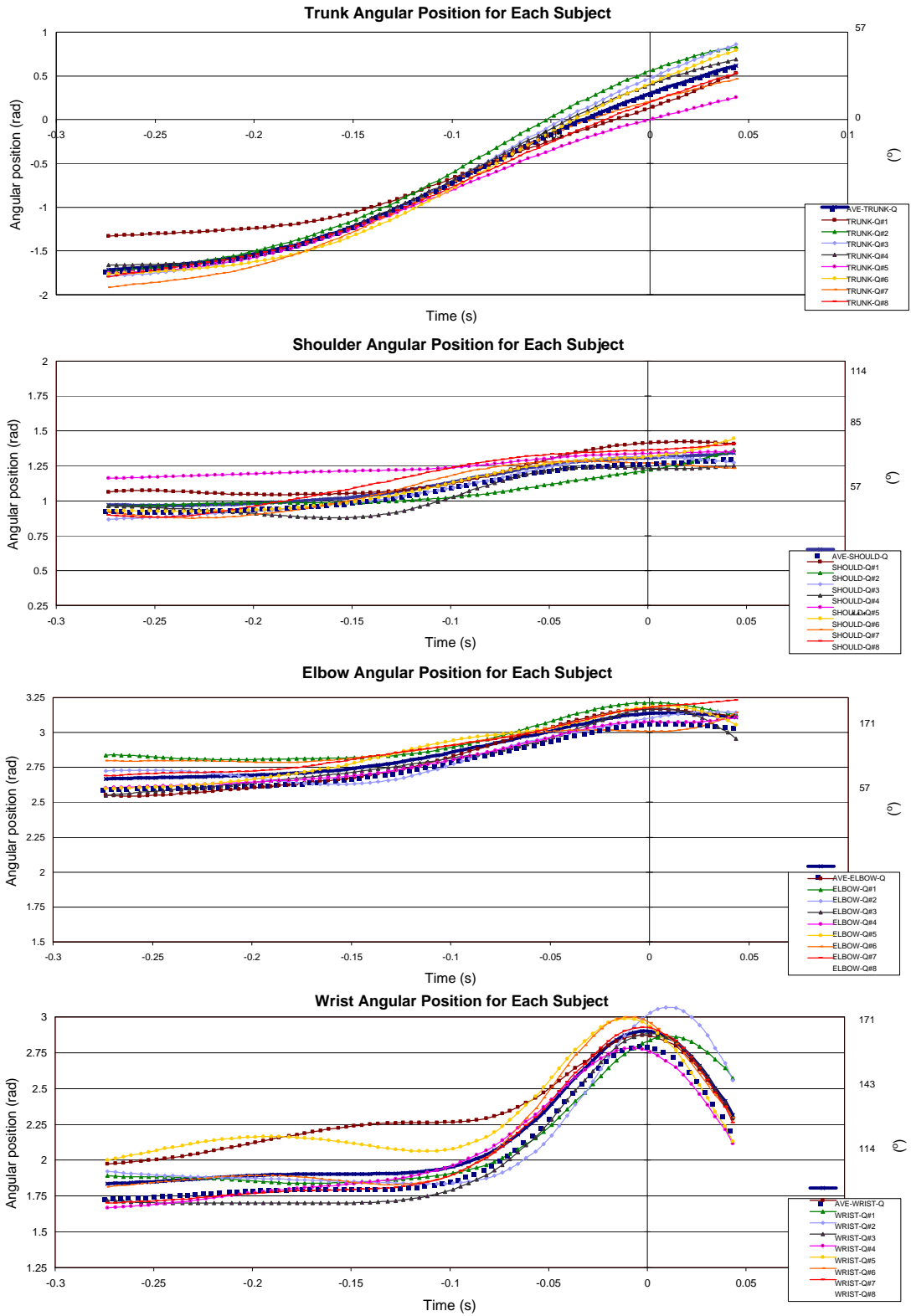


Figure 4.10 Angular position of trunk, shoulder, elbow, and wrist joints (all subjects and mean).

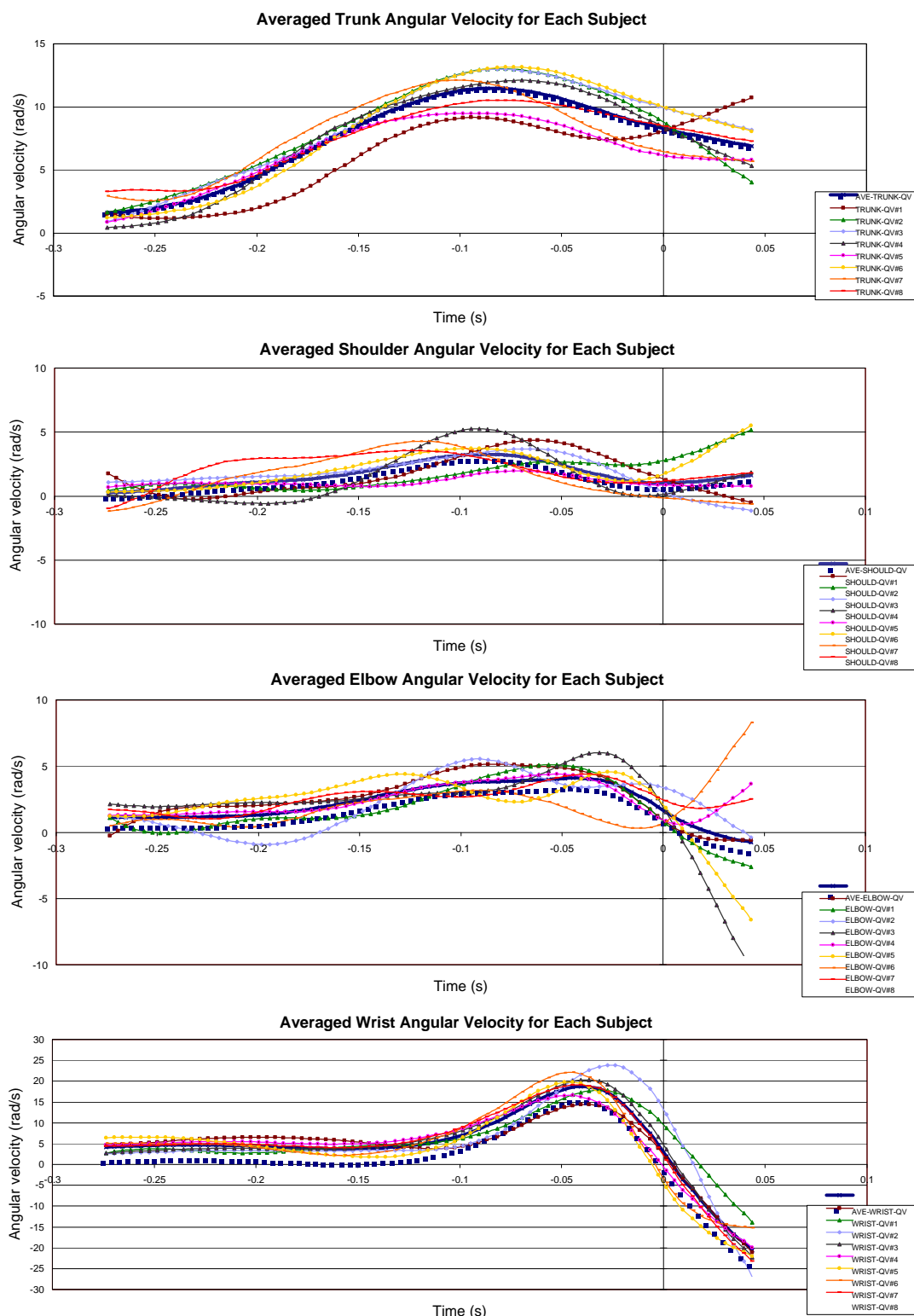


Figure 4.11 Angular velocity of trunk, shoulder, elbow, and wrist joints (all subjects and mean).



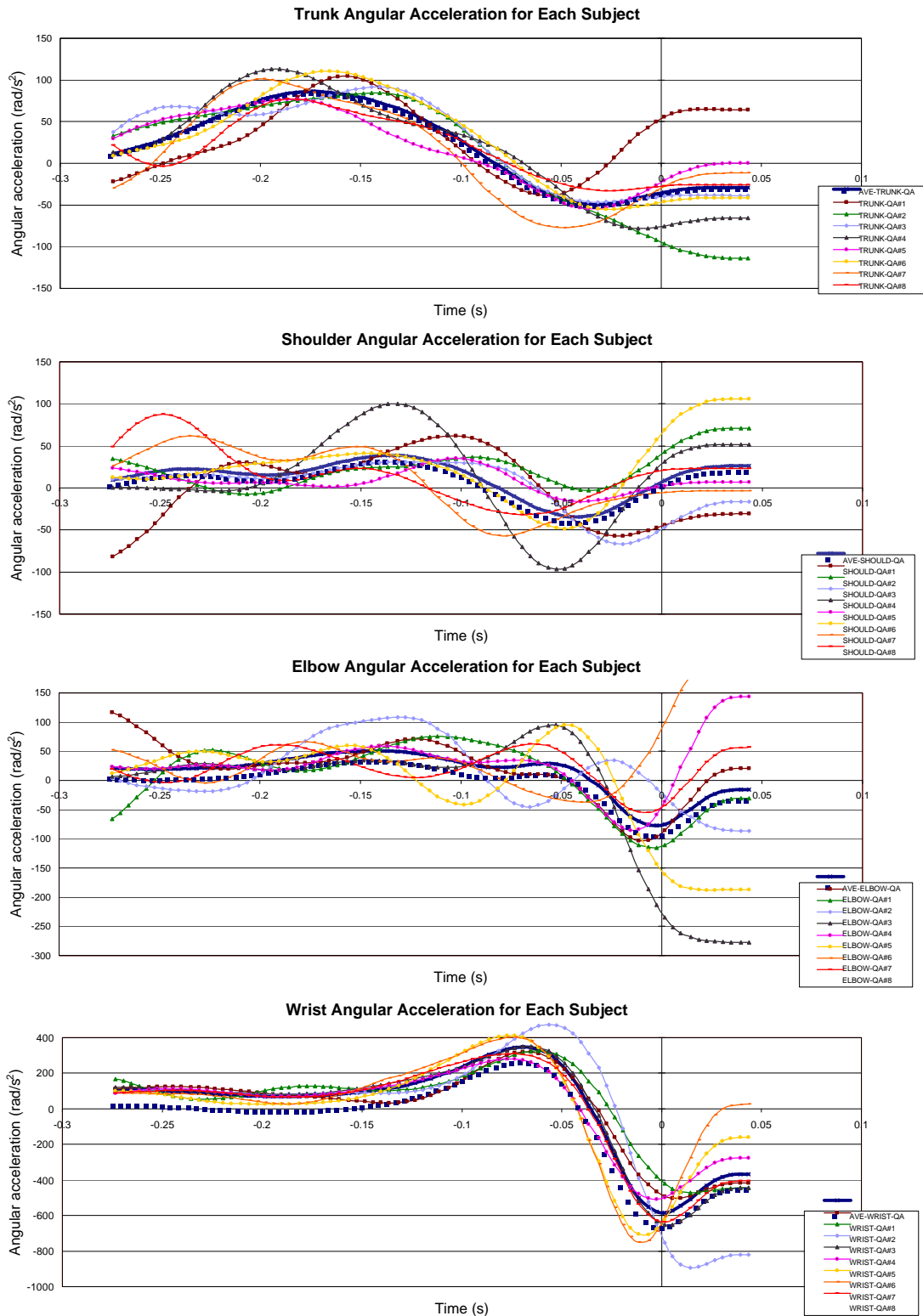


Figure 4.12 Angular acceleration of trunk, shoulder, elbow, and wrist joints (all subjects and mean).

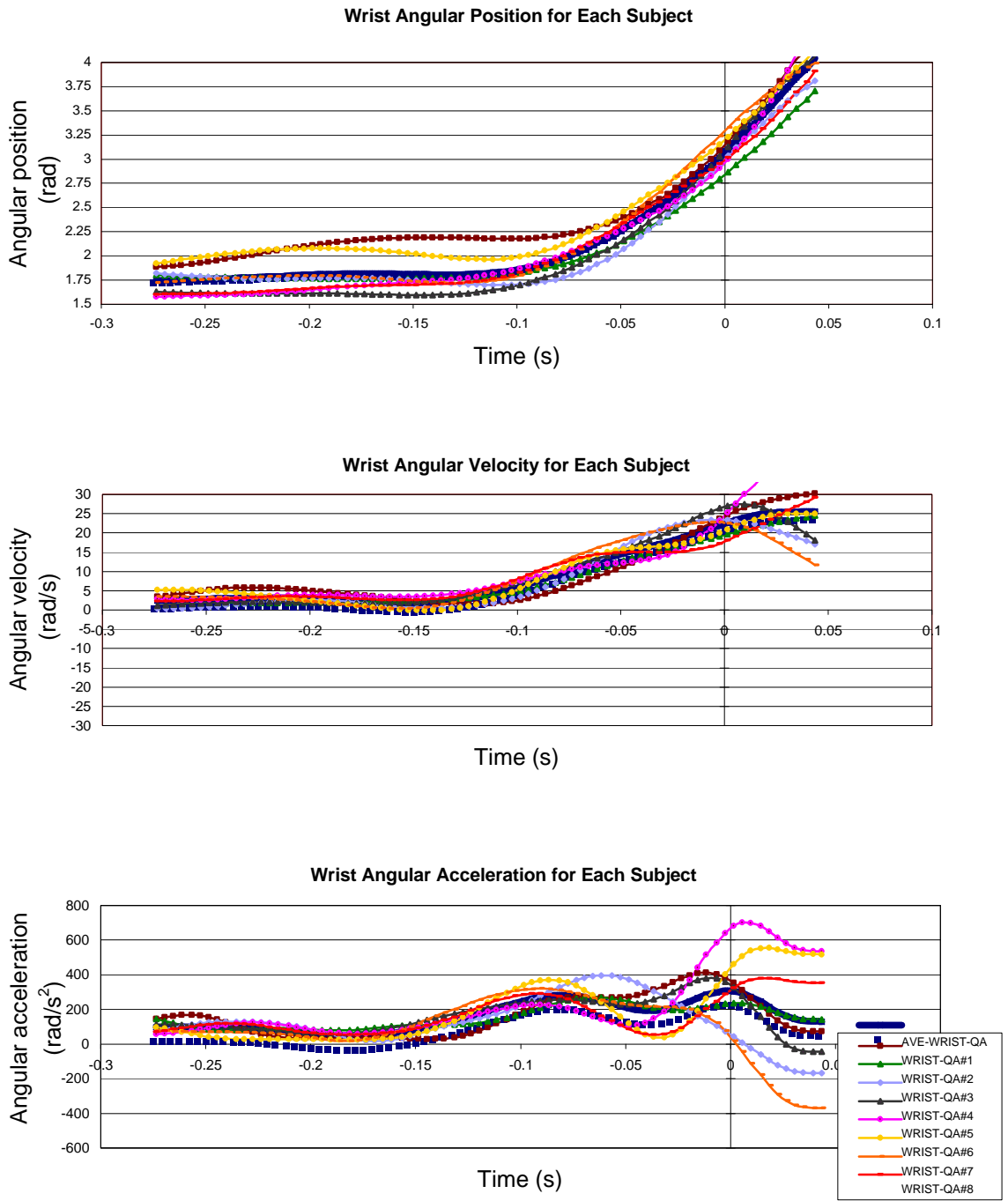


Figure 4.13 Angular position, velocity, and acceleration of wrist joint in projected plane (all subjects and mean).

