EXAMPLE ANSWERS

CHAPTER 3

EXAMPLE 3.1

$$m_{thigh} = P_{thigh}\, m_{total} = 0.100 \times 180.0 = 18.00 \text{ kg}$$

EXAMPLE 3.2

$$x_{cg} = -12.80 + 0.433\ (7.3 - (-12.80)) = -4.10 \text{ cm}$$

$$y_{cg} = 83.3 + 0.433\ (46.8 - 83.3) = 67.5 \text{ cm}$$

Notice that the coordinates of the thigh's center of gravity (–4.10, 67.5) must fall between the two endpoints. Carefully preserve the signs of the coordinates during the computations. These coordinates were taken from frame 10 of table 1.3.

EXAMPLE 3.3

$$l_{thigh} = \sqrt{(7.3 - (-12.80))^2 + (46.8 - 83.3)^2} = 41.67\text{cm}$$

$$k_{cg} = K_{cg(thigh)} \times l_{thigh} = 0.323 \times 41.67$$
$$= 13.46 \text{ cm} = 0.1346 \text{ m}$$

In example 3.1, the thigh mass was calculated to be 18.00 kg.

$$I_{cg} = mk^2 = 18.00 \times 0.1346^2 = 0.326 \text{ kg·m}^2$$

Notice that the radius of gyration *(k_{cg})* was calculated before the moment of inertia could be computed. This in turn required calculating the thigh segment's length *(l_{thigh})*. Also notice that the units of the radius of gyration were converted to meters before squaring.

To compute the moment of inertia about the proximal end of this thigh, we must apply the parallel axis theorem after first computing the distance from the thigh center to the proximal end. This distance is called $r_{proximal}$.

$$r_{proximal} = 0.433 \times l_{thigh} = 0.433 \times 41.67$$
$$= 18.04 \text{ cm} = 0.1804 \text{ m}$$

$$I_{proximal} = I_{cg} + m_{thigh}\, r_{proximal}^{\ 2}$$
$$= 0.326 + 18.00 \times 0.1804^2 = 0.912 \text{ kg·m}^2$$

Notice that the moment of inertia about the proximal end is larger than that about the center of gravity. The moment of inertia is always smallest about an axis through the center of gravity.

CHAPTER 4

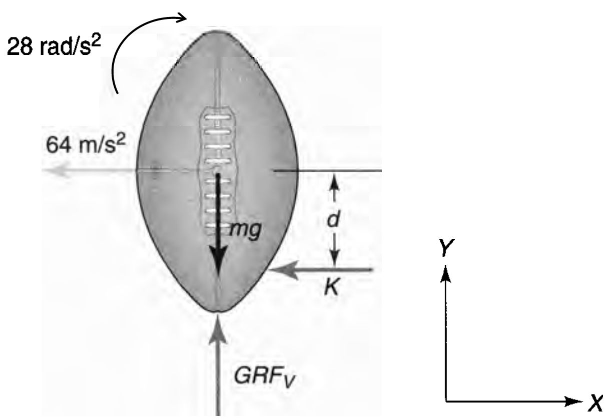
EXAMPLE 4.1

The issue with drawing the wind is that we are unable to locate its center of pressure. This is the same problem with FBDs of swimmers and cyclists.

CHAPTER 5

EXAMPLE 5.1a

The FBD for this example is almost the same as figure 5.10d, except that in the FBD for this example we included the forces of gravity and the vertical GRF. Note that X- and Y-axes have been drawn to indicate positive direction. For the sake of example, note that the horizontal acceleration (–64 m/s²) is drawn with an arrow in the negative horizontal direction (to the left) with a positive 64 m/s². Similarly note that the angular acceleration is drawn in a negative (clockwise) direction with a positive 28 rad/s² value. In these two cases, it is also acceptable to draw them pointing in the opposite directions with negative values.



As indicated by the grey arrows, the unknowns in this diagram are the vertical GRF, the force, *K*, and the distance, *d*, between *K*'s line of action and the center of the ball. We start by solving for the forces in the vertical direction because they are the simplest in this case.

$$\Sigma F_y = ma_y:$$

$$GRF_y - mg = ma_y$$

$$\square\ GRF_y - (0.25\ \text{kg})(9.81\ \text{m/s}^2) = 0$$

$$\square\ GRF_y = 2.45\ \text{N}$$

This is the expected result when the ground is simply supporting the weight of the ball.

When solving for the horizontal force K, note that the FBD is used to determine that K should have a minus sign because it points to the left (−x). That we calculated a positive K means that the force acts in the direction drawn in the FBD.

$$\Sigma F_x = ma_x:$$

$$-K = ma_x$$

$$\square -K = (0.25\ \text{kg})(-64\ \text{m/s}^2)$$

$$\square\ K = 16.0\ \text{N}$$

As is standard in human-movement problems, we calculate moments about the ball's mass center. The FBD shows that there is only one force, K, that does not act through the mass center. Its moment equals the product Kd. We write this term down and then put a negative sign in front of it because this moment causes a *clockwise* (negative) effect. This somewhat tricky part requires visualization. Imagine the mass center as a fixed point and observe that the force K turns about the mass center in a clockwise fashion.

$$\Sigma M = I\alpha:$$

$$-K(d) = I\alpha$$

$$\Rightarrow d = \frac{(0.04\ \text{kg}\cdot\text{m}^2)(-28\ \text{rad/s}^2)}{16\ \text{N}} = -0.070\ \text{m}$$

The distance, d, to the force is negative, indicating that the ball was kicked below the mass center.

EXAMPLE 5.1b

The FBD is almost the same as in the previous example, except for the addition of the horizontal tee force. Note that the tee force has a positive sign:

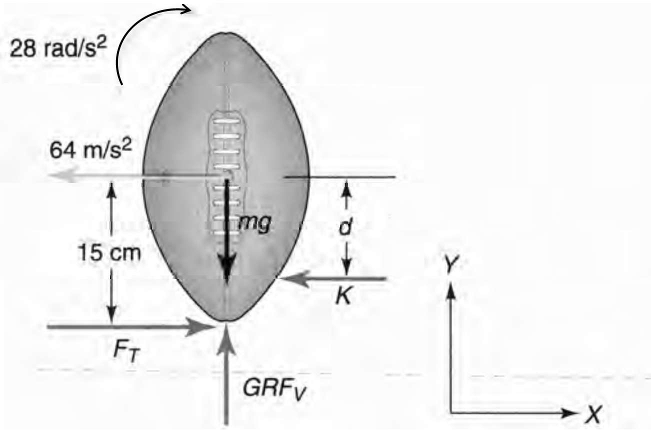
$$\Sigma F_x = ma_x:$$

$$-K + F_T = ma_x$$

$$-K + 4\ \text{N} = (0.25\ \text{kg})(-64\ \text{m/s}^2) = -16\ \text{N}$$

$$\square\ K = 20.0\ \text{N}$$

This is the logical result: The kicking force was the sum of the tee force and the ball's reaction.