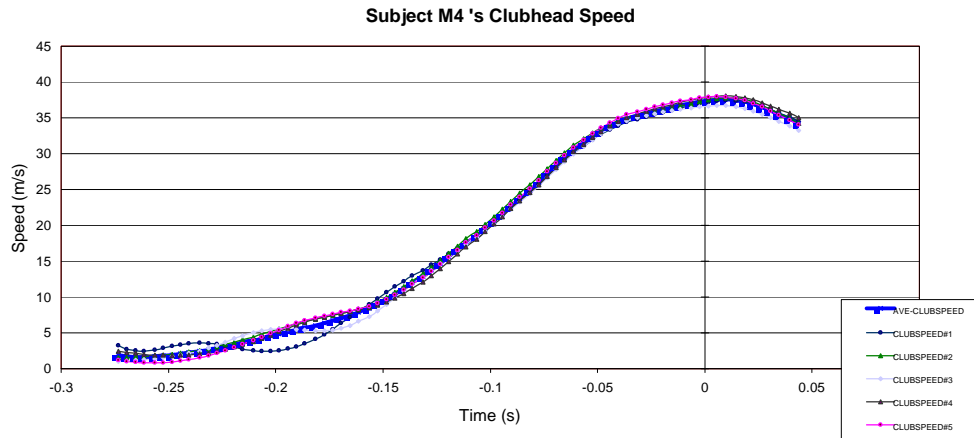


Figure 4.14 Subject M4 's clubhead speed by trial(s) and mean.

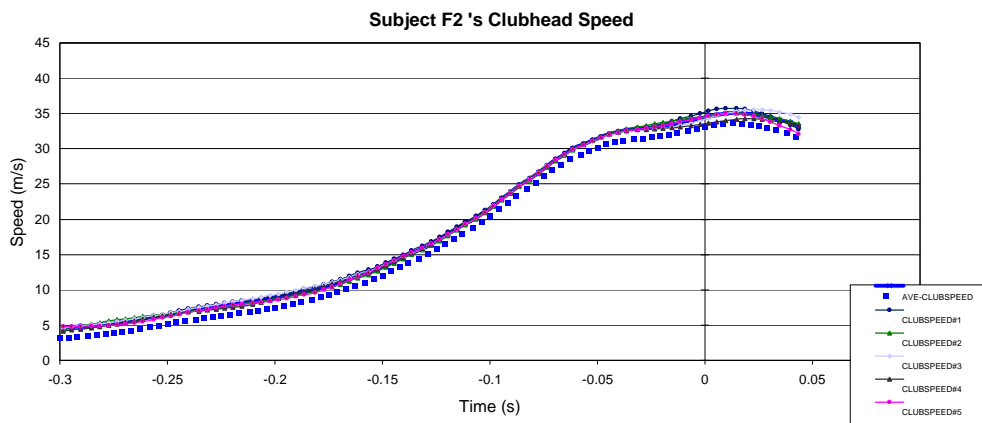


Figure 4.15 Subject F2 's clubhead speed by trial(s) and mean.

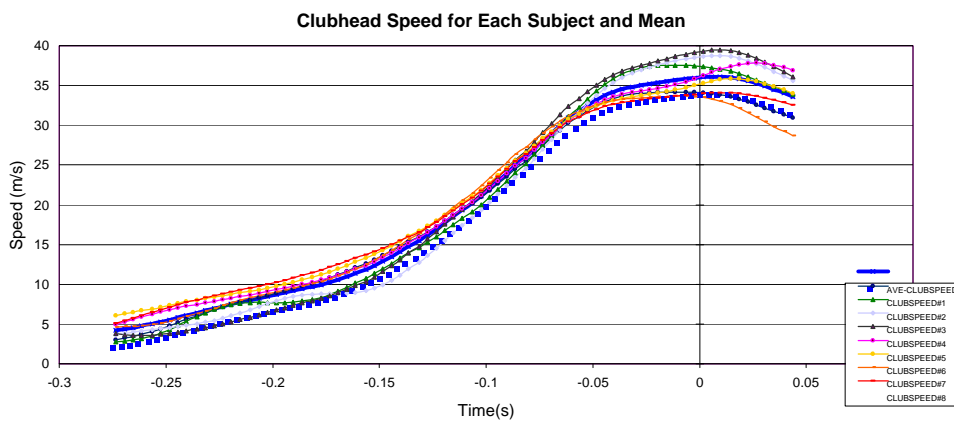


Figure 4.16 Clubhead speed (all subjects and mean).

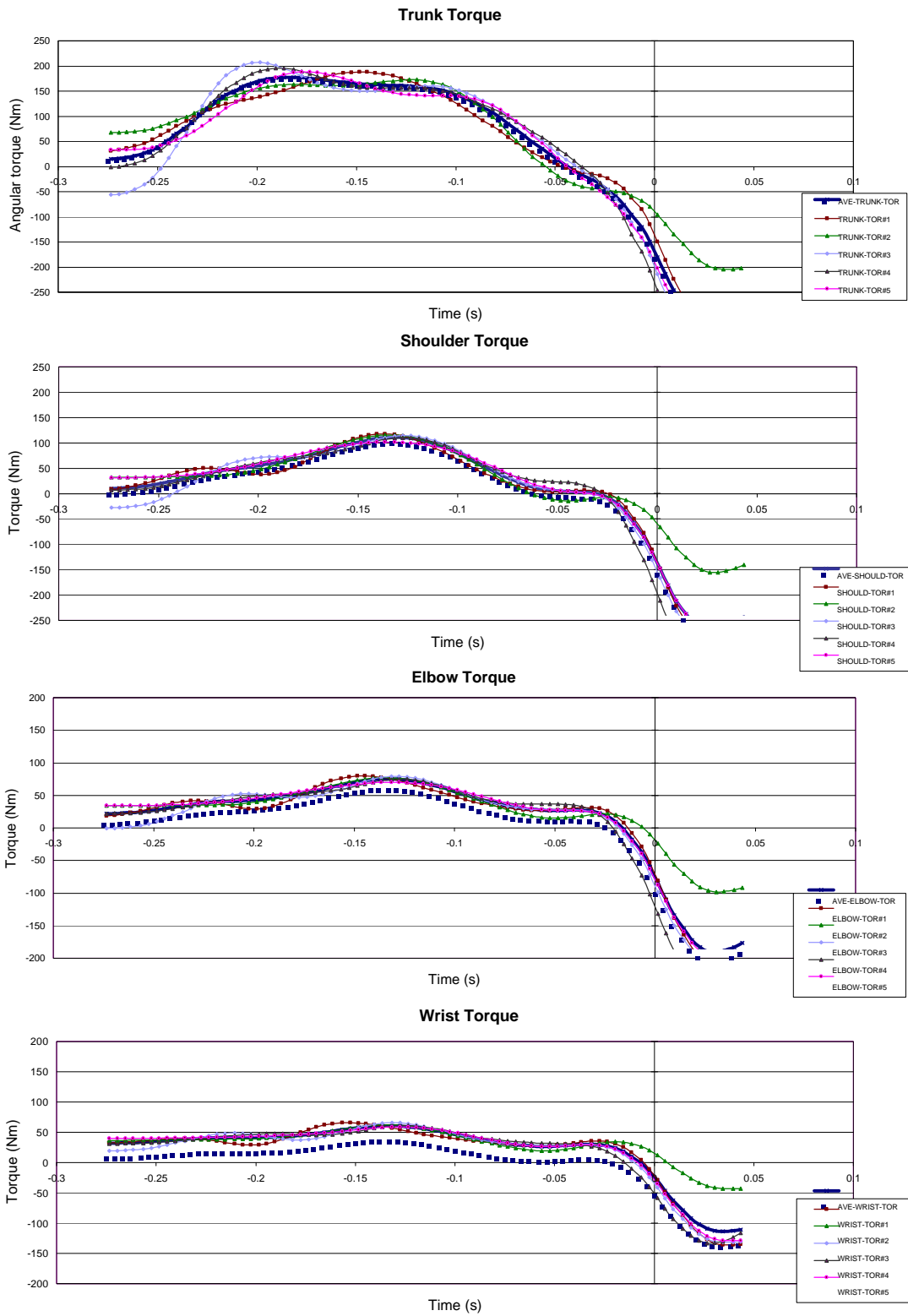


Figure 4.17 Subject M4 's torque by trial(s) and mean for trunk, shoulder, elbow, and wrist joints.

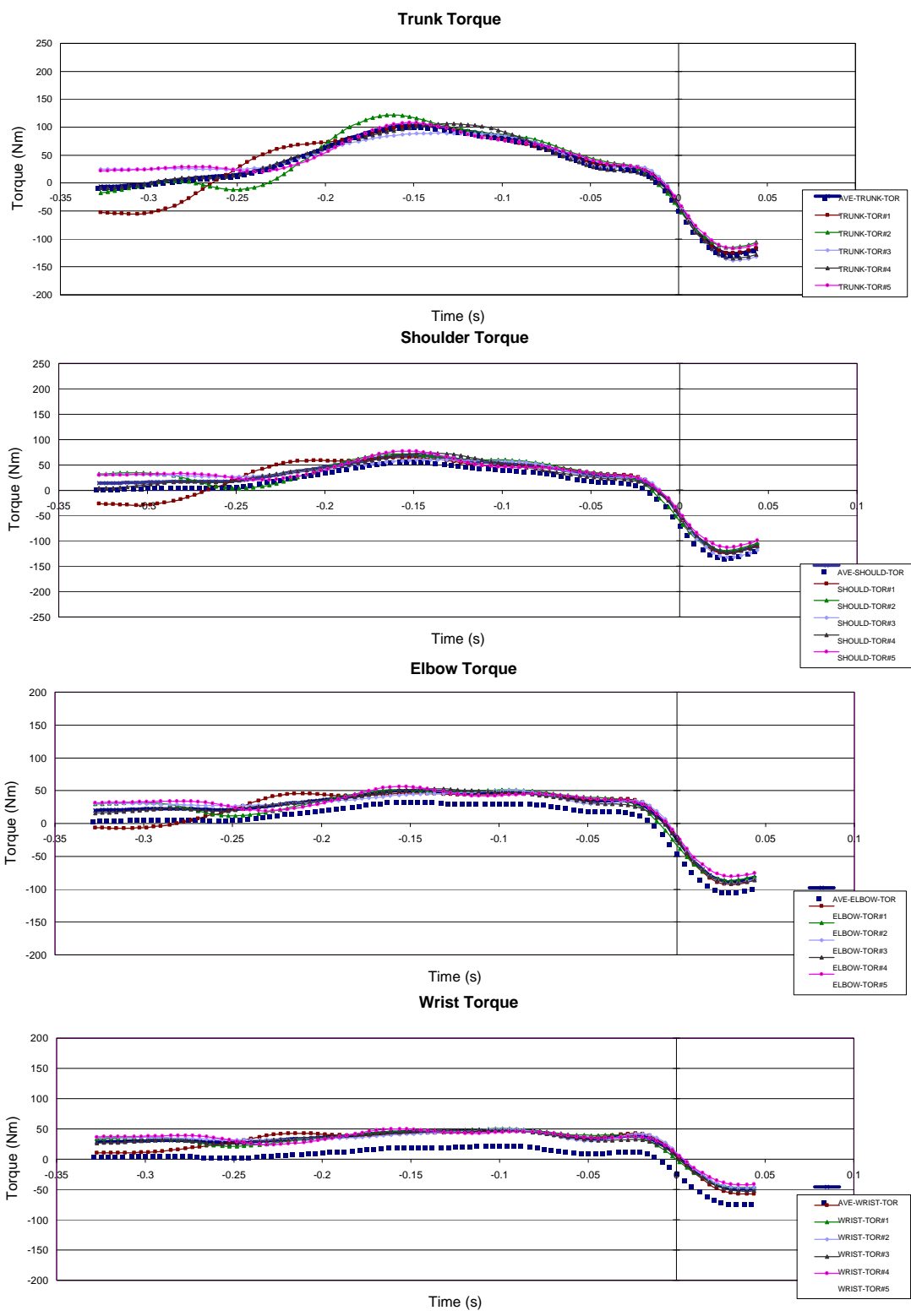


Figure 4.18 Subject F2 's torque by trial(s) and mean for trunk, shoulder, elbow, and wrist joints.

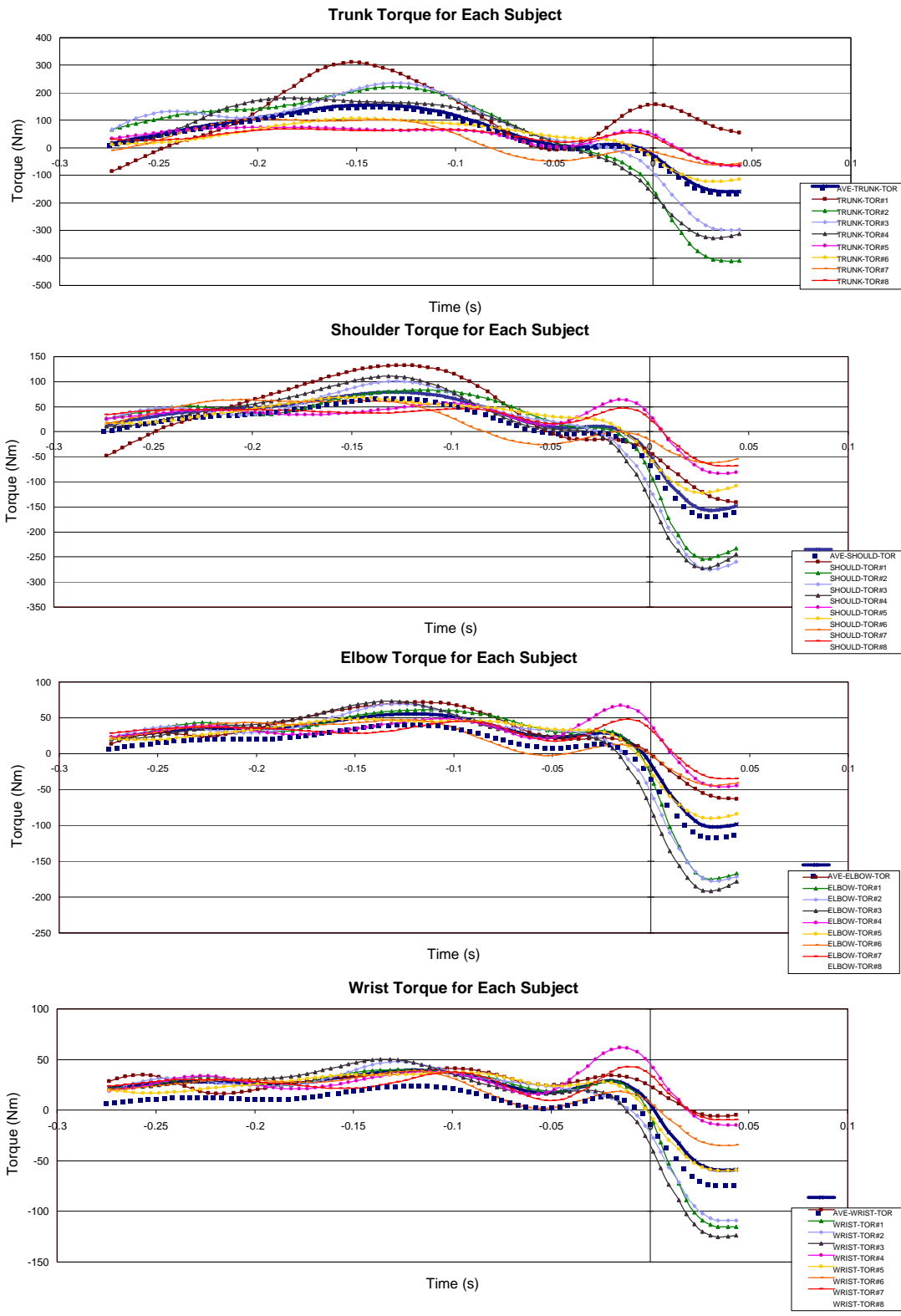


Figure 4.19 Torques of trunk, shoulder, elbow, and wrist joints (all subjects and mean).

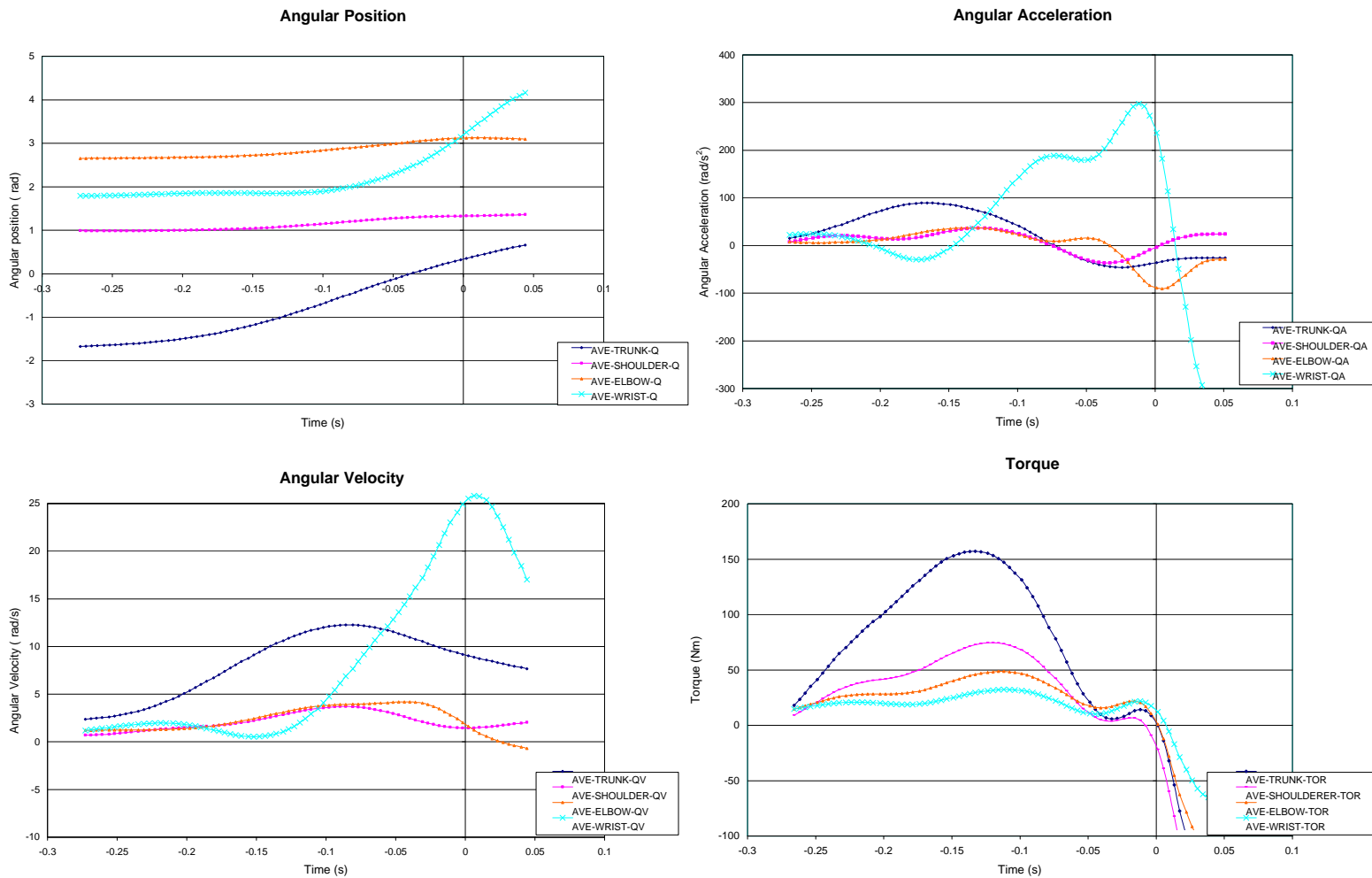


Figure 4.20 Averaged angular position, velocity, acceleration, and torque from all subjects (projected plane).

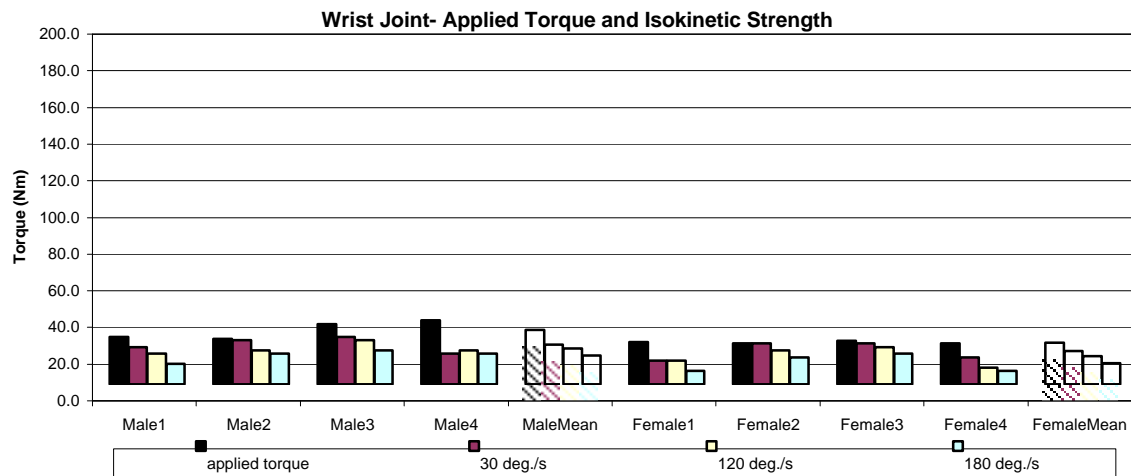
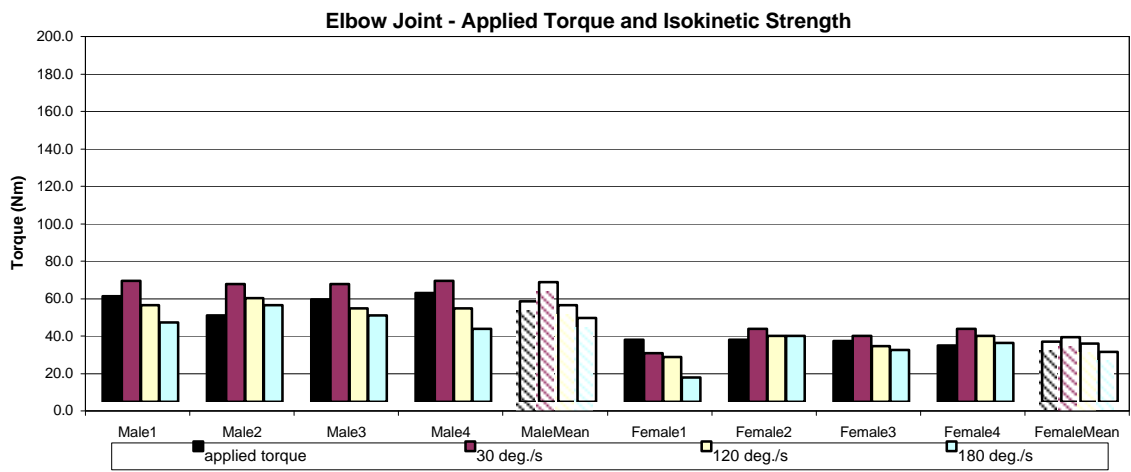
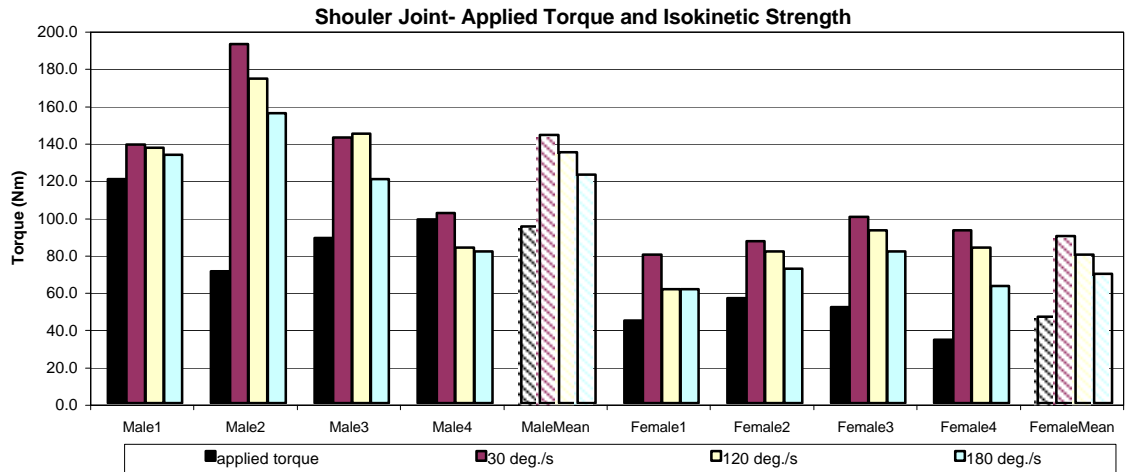


Figure 4.21 Applied joint torque and isokinetic strength at shoulder, elbow, and wrist (projected plane).



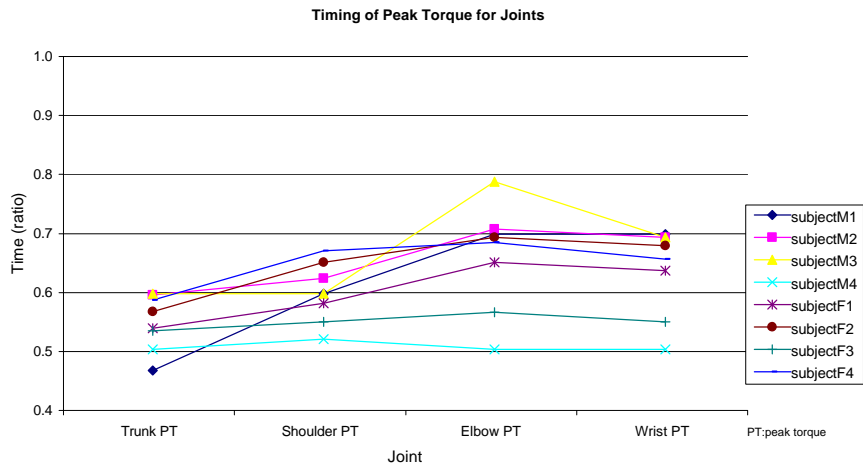


Figure 4.22 The pattern of timing of peak torque for joints (all subjects).

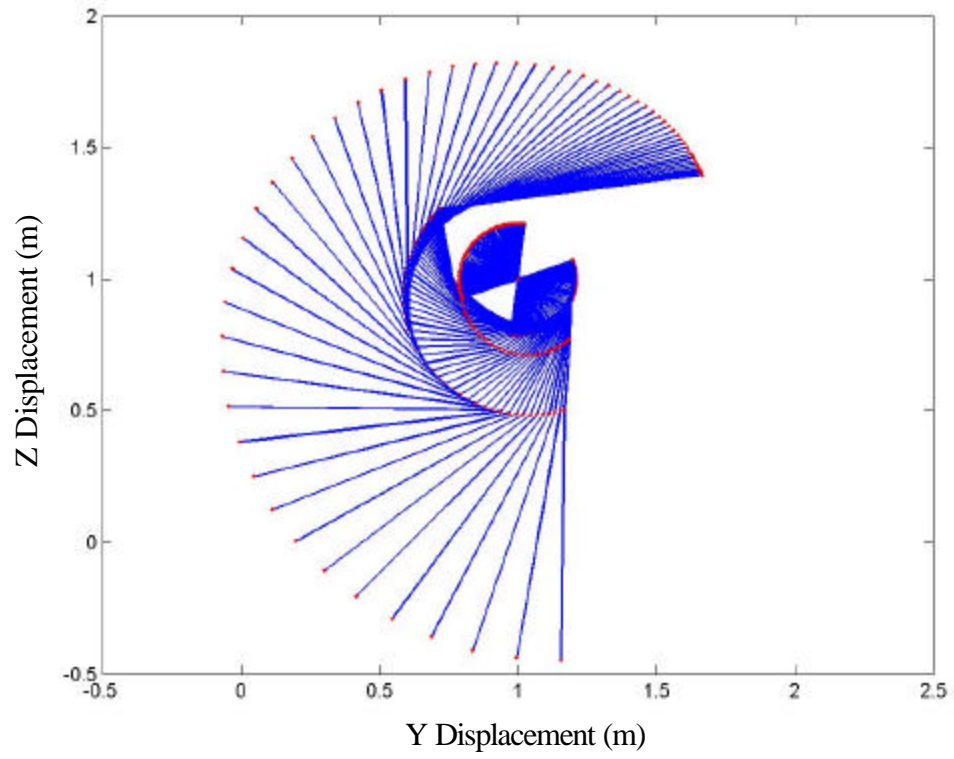


Figure 4.23 Experimental animation of subject M4 in projected plane.

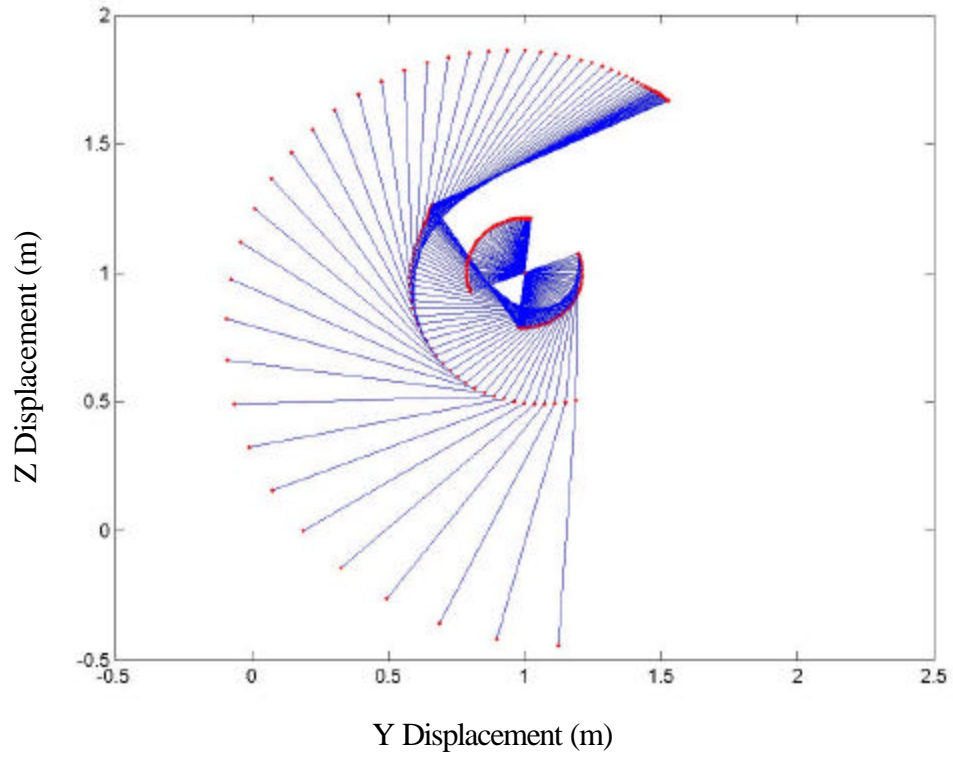


Figure 4.24 Optimal simulation of three-segment model in projected plane.