In summing moments about the mass center, note that the moment of the force K is negative, as before, but that the moment of the force F_T is positive:

$$\Sigma M = I\alpha:$$

$$-K(d) + F_T(0.15\ \text{m}) = I\alpha$$

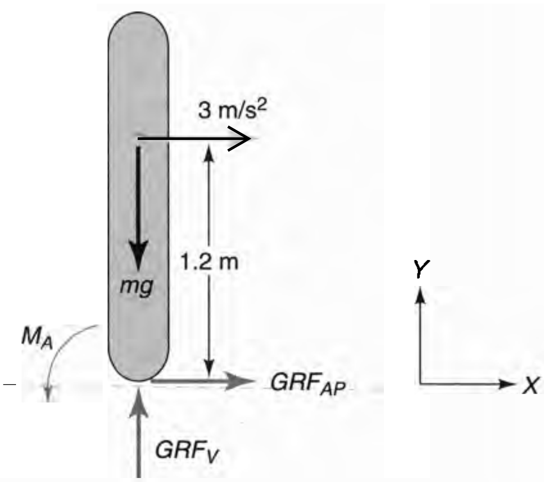
$$\Rightarrow d = \frac{(0.04\ \text{kg}\cdot\text{m}^2)(-28\ \text{rad/s}^2) - 4(0.15\ \text{m})}{16\ \text{N}}$$

$$= -0.1075\ \text{m}$$

Thus, the force was lower on the ball in this instance.

EXAMPLE 5.2

This is an apparently complex situation resolvable with an FBD; in fact, it is very similar to our football example. To draw this correctly, it helps to first identify the forces involved: body weight (gravity), the unknown GRFs, and the reaction of a body mass being accelerated. There is also an unknown ankle moment, M_A. In the FBD, they look like this:



Because the commuter maintains a still posture, her body's angular acceleration is 0 rad/s. The unknown reactions GRF_x, GRF_y, and M_A are at the floor; they are drawn in positive coordinate system directions and determined with our three equations of motion:

$$\Sigma F_x = ma_x:$$
$$GRF_x = ma_x$$
$$\square\ GRF_x = (60\ \text{kg})(3\ \text{m/s}^2) = 180.0\ \text{N}$$
$$\Sigma F_y = ma_y:$$
$$GRF_y - mg = ma_y$$
$$\square\ GRF_y - (60\ \text{kg})(9.81\ \text{m/s}^2) = 0$$
$$\square\ GRF_y = 589\ \text{N}$$

Both of these reactions are positive, indicating that they act in the directions drawn.

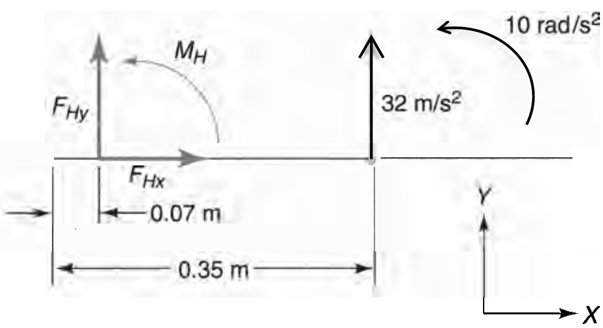
We will again sum moments about the mass center. Note that we have drawn the unknown moment M_A in a counterclockwise direction, so it is positive in the first equation below. The moment of the vertical GRF_y is zero and the moment of the horizontal GRF_x is positive because it points counterclockwise about the mass center.

$$\Sigma M = I\alpha:$$
$$M_A + GRF_x(1.2\ \text{m}) = I\alpha$$
$$\square\ M_A + (180\ \text{N})(1.2\ \text{m}) = (130\ \text{kg}\cdot\text{m}^2)(0\ \text{rad/s}^2)$$
$$\square\ M_A = -216\ \text{N}\cdot\text{m}$$

This is a large ankle moment, which is why sudden subway starts usually cause people to either take a step or grab a handle.

EXAMPLE 5.3

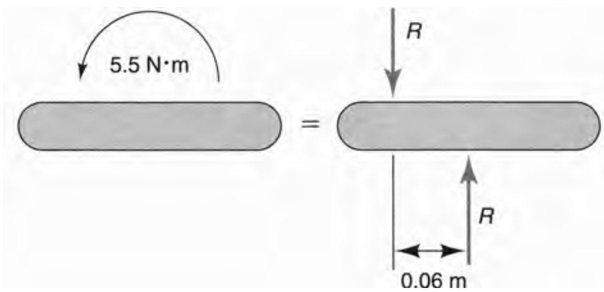
The FBD of the racket is quite simple. The only tricky part is that the hand has an unknown action. Therefore, we must draw unknown actions in each coordinate system direction:



Solving our three equations of motion:

$$\Sigma F_x = ma_x:$$
$$F_{Hx} = (0.5\ \text{kg})(0\ \text{m/s}^2)$$
$$\square\ F_{Hx} = 0\ \text{N}.$$
$$\Sigma F_y = ma_y:$$
$$F_{Hy} = (0.5\ \text{kg})(32\ \text{m/s}^2) = 16\ \text{N}.$$
$$\Sigma M = I\alpha:$$
$$M_H - F_{Hy}(0.35\ \text{m} - 0.07\ \text{m}) = (0.1\ \text{kg}\cdot\text{m}^2)(10\ \text{rad/s}^2)$$
$$\square\ M_H = (16\ \text{N})(0.35\ \text{m} - 0.07\ \text{m}) + (0.1\ \text{kg}\cdot\text{m}^2)(10\ \text{rad/s}^2) = 5.5\ \text{N}\cdot\text{m}$$

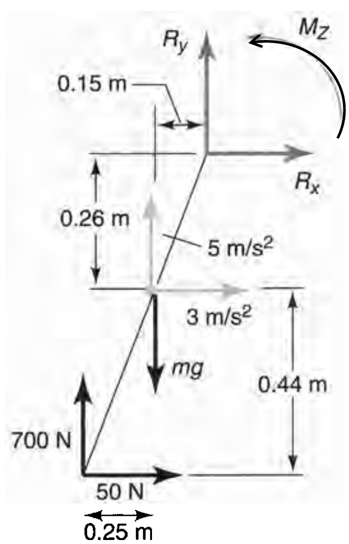
A moment is the result of a force couple, that is, two equal, opposite, noncollinear forces. We can assume that the hand area by the forefinger is pushing the racket handle forward and the area of the little finger is pulling it backward:



If these points are about 6 cm apart, we can estimate that each member of the force couple is about 91.7 N.