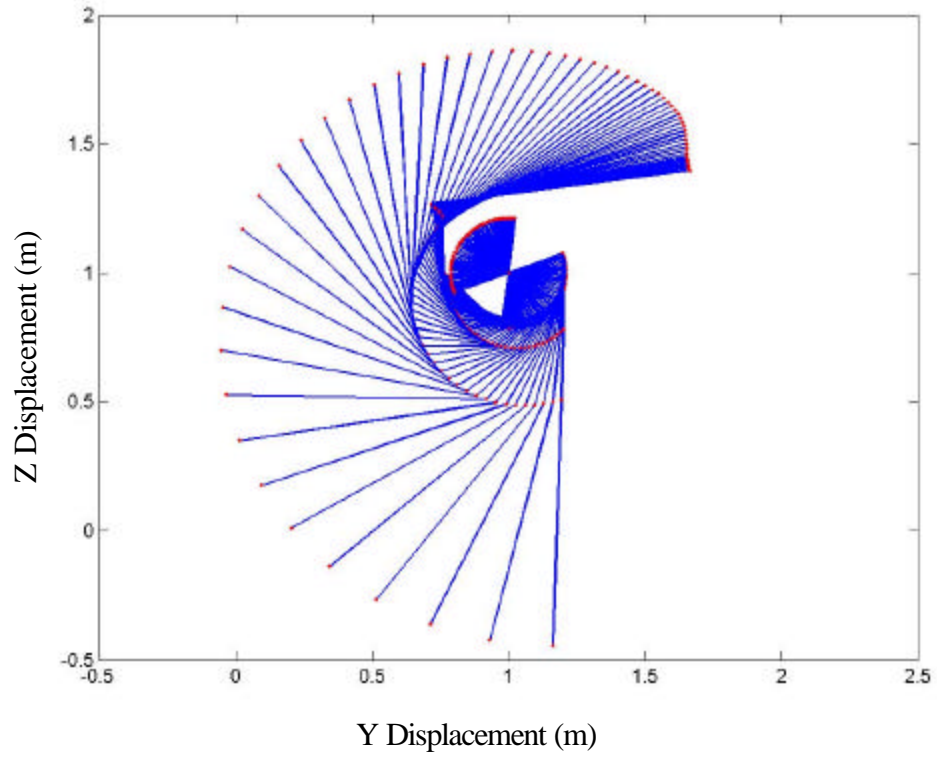


Figure 4.25 Optimal simulation of four-segment model in projected plane.

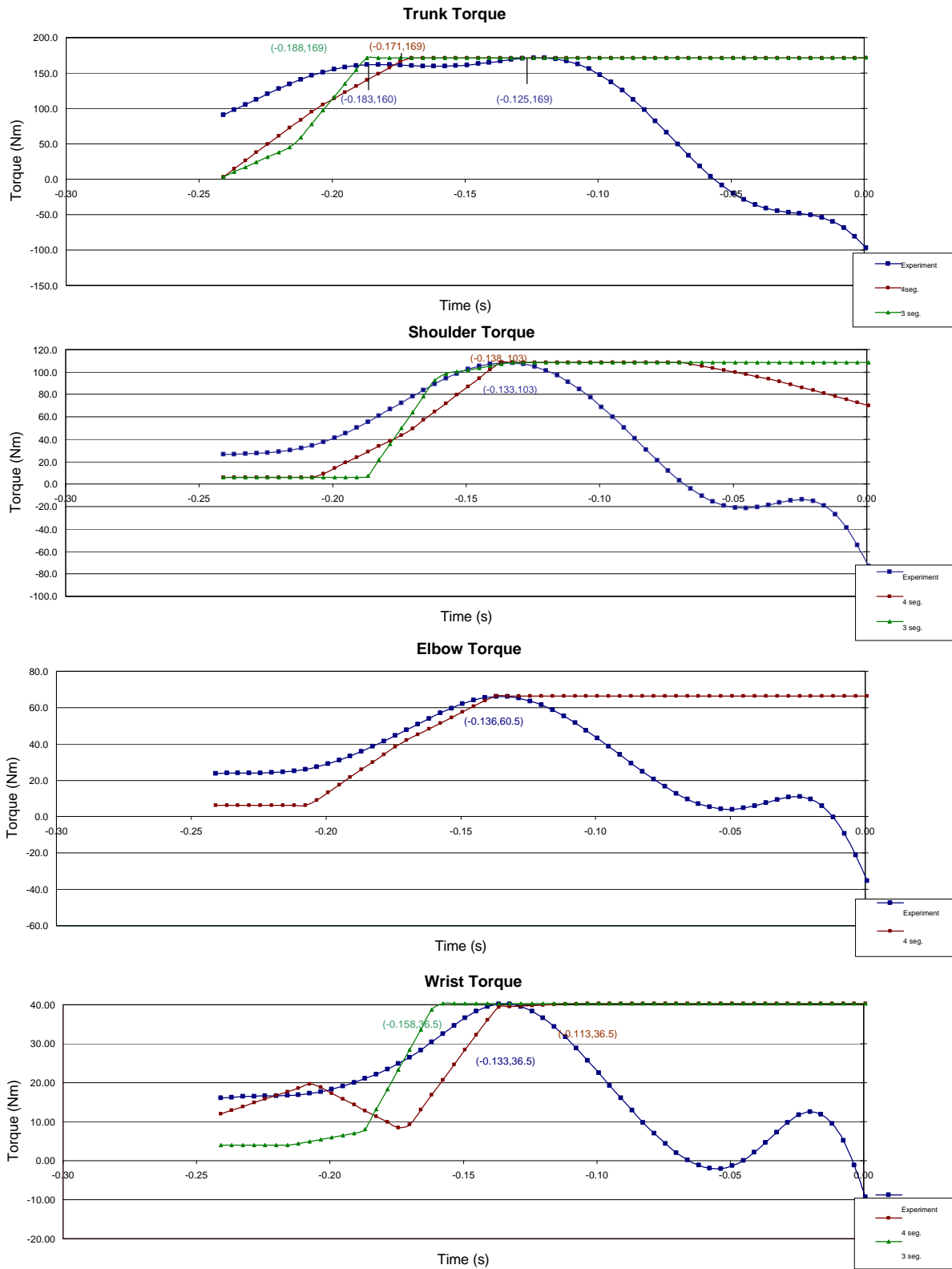


Figure 4.26 Experimental and optimal torques for trunk, shoulder, elbow, and wrist joints.

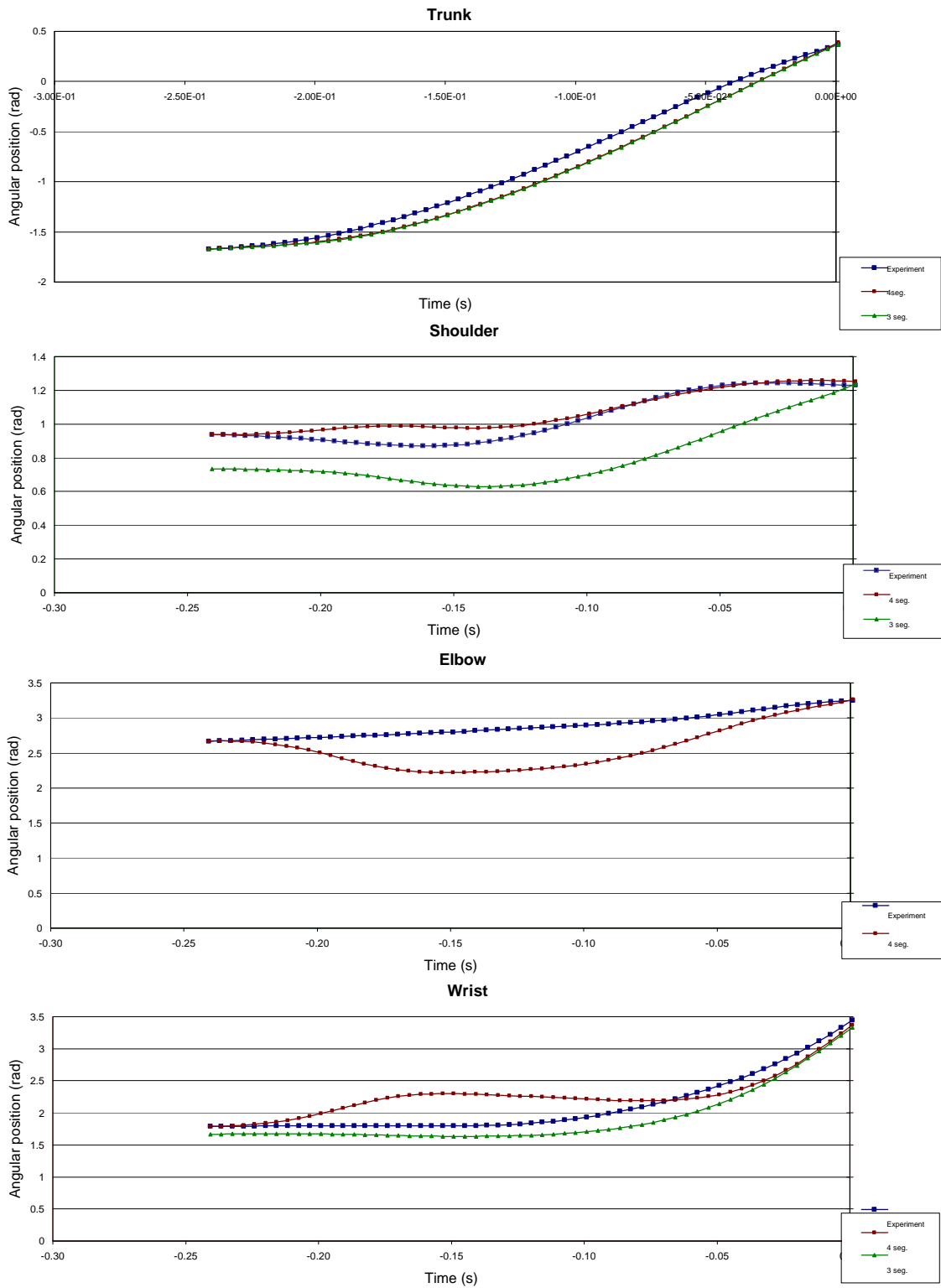


Figure 4.27 Experimental and optimal angular position for trunk, shoulder, elbow, and wrist joints.

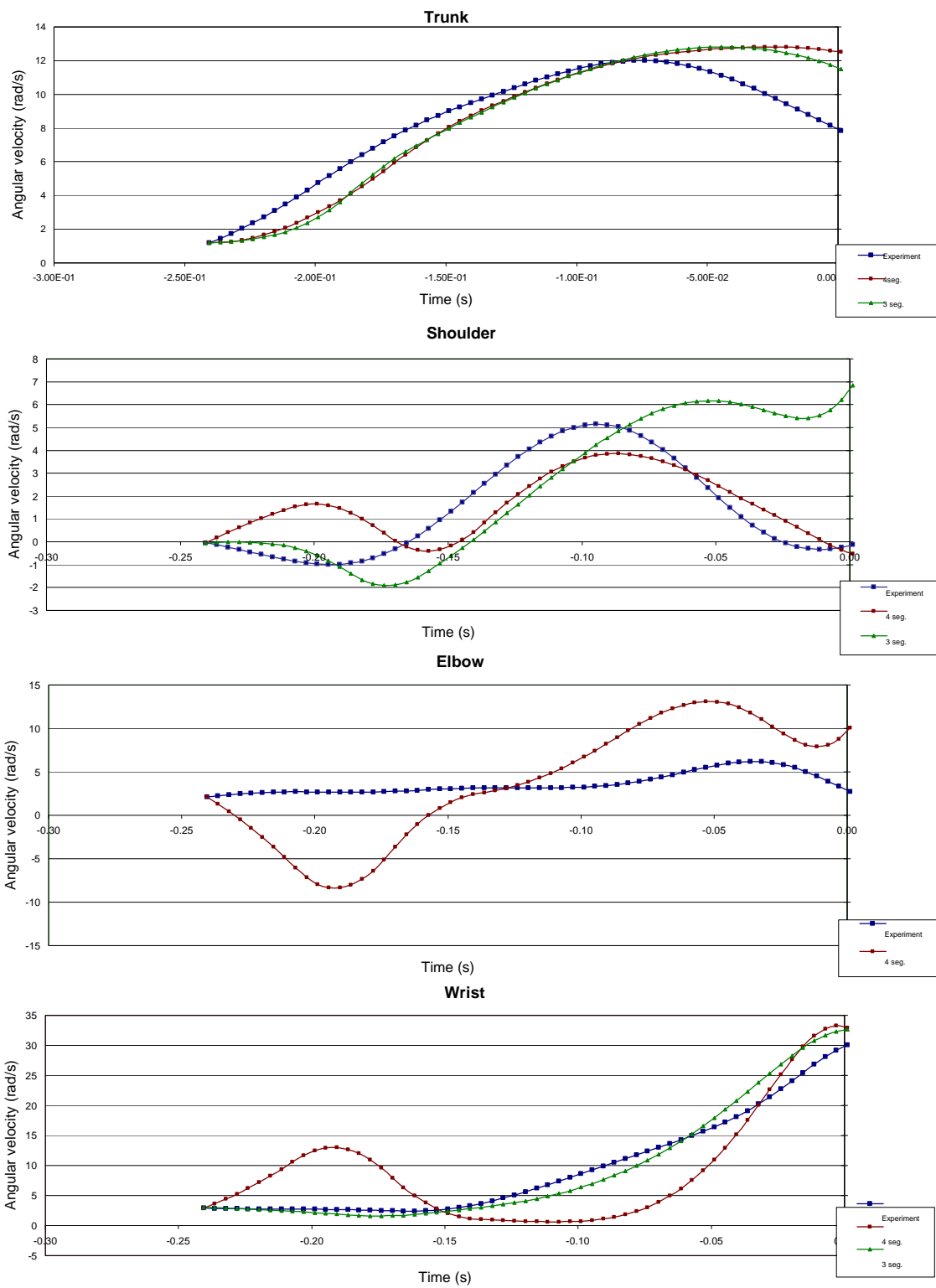


Figure 4.28 Experimental and optimal angular velocity for trunk, shoulder, elbow, and wrist joints.

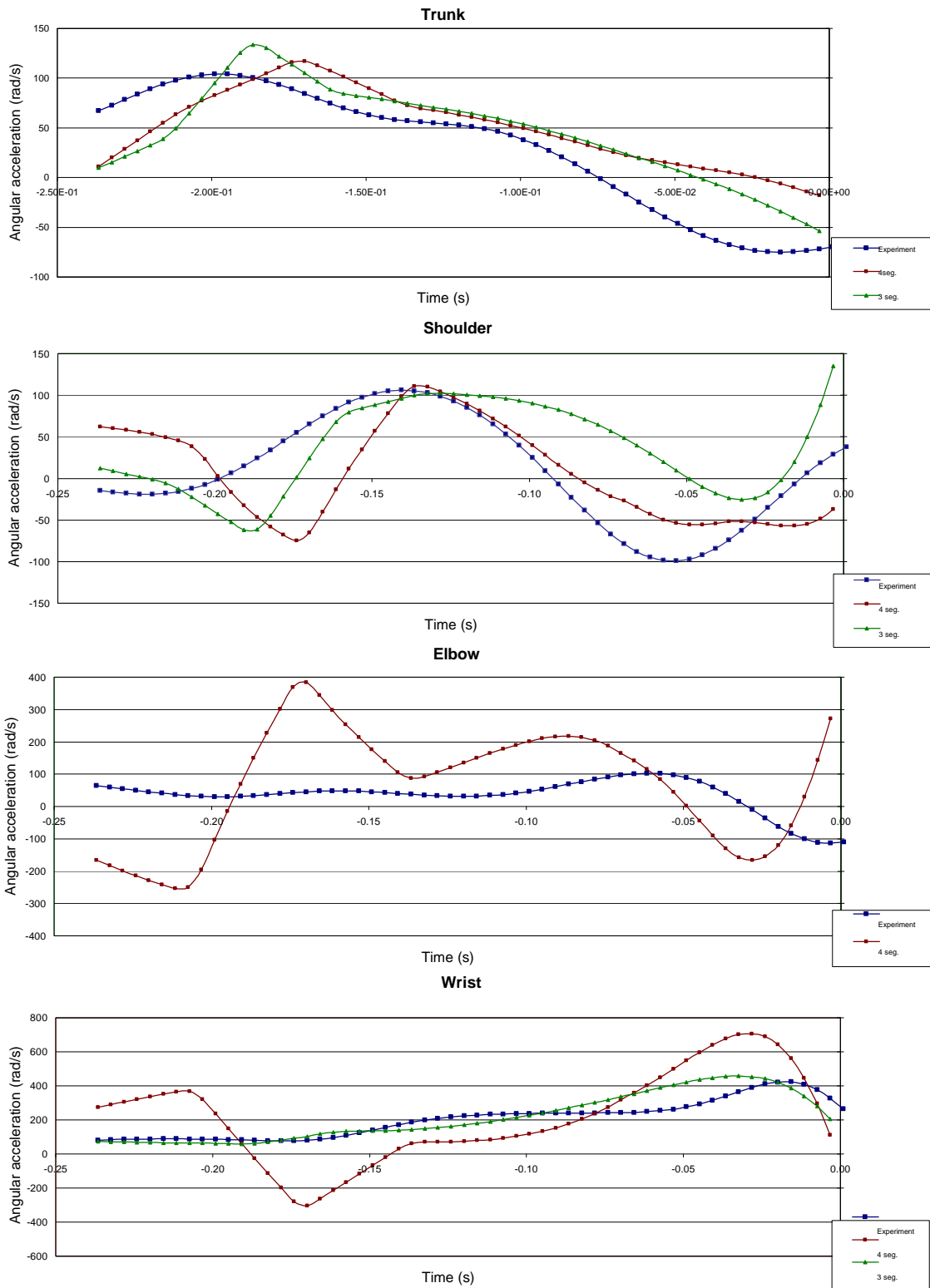


Figure 4.29 Experimental and optimal angular acceleration for trunk, shoulder, elbow, and wrist joints.



## Performance

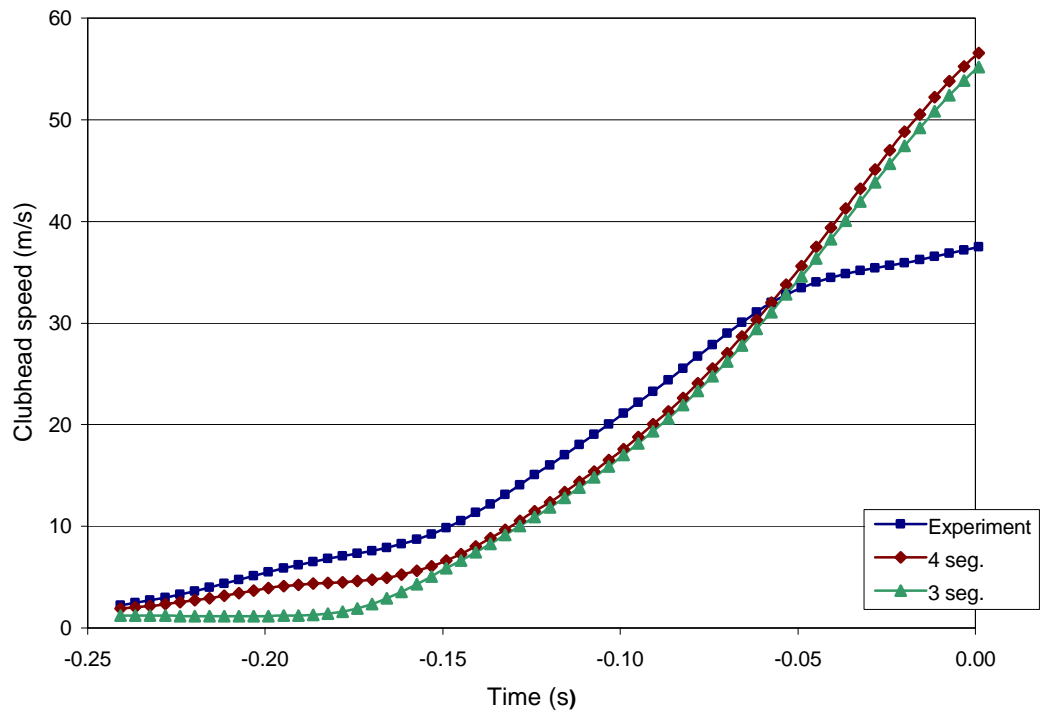


Figure 4.30 Experimental and optimal clubhead speed for trunk, shoulder, elbow, and wrist joints.

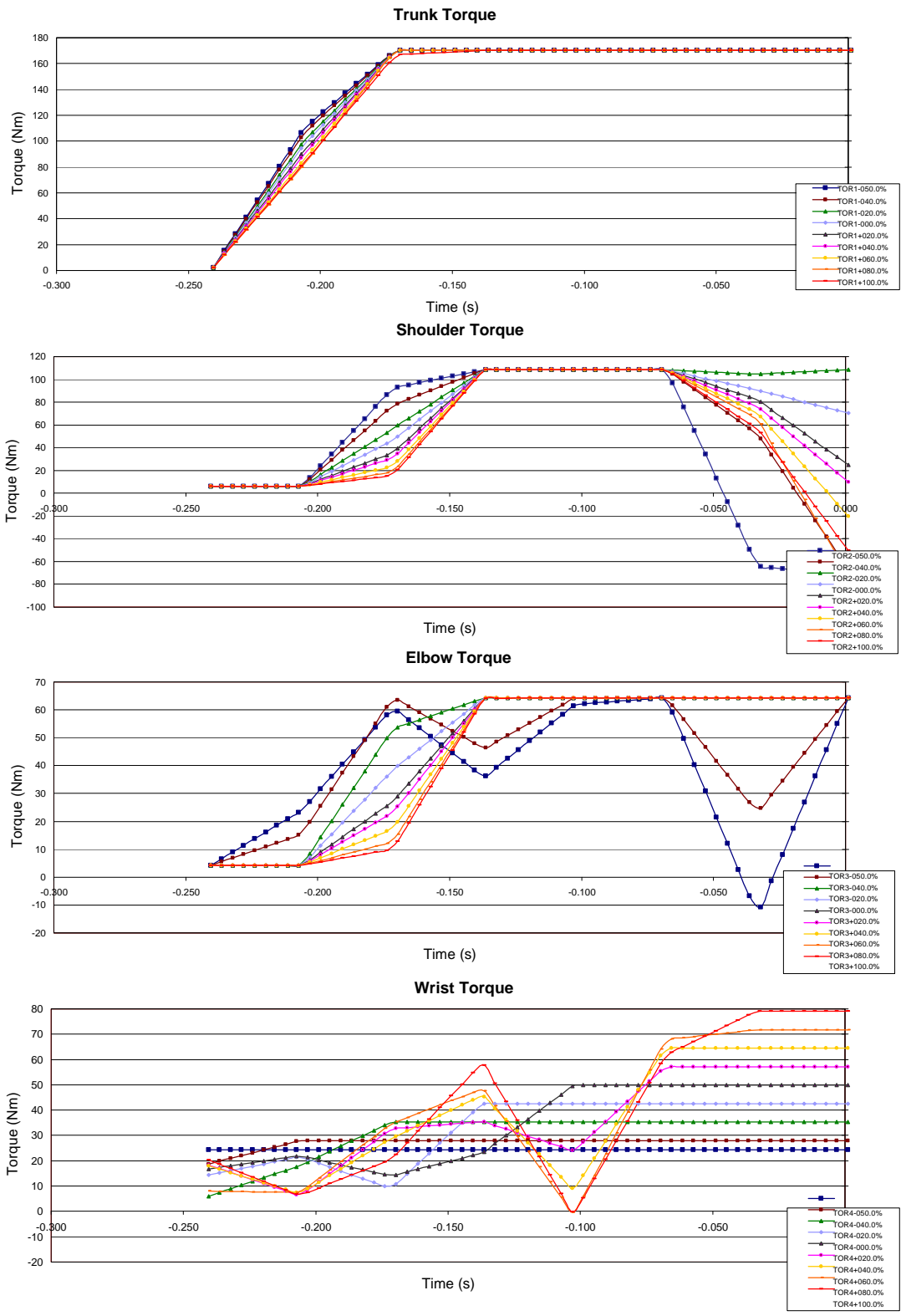


Figure 4.31 Torques of trunk, shoulder, elbow, and wrist joints over varied wrist strength boundaries.

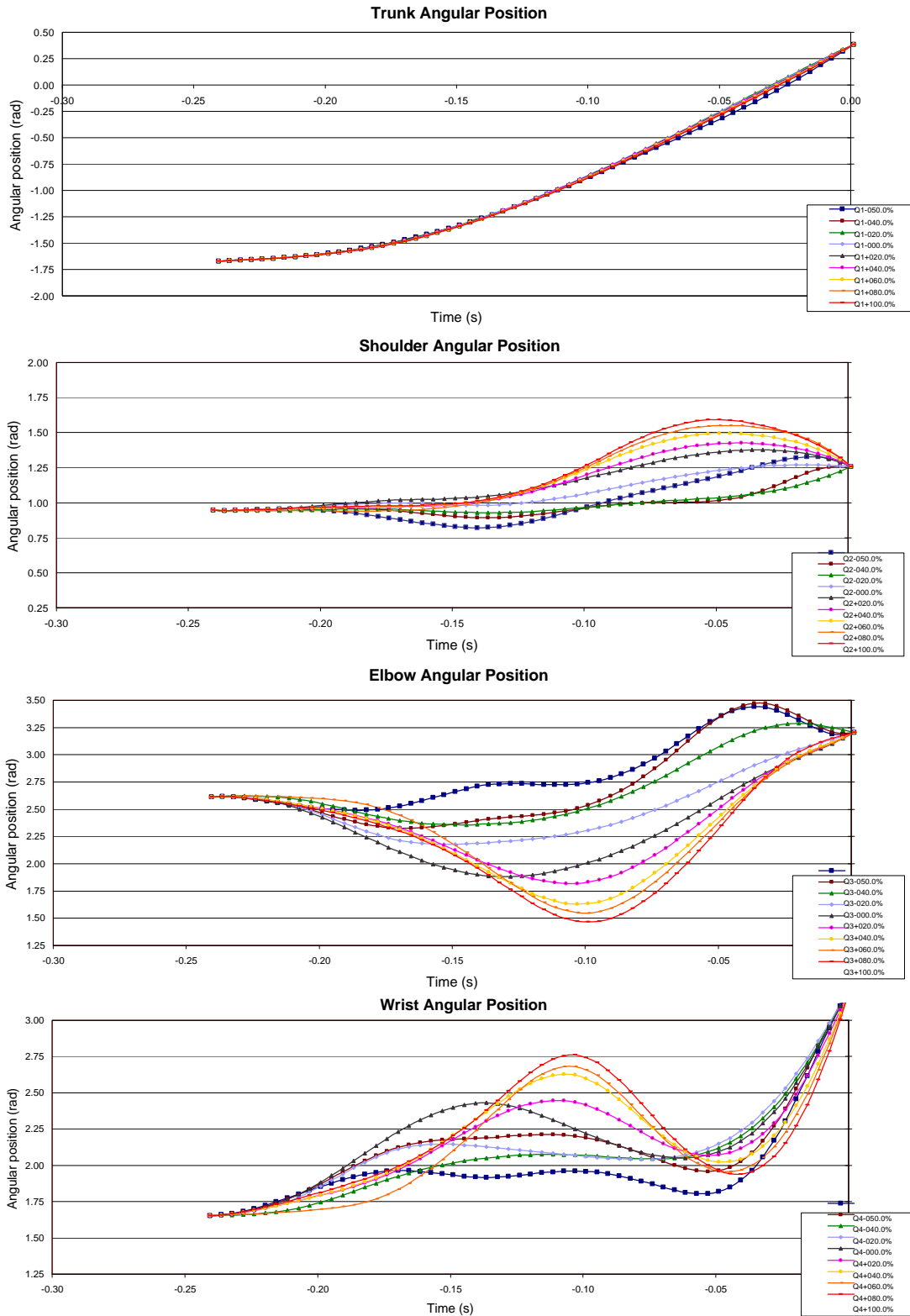


Figure 4.32 Angular position data of trunk, shoulder, elbow, and wrist joints over varied wrist strength boundaries.