EXAMPLE 5.4

We have measured the endpoints of the object with our camera and determined where the object's mass center is located. We start by redrawing our FBD with the distances in the X and Y directions between the forces and the mass center, about which we are going to calculate the moments of force:



We then apply the three equations of 2-D motion:

$$\Sigma F_x = ma_x:$$
$$R_x + 50\ \text{N} = (8\ \text{kg})(3\ \text{m/s}^2)$$
$$\square\ R_x = -26.0\ \text{N}$$

Note that R_x is negative. This means that the force points in the direction opposite to that shown in the FBD.

$$\Sigma F_y = ma_y:$$

$$R_y + 700\ \text{N} - mg = (8\ \text{kg})(5\ \text{m/s}^2)$$

$$\square\ R_y = -700\ \text{N} + (8\ \text{kg})(9.81\ \text{m/s}^2) + (8\ \text{kg})(5\ \text{m/s}^2) = -581.5\ \text{N}$$

As with R_x, our calculated value for R_y is negative, indicating that it points downward, the opposite of what we drew in the FBD.

In writing our moment equation, we first write down all the terms and then decide if each term is positive or negative according to the right-hand rule:

$$\Sigma M_z = I\alpha:$$

$$M_z + (50\ \text{N})(0.44\ \text{m}) - (700\ \text{N})(0.25\ \text{m}) - R_x(0.26\ \text{m}) + R_y(0.15\ \text{m})$$
$$= (0.2\ \text{kg·m}^2)(10\ \text{rad/s}^2)$$

We then substitute the numerical values for R_x and R_y that we calculated previously. Note that we put these in parentheses with their negative signs to preserve the signs that we had determined for the signs of the moments:

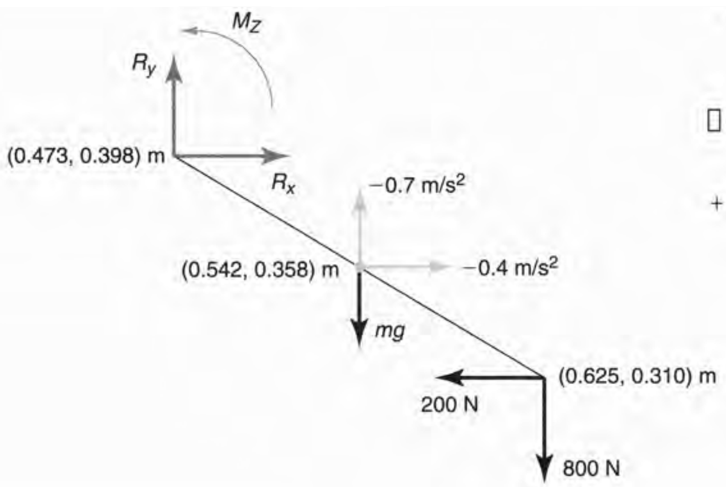
$$\square\ M_z + (50\ \text{N})(0.44\ \text{m}) - (700\ \text{N})(0.25\ \text{m}) - (-26\ \text{N})(0.26\ \text{m}) + (-582\ \text{N})(0.15\ \text{m})$$
$$= (0.2\ \text{kg·m}^2)(10\ \text{rad/s}^2)$$

We then bring all terms but our unknown moment M_z to the right-hand side of the equation and solve:

$$\square\ M_z = -(50\ \text{N})(0.44\ \text{m}) + (700\ \text{N})(0.25\ \text{m}) + (-26\ \text{N})(0.26\ \text{m})$$
$$- (-582\ \text{N})(0.15\ \text{m}) + (0.2\ \text{kg·m}^2)(10\ \text{rad/s}^2) = 235\ \text{N·m}$$

EXAMPLE 5.5

The FBD is:



$$\Sigma F_x = ma_x:$$

$$R_x - 200\ \text{N} = (0.1\ \text{kg})(-0.4\ \text{m/s}^2)$$

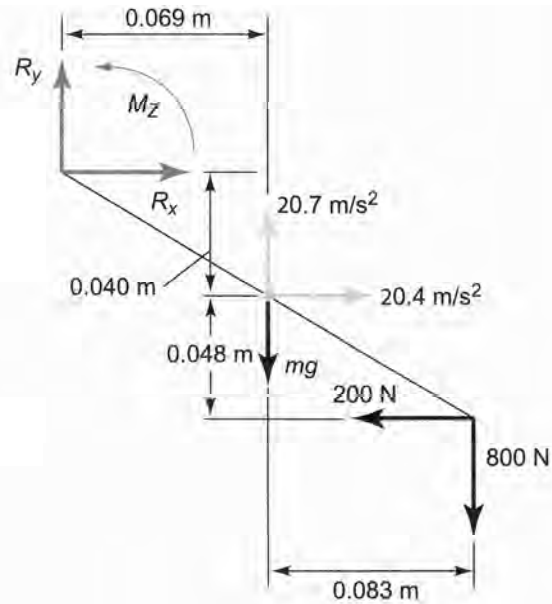
$$\square\ R_x = 200\ \text{N}$$

The exact value is 199.96 N, but it rounds to 200 N. The crank is so light that its mass-acceleration product is negligible. The same thing happens in the y-direction:

$$\Sigma F_y = ma_y:$$

$$R_y - 800\ \text{N} - (0.1\ \text{kg})(9.81\ \text{m/s}^2) = (0.1\ \text{kg})(-0.7\ \text{m/s}^2)$$
$$\square\ R_y = 801\ \text{N}$$

Again, it is helpful when calculating moments to redraw the FBD with distances between points:



$$\Sigma M = I\alpha:$$

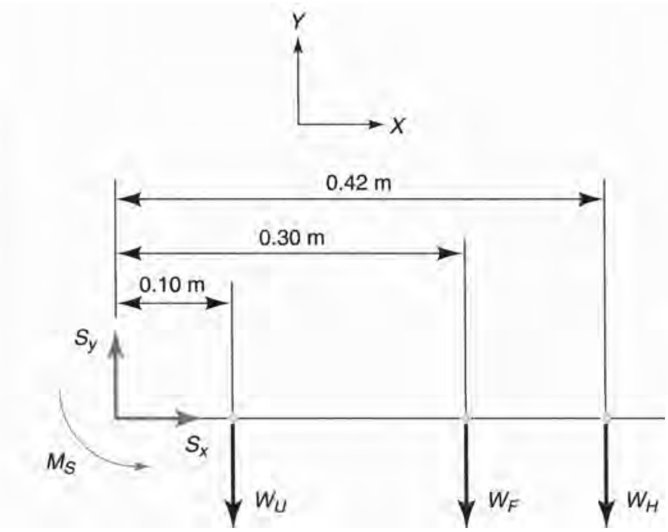
$$M_z - R_x(0.040\ \text{m}) - R_y(0.069\ \text{m}) - (200\ \text{N})(0.048\ \text{m}) - (800\ \text{N})(0.083\ \text{m})$$
$$= (0.003\ \text{kg·m}^2)(10\ \text{rad/s}^2)$$

$$\square\ M_z = (200\ \text{N})(0.040\ \text{m}) + (801\ \text{N})(0.069\ \text{m}) + (200\ \text{N})(0.048\ \text{m})$$
$$+ (800\ \text{N})(0.083\ \text{m}) + (0.1\ \text{kg·m}^2)(10\ \text{rad/s}^2)$$

$$\square\ M_z = 139.3\ \text{N·m}$$

EXAMPLE 5.6

In this example we need only to section the arm at the shoulder and analyze that one piece. Because this is a static case, the *ma* and *Iα* terms are zero. To construct the FDB, we section the arm and draw the three unknowns at the shoulder joint. We also draw the weight of the upper arm, forearm, and hand:



To solve for the three unknowns:

$$\Sigma F_x = ma_x:$$

$$S_x = 0 \text{ N}$$

$$\Sigma F_y = ma_y:$$

$$S_y - W_U - W_F - W_H = 0$$

$$\square\ S_y = W_U + W_F + W_H$$

$$\square\ S_y = (4 \text{ kg})(9.81 \text{ m/s}^2) + (3 \text{ kg})(9.81 \text{ m/s}^2) + (1 \text{ kg})(9.81 \text{ m/s}^2) = 78.5 \text{ N}$$

In this example, it is convenient to calculate moments about the shoulder. Once again, the procedure is first to write down the products of forces and distances and then establish the sign of each moment.

$$\Sigma M_z = 0:$$

$$M_S - W_U(0.10 \text{ m}) - W_F(0.30 \text{ m}) - W_H(0.42 \text{ m}) = 0$$

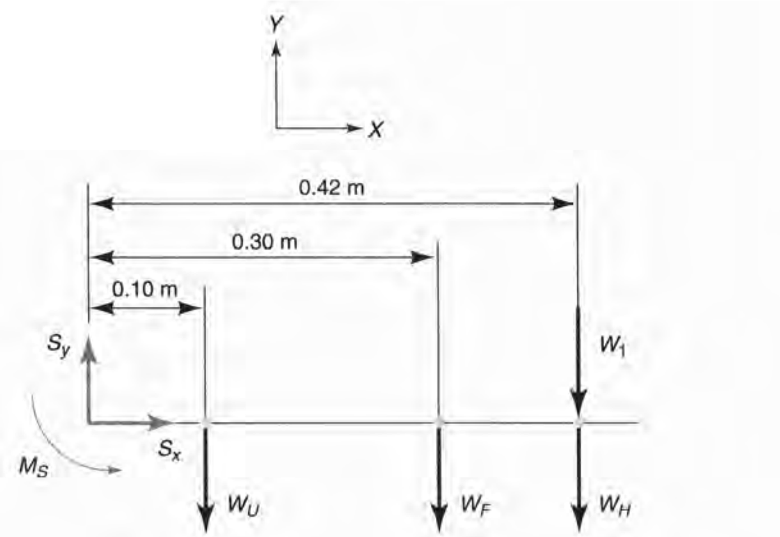
$$\square\ M_S = W_U(0.10 \text{ m}) + W_F(0.30 \text{ m}) + W_H(0.42 \text{ m})$$

$$\square\ M_S = (4 \text{ kg})(9.81 \text{ m/s}^2)(0.10 \text{ m}) + (3 \text{ kg})(9.81 \text{ m/s}^2)(0.30 \text{ m})$$

$$+ (1 \text{ kg})(9.81 \text{ m/s}^2)(0.42 \text{ m}) = 16.9 \text{ N·m}$$

EXAMPLE 5.7

The FBD is almost identical to the previous example, except for the added weight W_1:



Solving for the three unknowns:

$$\Sigma F_x = ma_x:$$

$$S_x = 0 \text{ N}$$

$$\Sigma F_y = ma_y:$$

$$S_y - W_U - W_F - W_H - W_1 = 0$$

$$\square\ S_y = W_U + W_F + W_H + W_1$$

$$\square\ S_y = (4 \text{ kg})(9.81 \text{ m/s}^2) + (3 \text{ kg})(9.81 \text{ m/s}^2) + (1 \text{ kg})(9.81 \text{ m/s}^2)$$

$$+ (2 \text{ kg})(9.81 \text{ m/s}^2) = 98.1 \text{ N}$$

This is the expected change (compared to example 5.6) of about 20 N.