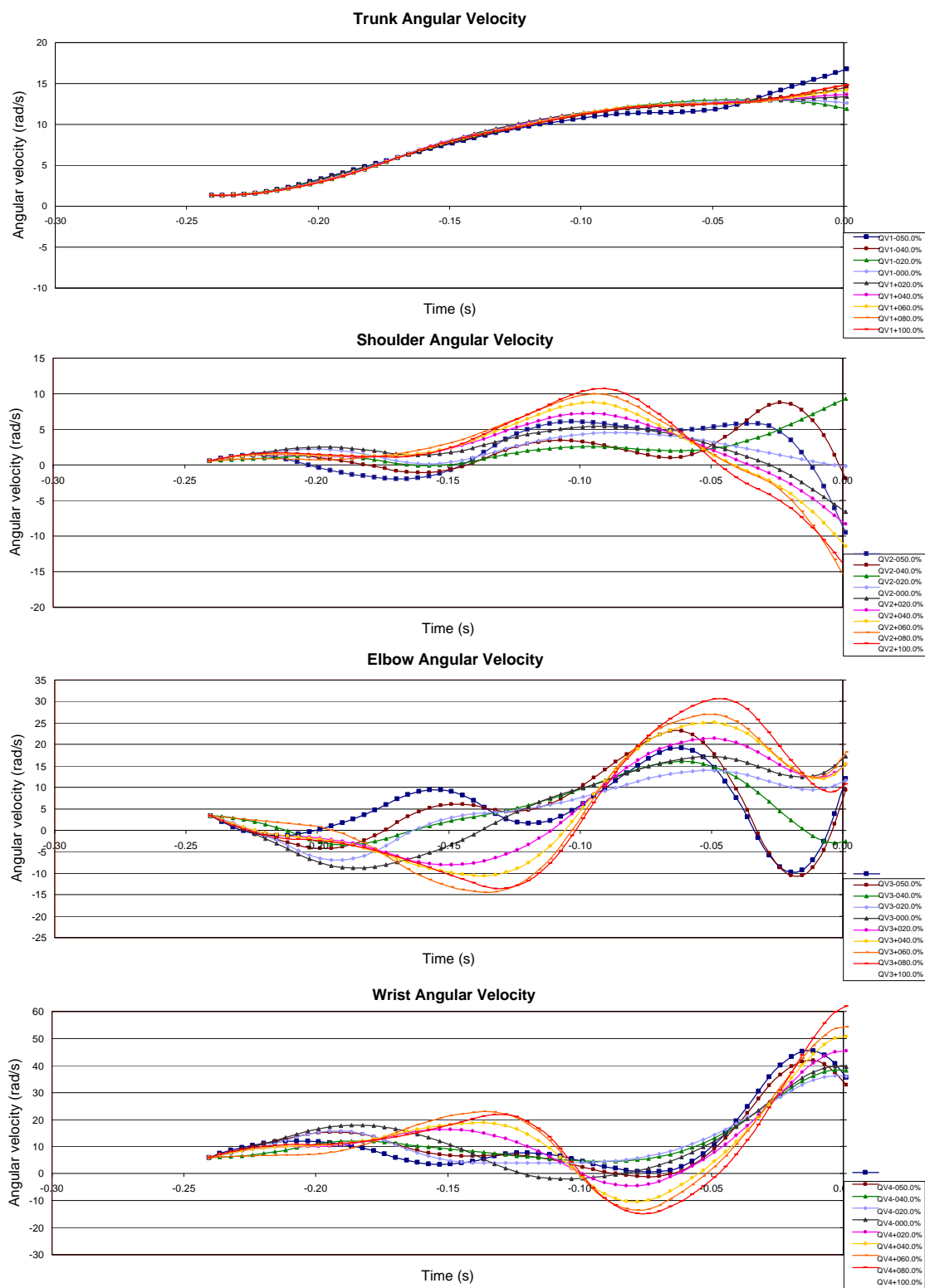


Figure 4.33 Angular velocity of trunk, shoulder, elbow, and wrist joints over varied wrist strength boundaries.

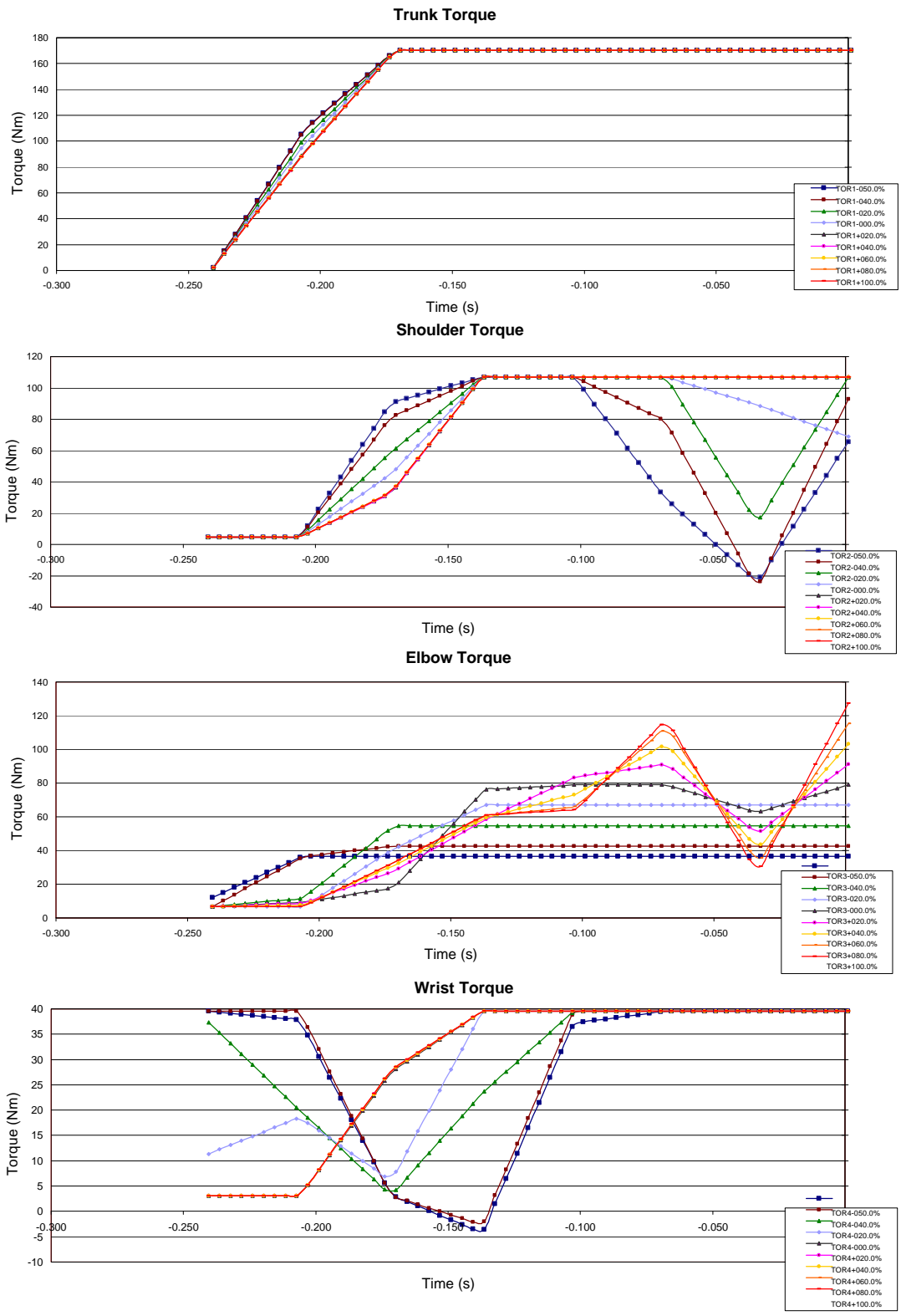


Figure 4.34 Torques of trunk, shoulder, elbow, and wrist joints over varied elbow strength boundaries.

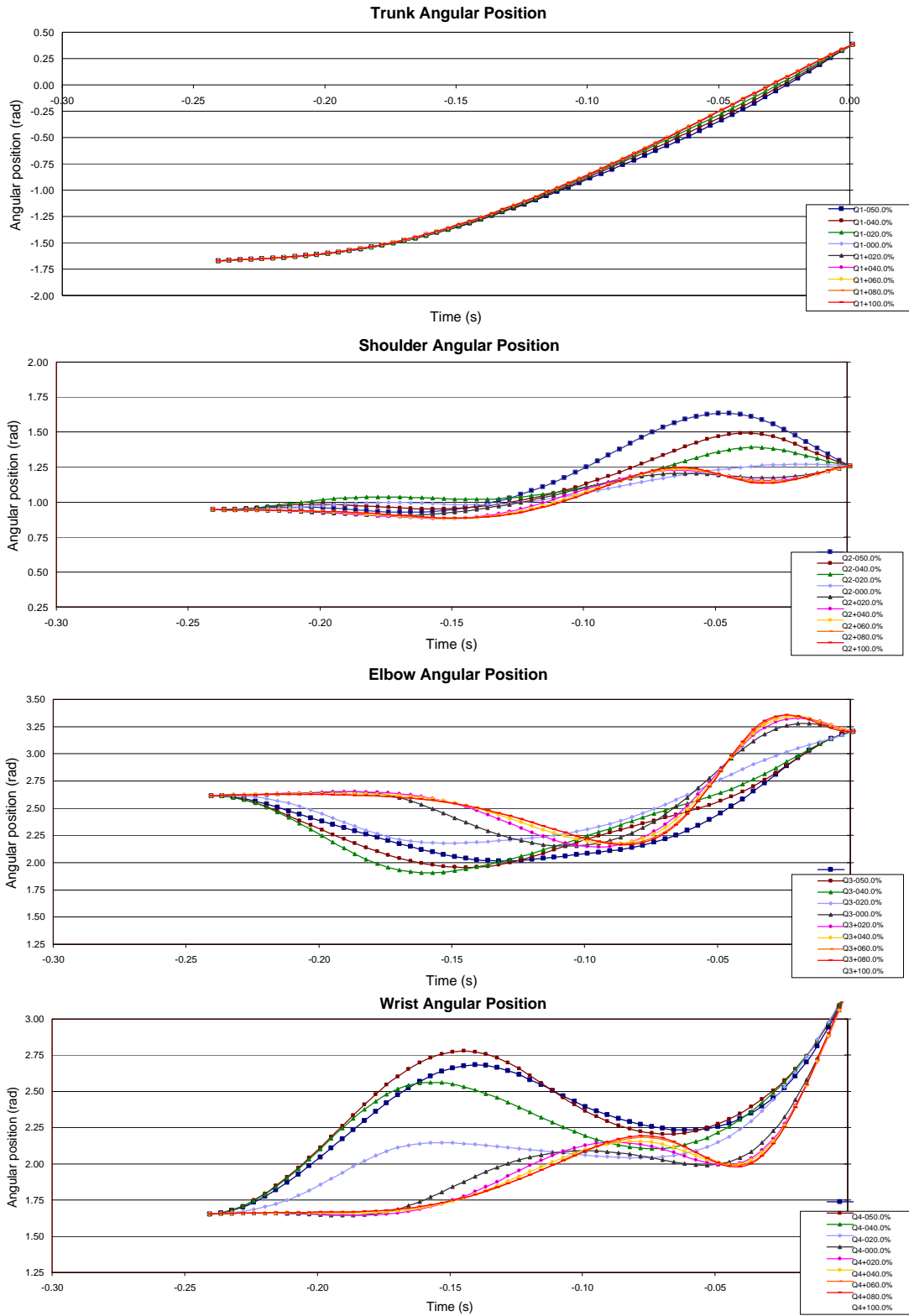


Figure 4.35 Angular position data of trunk, shoulder, elbow, and wrist joints over varied elbow strength boundaries.

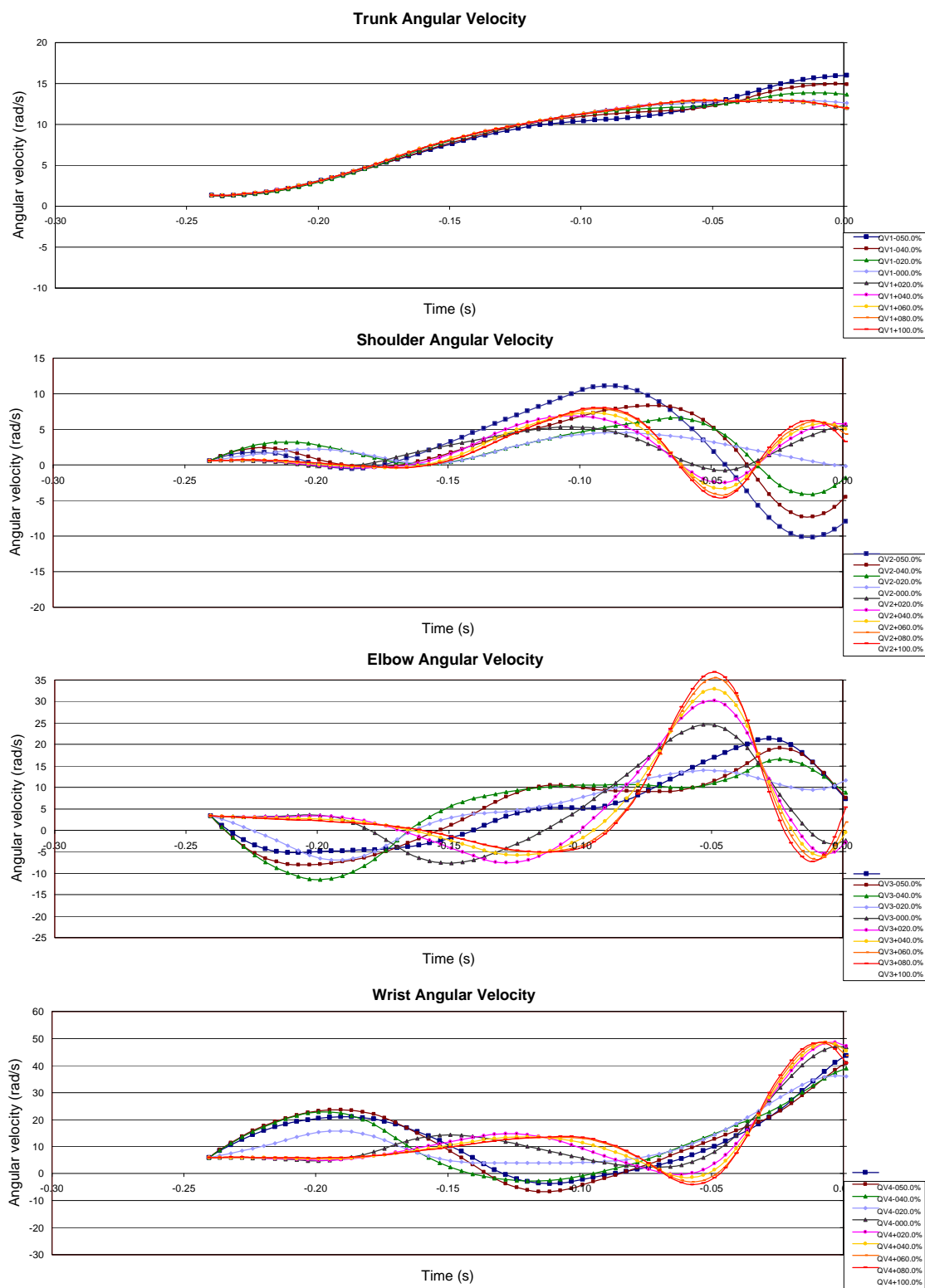


Figure 4.36 Angular velocity of trunk, shoulder, elbow, and wrist joints over varied elbow strength boundaries.