Once again, to sum the moments, we first write down each moment, then establish its sign:

$$\Sigma M = 0:$$

$$M_S - W_U(0.10 \text{ m}) - W_F(0.30 \text{ m}) - (W_H + W_1)(0.42 \text{ m}) = 0$$

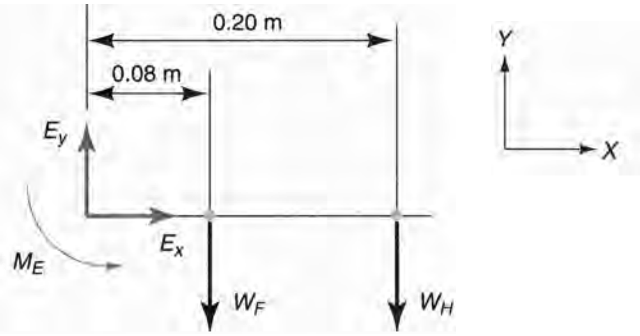
$$\square\ M_S = W_U(0.10 \text{ m}) + W_F(0.30 \text{ m}) + (W_H + W_1)(0.42 \text{ m})$$

$$\square\ M_S = (4 \text{ kg})(9.81 \text{ m/s}^2)(0.10 \text{ m}) + 3 \text{ kg } (9.81 \text{ m/s}^2)(0.30 \text{ m})$$

$$+ (1 \text{ kg} + 2 \text{ kg})(9.81 \text{ m/s}^2)(0.42 \text{ m}) = 25.1 \text{ N·m}$$

EXAMPLE 5.8

The FBD of the forearm is again similar to the previous example. We draw the new unknowns at the elbow joint in a positive sense and calculate the distances from the forces to the elbow:



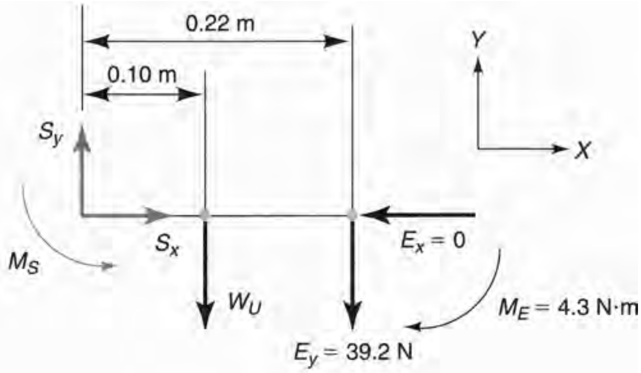
Solving for the three unknowns:

$$\Sigma F_x = ma_x:$$
$$E_x = 0\ \text{N}$$
$$\Sigma F_y = ma_y:$$
$$E_y - W_F - W_H = 0$$
$$\square\ E_y = W_F + W_H$$
$$\square\ E_y = (3\ \text{kg})(9.81\ \text{m/s}^2) + (1\ \text{kg})(9.81\ \text{m/s}^2) = 39.2\ \text{N}$$

It is convenient to calculate moments about the elbow:

$$\Sigma M = 0:$$
$$M_E - W_F(0.08\ \text{m}) - W_H(0.20\ \text{m}) = 0$$
$$\square\ M_E = W_F(0.08\ \text{m}) + W_H(0.20\ \text{m})$$
$$\square\ M_E = (3\ \text{kg})(9.81\ \text{m/s}^2)(0.08\ \text{m}) + (1\ \text{kg})(9.81\ \text{m/s}^2)(0.20\ \text{m}) = 4.3\ \text{N·m}$$

To solve for the upper arm, the method of sections requires that we draw the upper arm with forces and moments at the elbow having directions equal but opposite to those on the forearm:



$$\Sigma F_x = ma:$$
$$S_x - E_x = 0$$
$$\square\ S_x = 0\ \text{N}$$
$$\Sigma F_Y = ma:$$
$$S_y - W_U - E_y = 0$$
$$\square\ S_Y = W_U + E_y$$
$$\square\ S_Y = (4\ \text{kg})(9.81\ \text{m/s}^2) + 39.2\ \text{N} = 78.4\ \text{N}$$

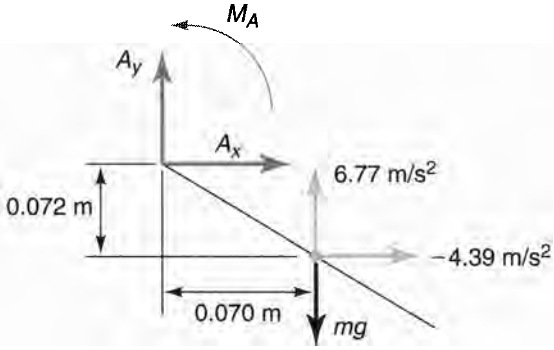
This is the same value we had calculated earlier. Summing the moments about the shoulder:

$$\Sigma M = 0:$$
$$M_S - M_E - W_U(0.10\ \text{m}) - E_y(0.22\ \text{m}) = 0$$
$$\square\ M_S = M_E + W_U(0.10\ \text{m}) + E_y(0.22\ \text{m})$$
$$\square\ M_S = 4.32\ \text{N·m} + (4\ \text{kg})(9.81\ \text{m/s}^2)(0.10\ \text{m}) + (39.2\ \text{N})(0.22\ \text{m}) = 16.9\ \text{N·m}$$

This is also the same value that we had calculated earlier.

EXAMPLE 5.9

The FBD of the foot:



Solving for the reaction forces:

$$\Sigma F_x = ma_x:$$
$$A_x = m_f a_f$$
$$\square\ A_x = (1.2\ \text{kg})(-4.39\ \text{m/s}^2) = -5.27\ \text{N}$$
$$\Sigma F_y = ma_y:$$
$$A_y - m_f g = m_f a_f$$
$$\square\ A_y = m_f g + m_f a_f$$
$$\square\ A_y = (1.2\ \text{kg})(9.81\ \text{m/s}^2) + (1.2\ \text{kg})(6.77\ \text{m/s}^2) = 19.9\ \text{N}$$

Because the foot mass is small, the reactions are small.

Solving for the ankle joint moment, we sum moments about the mass center. There are three moments; of these, the ankle moment M_A is positive because that is how it was drawn. The moments of the reaction forces are both negative because they turn clockwise about the mass center:

$$\Sigma M = I\alpha:$$

$$M_A - A_x(0.072 \text{ m}) - A_y(0.070 \text{ m}) = I_f\alpha_f$$

$$\square\ M_A = A_x(0.072 \text{ m}) + A_y(0.070 \text{ m}) + I_f\alpha_f$$

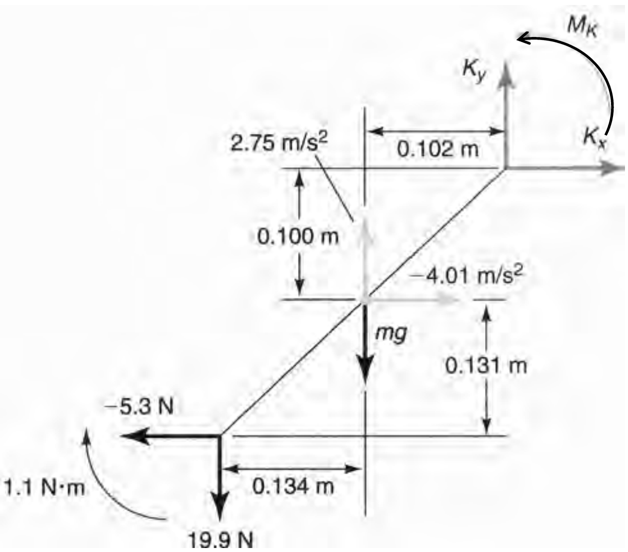
We now substitute the numerical values of the reaction forces A_x and A_y in parentheses so that we preserve their signs:

$$\square\ M_A = (-5.27 \text{ N})(0.072 \text{ m}) + (19.9 \text{ N})(0.070 \text{ m}) + (0.011 \text{ kg·m}^2)(5.12 \text{ rad/s}^2)$$

$$\square\ M_A = 1.1 \text{ N·m}$$

This is a very small joint moment. Although it is essentially zero, because it is positive it is a dorsiflexor moment, assuming the person is facing the right.

The FBD of the leg is as follows. Note that we placed the numerical values for the ankle reactions into this diagram and retained their original signs.



$$\Sigma F_x = ma_x:$$

$$K_x - A_x = m_l a_l$$

$$\square\ K_x = A_x + m_l a_l$$

$$\square\ K_x = -5.3 \text{ N} + (2.4 \text{ kg})(-4.01 \text{ m/s}^2) = -14.9 \text{ N}$$

$$\Sigma F_y = ma_y:$$

$$K_y - A_y - m_l g = m_l a_l$$

$$\square\ K_y = A_y + m_l g + m_l a_l$$

$$\square\ K_y = 19.9 \text{ N} + (2.4 \text{ kg})(9.81 \text{ m/s}^2) + (2.4 \text{ kg})(2.75 \text{ m/s}^2) = 50.0 \text{ N}$$

When summing leg moments, note that the moments of the vertical (y) forces are positive, whereas the moments of the horizontal (x) forces are negative.

$$\Sigma M = I\alpha:$$

$$M_K - M_A - K_x(0.100 \text{ m}) + K_y(0.102 \text{ m}) - A_x(0.131 \text{ m}) + A_y(0.134 \text{ m}) = I_l\alpha_l$$

$$\square\ M_K = M_A + K_x(0.100 \text{ m}) - K_y(0.102 \text{ m}) + A_x(0.131 \text{ m}) - A_y(0.134 \text{ m}) + I_l\alpha_l$$

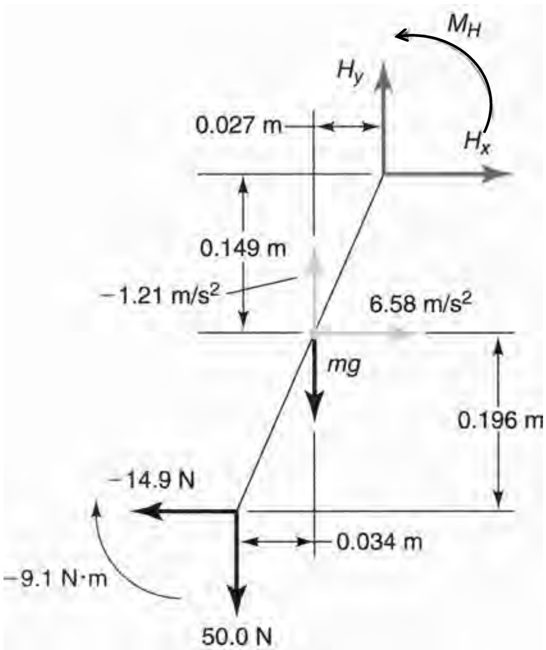
Note how we again substitute for the reaction forces the numerical values in parentheses to preserve their signs:

$$\square\ M_K = 1.1 \text{ N·m} + (-14.9 \text{ N})(0.100 \text{ m}) - (50.0 \text{ N})(0.102 \text{ m}) + (-5.3 \text{ N})(0.131 \text{ m}) - (19.90 \text{ N})(0.134 \text{ m}) + (0.064 \text{ kg·m}^2)(-3.08 \text{ rad/s}^2)$$

$$\square\ M_K = -9.1 \text{ N·m}$$

This is a small knee flexor moment, assuming the person is facing to the right.

This is the FBD of the thigh. Again note that we placed the numerical values for the knee reactions into this diagram with the same signs that they were calculated as having.



$$\Sigma F_x = ma_x:$$

$$H_x - K_x = m_t a_t$$

□ $H_x = K_x + m_t a_t$

□ $H_x = -14.9\text{ N} + (6.0\text{ kg})(6.58\text{ m/s}^2) = 24.6\text{ N}$

$$\Sigma F_y = ma_y:$$

$$H_y - K_y - m_t g = m_t a_t$$

□ $H_y = K_y + m_t g + m_t a_t$

□ $H_y = 50.0\text{ N} + (6.0\text{ kg})(9.81\text{ m/s}^2) + (6.0\text{ kg})(-1.21\text{ m/s}^2) = 101.6\text{ N}$

$$\Sigma M = I\alpha:$$

$$M_H - M_K - H_x(0.149\text{ m}) + H_y(0.027\text{ m}) - K_x(0.196\text{ m}) + K_y(0.034\text{ m}) = I_t(\alpha_t)$$

□ $M_H = M_K + H_x(0.149\text{ m}) - H_y(0.027\text{ m}) + K_x(0.196\text{ m}) - K_y(0.034\text{ m}) + I_t(\alpha_t)$

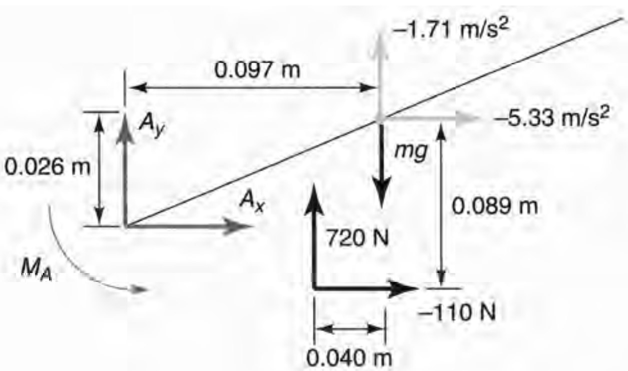
□ $M_H = -9.1\text{ N·m} + (24.6\text{ N})(0.149\text{ m}) - (101.6\text{ N})(0.027\text{ m}) + (-14.9\text{ N})(0.196\text{ m}) - (50.0\text{ N})(0.034\text{ m}) + (0.130\text{ kg·m}^2)(8.62\text{ rad/s}^2)$

□ $M_H = -11.7\text{ N·m}$

Because this is a negative result, it is a hip extensor moment, assuming the person is facing to the right.