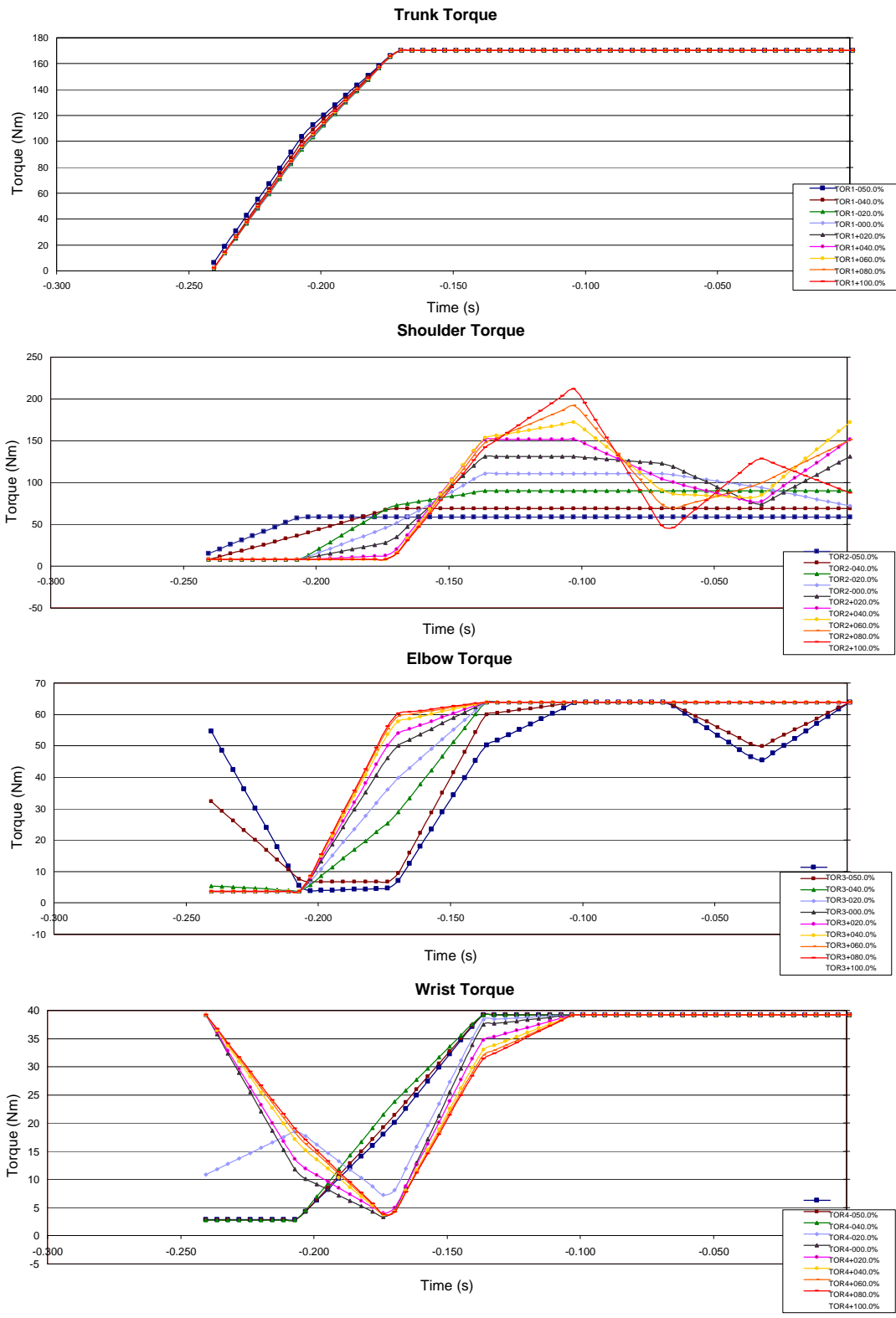


Figure 4.37 Torques of trunk, shoulder, elbow, and wrist joints over varied shoulder strength boundaries.



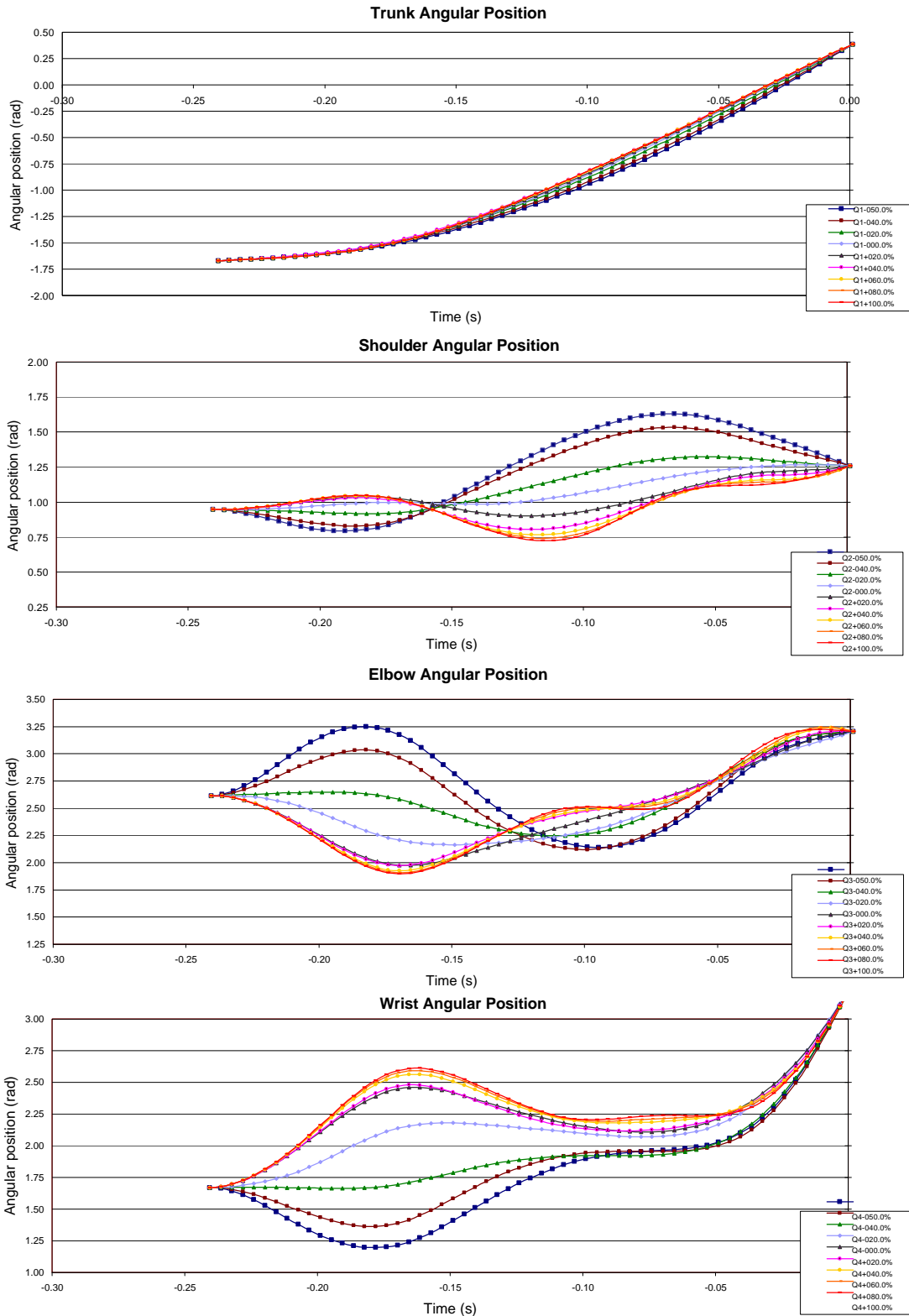


Figure 4.38 Angular position data of trunk, shoulder, elbow, and wrist joints over varied shoulder strength boundaries.

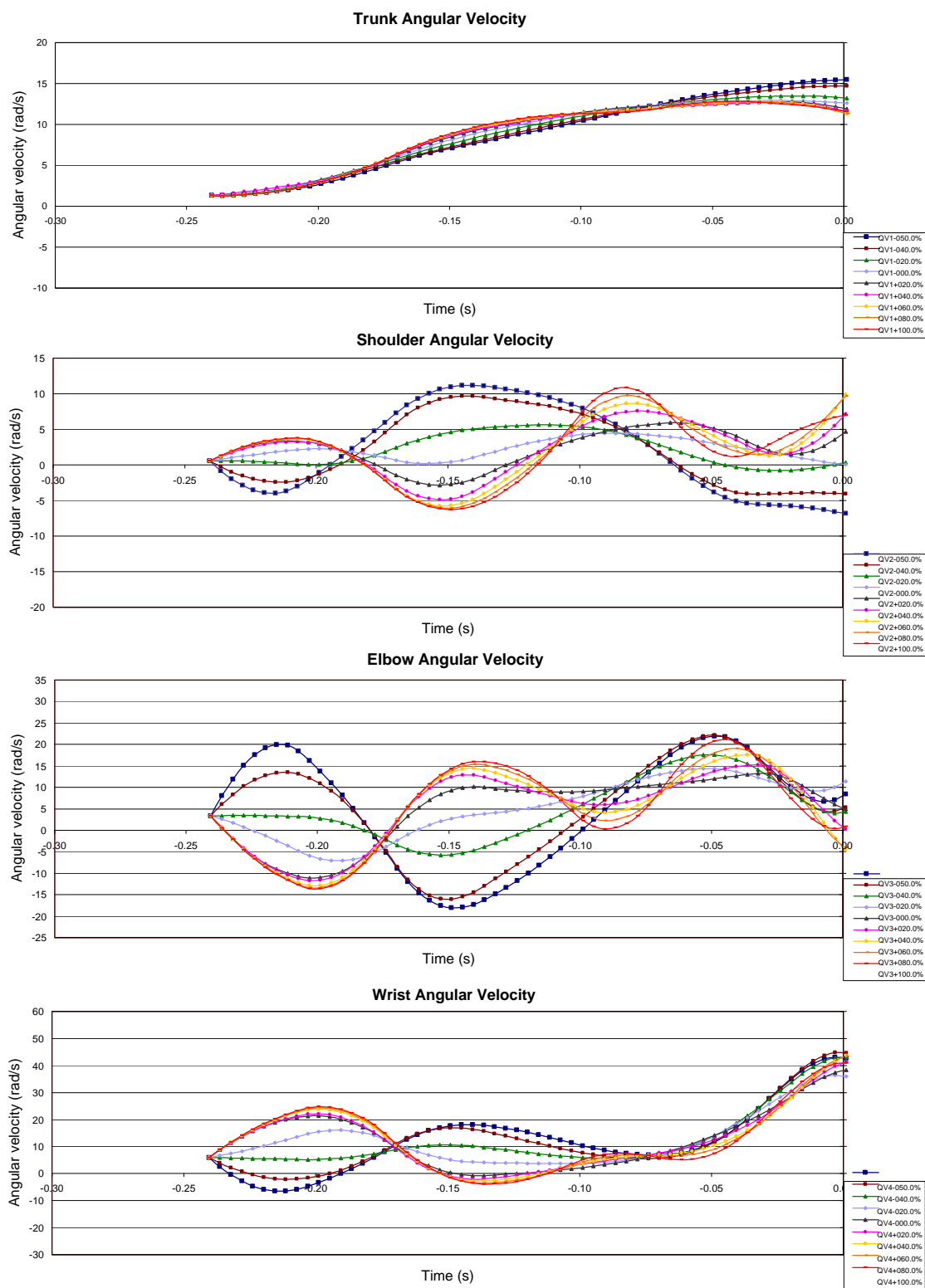


Figure 4.39 Angular velocity of trunk, shoulder, elbow, and wrist joints over varied shoulder strength boundaries.

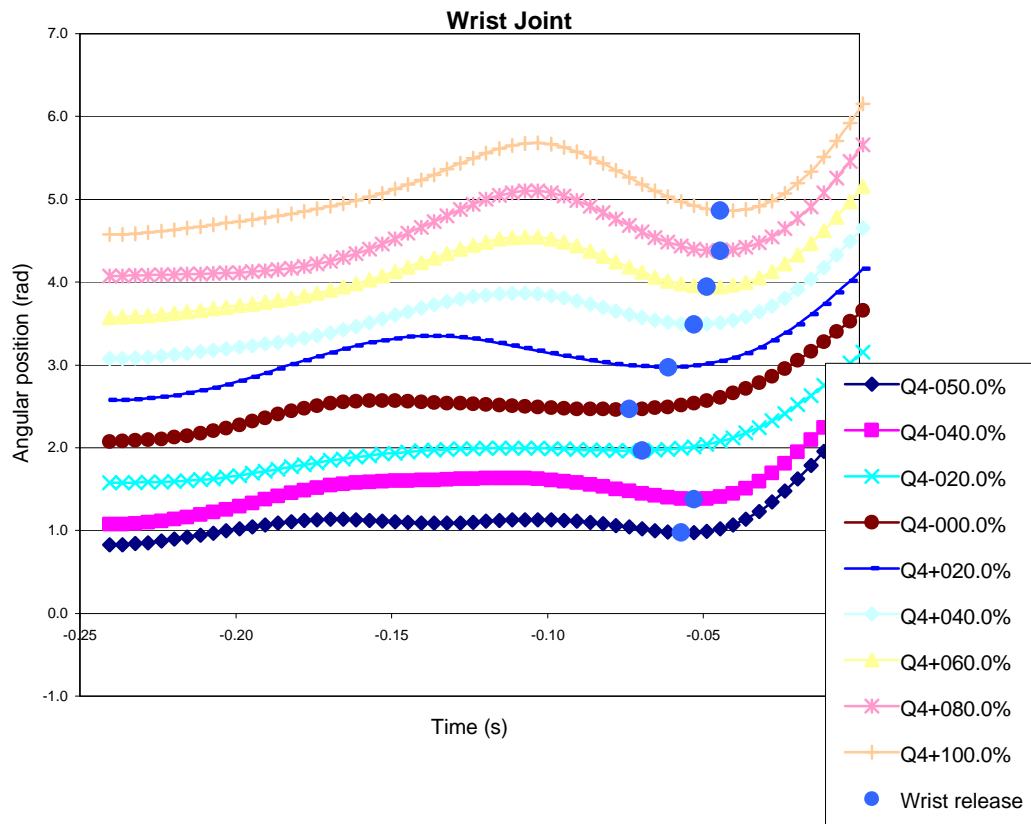


Figure 4.40 Wrist angular displacement and release timing for varied wrist strength boundaries.

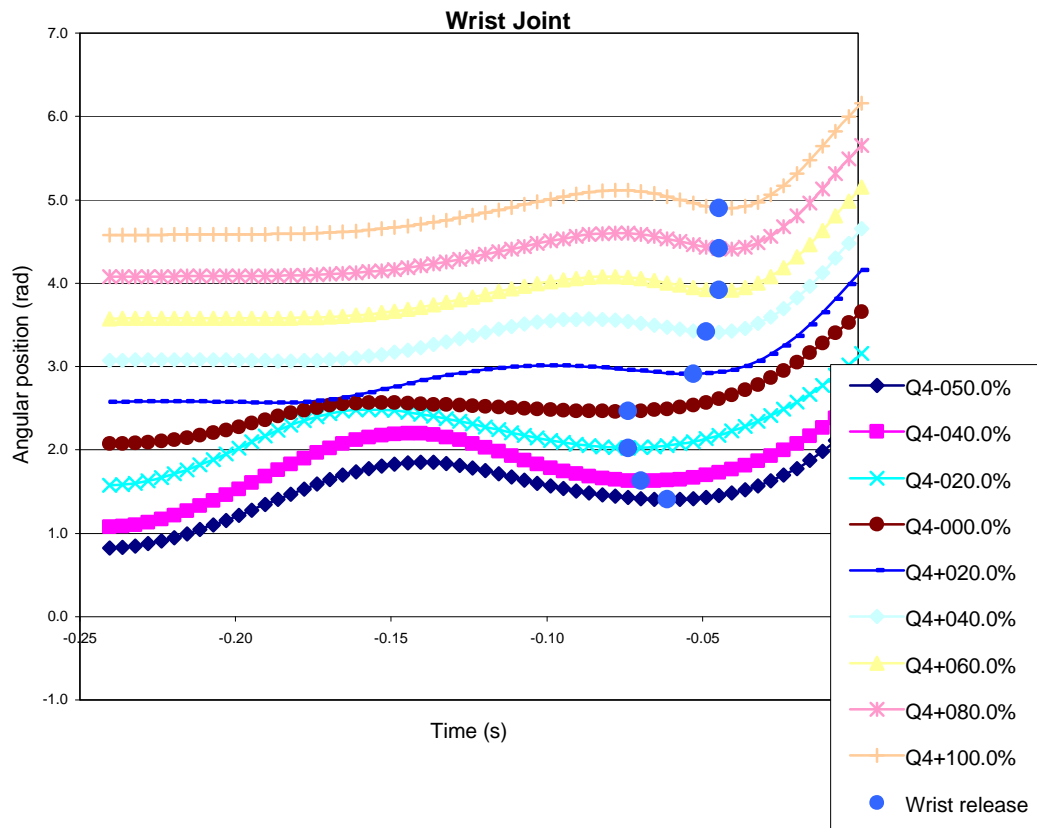


Figure 4.41 Wrist angular displacement and release timing for varied elbow strength boundaries.