EXAMPLE 5.10

The free-body diagram of the foot is the same as for the swing phase except for the ground reaction forces. With these we must be careful to place them in the proper location. It is also helpful to draw them in the positive direction and put their values on, positive or negative.



Solving for the reaction forces:

$$\Sigma F_x = ma_x:$$

$$A_x + GRF_x = m_f a_f$$

□ $A_x = -GRF_x + m_f a_f$

□ $A_x = -(-110\text{ N}) + 1.2\text{ kg}(-5.33\text{ m/s}^2)$

□ $A_x = 103.6\text{ N}$

$$\Sigma F_y = ma_y:$$

$$A_y + GRF_y - m_f g = m_f a_f$$

□ $A_y = -GRF_y + m_f g + m_f a_f$

□ $A_y = -(720.0\text{ N}) + 1.2\text{ kg}(9.81\text{ m/s}^2) + 1.2\text{ kg}(-1.71\text{ m/s}^2)$

□ $A_y = -710.3\text{ N}$

Because the foot mass is small, the ankle joint reaction forces are nearly equal and opposite to the ground reaction forces. Note that the vertical reaction A_y is negative; this means that the actual force is pushing down on the ankle joint, which is a logical result given that this joint is bearing body weight.

Solving for the ankle joint moment, we sum moments about the mass center. There are five moments; of these the ankle moment M_A is positive because of how it is drawn. The moment of the horizontal joint reaction force is positive because it runs counterclockwise around the mass center; however, the moment of vertical joint reaction force is negative because it turns clockwise about the mass center. Similarly, the moment of the horizontal ground reaction force is positive, and the moment of the vertical ground reaction force is negative. Note that the center-of-pressure location is critical in establishing the moment of the vertical GRF; also note that the moment arm of the moment of the horizontal GRF is the vertical position of the mass center because that force is located on the ground (i.e., at y = 0.0 m):

$$\Sigma M = I\alpha:$$

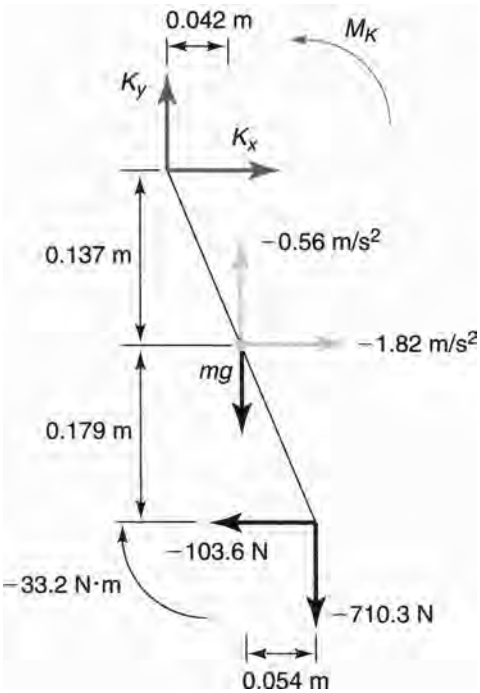
$$M_A - A_x(0.026\text{ m}) - A_y(0.097\text{ m}) + GRF_x(0.089\text{ m}) - GRF_y(0.040\text{ m}) = I_f\alpha_f$$

□ $M_A = A_x(0.026\text{ m}) + A_y(0.097\text{ m}) - GRF_x(0.089\text{ m}) + GRF_y(0.040\text{ m}) + I_f\alpha_f$

□ $M_A = (103.6\text{ N})(0.026\text{ m}) + (-710.3\text{ N})(0.097\text{ m}) - (-110\text{ N})(0.089\text{ m}) + (720\text{ N})(0.040\text{ m}) + (0.011\text{ kg·m}^2)(-20.2\text{ rad/s}^2)$

□ $M_A = -33.2\text{ N·m}$

This is a plantarflexor action. The free-body diagram of the leg is shown here.



$\Sigma F_x = ma_x$:

$K_x - A_x = m_l\, a_l$

□ $K_x = A_x + m_l\, a_l$

□ $K_x = 103.6\text{ N} + 2.4\text{ kg }(-1.82\text{ m/s}^2)$

□ $K_x = 99.2\text{ N}$

$\Sigma F_y = ma_y$:

$K_y - A_y - m_l\, g = m_l\, a_l$

□ $K_y = A_y + m_l\, g + m_l\, a_l$

□ $K_y = -710.3\text{ N} + 2.4\text{ kg }(9.81\text{ m/s}^2) + 2.4\text{ kg }(-0.56\text{ m/s}^2)$

□ $K_y = -688.1\text{ N}$

When summing leg moments, note that the moments of both the vertical and horizontal forces are negative.

$\Sigma M = I\alpha$:

$M_K - M_A - K_x\,(0.137\text{ m}) - K_y\,(0.042\text{ m}) - A_x\,(0.179\text{ m}) - A_y\,(0.054\text{ m}) = I_l\,\alpha_l$

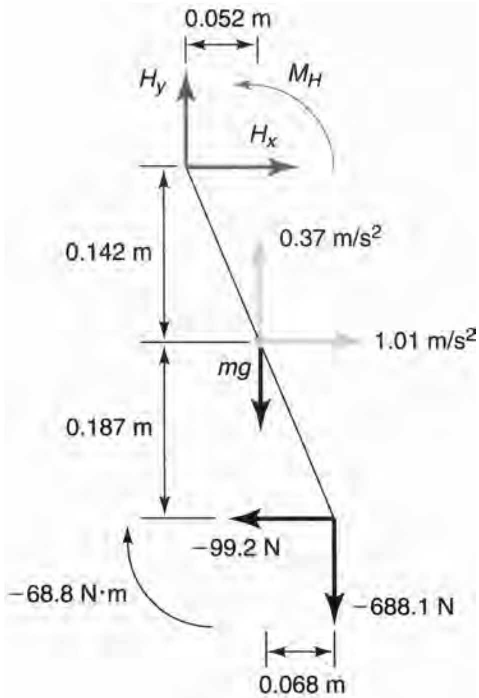
□ $M_K = M_A + K_x\,(0.137\text{ m}) + K_y\,(0.042\text{ m}) + A_x\,(0.179\text{ m}) + A_y\,(0.054\text{ m}) + I_l\,\alpha_l$

□ $M_K = -33.2\text{ N·m} + (99.2\text{ N})(0.137\text{ m}) + (-688.1\text{ N})(0.042\text{ m}) + (103.6\text{ N})(0.179\text{ m}) + (-710.3\text{ N})(0.054\text{ m}) + (0.064\text{ kg·m}^2)(-22.4\text{ rad/s}^2)$

□ $M_K = -69.8\text{ N·m}$

This is a knee flexor moment.

The free-body diagram of the thigh is as follows. Again note that we have placed the numerical values for the knee reactions into this diagram with the same signs that they were calculated as having:



$\Sigma F_x = ma_x$:

$H_x - K_x = m_l\, a_l$

□ $H_x = K_x + m_l\, a_l$

□ $H_x = 99.2\text{ N} + 6.0\text{ kg }(1.01\text{ m/s}^2)$

□ $H_x = 105.3\text{ N}$

$\Sigma F_y = ma_y$:

$H_y - K_y - m_l\, g = m_l\, a_l$

□ $H_y = K_y + m_l\, g + m_l\, a_l$

□ $H_y = -688.1\text{ N} + 6.0\text{ kg }(9.81\text{ m/s}^2) + 6.0\text{ kg }(0.37\text{ m/s}^2)$

□ $H_y = -627.0\text{ N}$

As when we summed the leg moments, note that the moments of both the vertical and horizontal forces are negative:

$\Sigma M = I\alpha$:

$M_H - M_K - H_x\,(0.142\text{ m}) - H_y\,(0.052\text{ m}) - K_x\,(0.187\text{ m}) - K_y\,(0.068\text{ m}) = I_t\,\alpha_t$

□ $M_H = M_K + H_x\,(0.142\text{ m}) + H_y\,(0.052\text{ m}) + K_x\,(0.187\text{ m})$

$$+ K_y\,(0.068\text{ m}) + I_t\,\alpha_t$$

$$\square\ M_H = -69.8\text{ N·m} + (105.3\text{ N})(0.142\text{ m}) + (-627.0\text{ N})(0.052\text{ m})$$

$$+ (99.2\text{ N})(0.187\text{ m}) + (-688.1\text{ N})(0.068\text{ m}) + (0.130\text{ kg·m}^2)(8.6\text{ rad/s}^2)$$

$$\square\ M_H = -114.6\text{ N·m}$$

This is a hip extensor moment. Note how all three of these lower-extremity joint moments are much larger than their swing-phase counterparts.

CHAPTER 6

EXAMPLE 6.1

First, convert the workload to newtons:

$$2.5 \times 9.81 = 24.53\text{ N}$$

Second, calculate the number of revolutions of the crank:

$$R = 60 \times 20 = 1200$$

Finally, calculate the work:

$$W = 24.53 \times 1200 \times 6 = 176\,616\text{ J} = 176.6\text{ kJ}$$

Notice, that the final answer was converted to kilojoules (kJ).

EXAMPLE 6.2

$$W = \Delta E = E_{final} - E_{initial}$$

Assuming that the only change in energy results from a change in speed:

$$\text{work} = 1/2\ m\,v^2 - 0 = 1/2 \times 180 \times 6^2 = 3240\text{ J}$$

$$\text{power} = \text{work/duration} = 3260/4 = 810\text{ W}$$

EXAMPLE 6.3

First, compute the potential energy:

$$E_{gpe} = 18.0 \times 9.81 \times 1.20 = 212.9\text{ J}$$

Second, calculate the translational kinetic energy:

$$E_{tke} = 1/2 \times 18.0 \times 8^2 = 576\text{ J}$$

Third, calculate the rotational kinetic energy:

$$E_{rke} = 1/2 \times 0.50 \times 20.0^2 = 100.0\text{ J}$$

Last, sum to obtain the total energy:

$$E_{tme} = 212.9 + 576 + 100 = 889\text{ J}$$

APPENDIX C

EXAMPLE C.1

Total resistance *(R)* is the product of the resistivity and the length:

$$R = (10^{-4}\ \Omega/\text{m})(10^{-2}\text{ m}),\ \text{which equals } 10^{-6}\ \Omega$$

EXAMPLE C.2a

In series, the total resistance is 10 Ω + 10 Ω, which equals 20 Ω. In parallel, the total resistance is

$$\frac{(10\Omega)(10\Omega)}{10\Omega + 10\Omega}$$

which equals 5 Ω.

EXAMPLE C.2b

In series, the total resistance is 10 Ω + 1 Ω, which equals 11 Ω. In parallel, the total resistance is

$$\frac{(10\Omega)(1\Omega)}{10\Omega + 1\Omega}$$

which equals 0.909 Ω.

Note that when resistors are in series, the total resistance must be greater than the largest resistor in the circuit. However, when resistors are connected in parallel, the total resistance must be less than the smallest resistor in the circuit.

EXAMPLE C.3a

By Ohm's law,

$$I = \frac{V}{R} \Rightarrow I = \frac{20\text{V}}{10\Omega} \Rightarrow I = 2\text{A}$$

EXAMPLE C.3b

By Ohm's law,

$$R = \frac{V}{I} \Rightarrow R = \frac{9\text{ V}}{0.002\text{A}} \Rightarrow R = 4.5\text{k}\Omega$$

EXAMPLE C.3c

By Ohm's law,

$$R = \frac{110\text{V}}{15\text{A}} \Rightarrow R = 7.3\Omega$$

EXAMPLE C.4a

Rearranging the power law,

$$R = \frac{V^2}{P} \Rightarrow R = 10.1\Omega$$