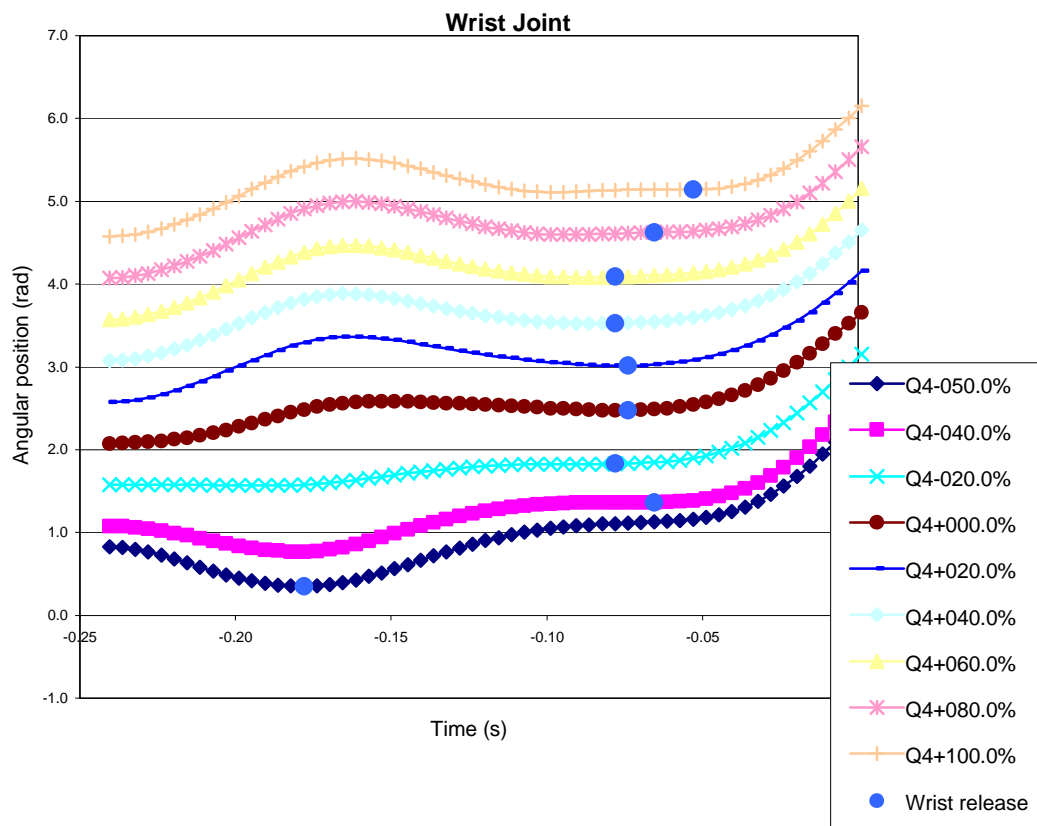


Figure 4.42 Wrist angular displacement and release timing for varied shoulder strength boundaries.

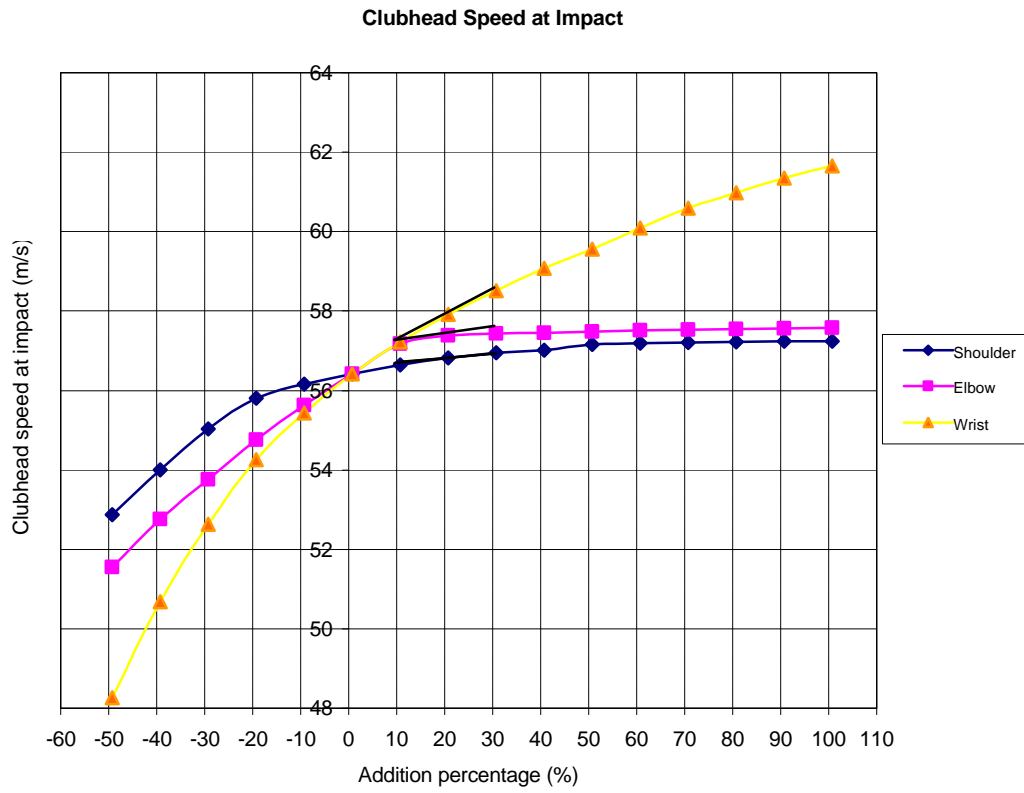


Figure 4.43 Clubhead speed at impact according to varied strength boundaries in wrist, elbow, and shoulder.

## **APPENDICES**

## Appendix A: Consent Form

The University of Texas at Austin  
**Participant Consent Form**

3D Kinematic and Kinetic Analyses of Golf Swing

Name: \_\_\_\_\_

Address: \_\_\_\_\_  
\_\_\_\_\_

Telephone: \_\_\_\_\_

You are invited to participate in a research study of optimal golf swing patterns. My name is Wen-Tzu Tang and I am a graduate student in Kinesiology at The University of Texas at Austin. This study will be used towards my Ph.D. dissertation research project under the supervision of Professor Lawrence Abraham, Ed.D.

**Wen-Tzu Tang, MS**  
546 D Belmont Hall,  
The University of Texas at  
Austin, Austin, TX  
(512) 232-2683

**Lawrence D. Abraham, Ed.D.**  
Chair, Dept. of Curriculum & Instruction  
Associate Dean, College of Education  
The University of Texas at Austin  
Austin, TX 78712 (512)471-3476

The purpose of this project is to examine dynamic patterns of movement used by skilled golfers. You were selected as a possible participant in the study because you are over 18 years of age and have performed well as an elite golfer. If you participate, you will be one of approximately 10-15 people in the study.

If you decide to participate, you agree to cooperate in the data collection session as described. Sixteen to twenty reflective markers will be attached with adhesive tape to the skin at your wrists, elbows, shoulders, trunk, hips, knees, and ankles. Next, you will stand on a mat and hit the ball. The ball might be attached to a string so the ball does not go too far or might be caught by a net. You will be instructed to swing as normally as possible but in some cases with these conditions: (1) with and (2) without bending your lead elbow, (3) limited weight shifting, (4) limited trunk rotation. You are allowed as many practice swings and practice hits as necessary to gain a comfortable feeling with the testing environment. Then, you will be instructed to swing the different test clubs



### 3 D Kinematic and Kinetic Analyses of Golf Swing

(chosen from 4 iron to 9 iron, and driver) and your motion will be recorded by a video system with high-speed cameras. The entire data collection procedure will require one session lasting approximately one to two hours.

Signing the consent form indicates an understanding of associated risks and possible side effects. These include the possibility of some discomfort or minor muscle soreness but no more than typical for a regular practice session.

No treatment will be provided for research related injury and no payment can be provided in the event of a medical problem. If you are a University of Texas student, you may be treated at the usual level of care with the usual cost for services at the Student Health Center, but no payment can be provided in the event of a medical problem.

Your decision to participate or to decide not to participate will not affect your present or future relationship with The University of Texas at Austin or Barton Creek Country Club.

If you have any questions about the study, please ask me. If you have any questions later, call me, Wen-Tzu Tang, at (512)232-2683 or call my supervisor, Professor Lawrence D. Abraham at (512)471-3476.

You will be given a copy of this consent form for your records.

You are making a decision whether or not to participate. Your signature below indicates that you have completely read the information provided above and have decided to participate in the study. If you later decide that you do not want to participate in the study, simply tell me. You may discontinue your participation in this study at any time.

---

Printed Name of Participant

---

Signature of Participant

---

Date

---

Signature of Investigator

---

Date

We may wish to present some of the tapes we will make in this study to scientific conventions or as demonstrations in classrooms. Please sign below if you are willing to allow us to do so with the tape of your performance.

\_\_\_\_\_ “I hereby give permission for the video tape made for this research study to be also used for educational purposes.”

## **Appendix B: USGA Handicap System**

(Abstracted from USGA Handicap System Manual)

A Handicap Index is the USGA's mark which is used to indicate a measurement of a player's potential scoring ability on a course of standard difficulty. Potential scoring ability is measured by a player's best scores, and is expressed as a number taken to one decimal place. These scores are identified by calculating the handicap differential for each score. The USGA Handicap Index is calculated by taking 96 percent of the average of the best handicap differentials, and applying the handicap differential usage (Table B.1) for golfers with two or more eligible tournament scores.

### **B.1 Determine Handicap Differentials**

A handicap differential is computed from four elements: adjusted gross score, USGA Course Rating, USGA Slope Rating and 113 (the Slope Rating of a course of standard difficulty). The formula is as follows:

$$\text{Handicap Differential} = \frac{(\text{Adjusted Gross Score} - \text{USGA Course Rating}) \times 113}{\text{USGA Slope Rating}}$$

Where:

Course rating is on the score card of almost every course. This number will always be close to par, it represents what a scratch player should

theoretically score on the course. For an example, an easier course might have a low rating like "68.9", versus a very difficult course might be rated "75.9".

Course slope is a number used in junction with the course rating to calculate your handicap index. This number will typically range from approximately 110 for a relatively forgiving course to 150 for a very difficult course.

## **B.2. USGA Handicap Index Formula**

The USGA Handicap Index Formula is based on the best handicap differentials in a player's scoring record. If a player's scoring record contains 20 or more scores, then the best 10 handicap differentials of the most recent 20 scores are used to calculate his USGA Handicap Index. The percentage of scores used in a scoring record decreases from the maximum of the best 50 percent as the number of scores in the scoring record decreases. If the scoring record contains 9 or 10 scores, then only the best three scores (30 to 33 percent) in the scoring record will be used. Thus, the accuracy of a player's Handicap Index is directly proportional to the number of acceptable scores he has posted. A USGA Handicap Index shall not be issued to a player who has returned fewer than five acceptable scores. The following procedure illustrates how authorized golf associations and golf clubs calculate a player's Handicap Index if the number of acceptable scores in the player's record is fewer than 20.

The procedure for calculating Handicap Indexes is as follows:

- (a) Use table B.1 to determine the number of handicap differentials to use;
- (b) Determine handicap differentials;
- (c) Average the handicap differentials being used;
- (d) Multiply the average by .96;
- (e) Delete all numbers after the tenths digit. Do not round off to the nearest tenth.

Table B.1 Handicap differential Usage

Number of Acceptable Scores	Differentials To Be Used
5 or 6	Lowest 1
7 or 8	Lowest 2
9 or 10	Lowest 3
11 or 12	Lowest 4
13 or 14	Lowest 5
15 or 16	Lowest 6
17	Lowest 7
18	Lowest 8
19	Lowest 9
20	Lowest 10

## **Appendix C: Marker Placement**

The reflective markers are spheres and are attached to the test participants with adhesive tapes. For the present study, sixteen markers were used to record the downswing motion of the body and three markers were used to record the motion of the club. The sixteen body markers were attached to the points listed below:

1. Left wrist (distal process of the ulna, left arm)
2. Right wrist (distal process of the ulna, right arm)
3. Left elbow (left lateral epicondyle of the humerus)
4. Right elbow (right lateral epicondyle of the humerus)
5. Left shoulder (superior aspect of the deltoid, left side)
6. Right shoulder (superior aspect of the deltoid, right side)
7. Upper back (second thoracic vertebra, T2)
8. Lower back ( fifth lumbar vertebra, L5)
9. Left hip (left iliac crest)
10. Right hip (right iliac crest)
11. Left knee (left lateral condyle of the femur)
12. Right knee (right lateral condyle of the femur)
13. Left ankle (left lateral malleolus of the fibula)

14. Right ankle (right lateral malleolus of the fibula)

15. Left foot (left distal phalanges)

16. Right foot (right distal phalanges)

## Appendix D: Parameters for Optimal Model

Description	Value	
Swing Plane angle ( $^{\circ}$ )		35.8
Mass Initial at Center of Mass ( $kg \cdot m^2$ )		
Upper Torso	IA1	1.1109
Upperarm	IB1	0.0687
Forearm	IC1	0.0330
Hand and club	ID1	0.1502
Segment length ( $m$ )		
Upper Torso	LA	0.4230
Upper Arm:	LB	0.2970
Forearm	LC	0.2711
Club	LD	0.9510
Segment Mass ( $kg$ )		
Upper Torso	MA	15.9875
Upper Arm	MB	2.5851
Forearm	MC	1.4874
Club and Hand:	MD	0.5170
Center of Gravity from proximal ( $m$ )		
Upper Torso:	RA	0.2115
Upper Arm:	RB	0.1390
Forearm:	RC	0.1269
Club	RD	0.3192
Joint angle ( $rad$ )		
Initial Value:	Q1	-1.687
Initial Value:	Q2	0.901 (0.698 for 3-segment)
Initial Value:	Q3	2.515 (1.425 for 3-segment)
Initial Value:	Q4	1.548
Joint angular velocity ( $rad/s$ )		
Initial Value:	U1	1.079
Initial Value:	U2	-0.320
Initial Value:	U3	0.882
Initial Value:	U4	0.318
Time ( $s$ )		
Initial Time:	TINITIAL	0
Final Time:	TFINAL	0.2416667
Integration Step:	INTEGSTP	0.00416667
Absolute Error:	ABSERR	1.0E-05
Relative Error:	RELERR	1.0E-10



Note:

ABSERR should be set equal to  $10^{(-8)} * X_{\text{small}}$ , where  $X_{\text{small}}$  is the estimated smallest maximum absolute value of the variables being integrated.

RELERR should be set equal to  $10^{(-d)}$ , where  $d$  is the desired number of significant digits of numerical integration results.

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