The principal critique against Keynesian conception of the macroeconomy is its reliance on price stickiness, thereby slowing down nominal wages’ and prices’ responses to shocks. These ideas are not consistent with our conception of microeconomic behavior. Yet, the inclusion of microeconomic theory is vital for welfare evaluations, and for considering the efficacy of various policies. The crux of the matter that there are various ways, in our arsenal, for modelling incomplete nominal price and wage adjustments, such as uncertainty, imperfect information and renegotiation costs, incentives, etc. The direct injection of departures from perfect markets and concluding that it matters, is simply loading the model to create the conclusions you want. This is not to say that the departure does not matter, but that we need to fully understand, using our stock of models, the mechanics behind it, so that we can understand the implications due to differing policy alternatives. We will however not proceed beyond Lucas’ model, leaving it for the future.

1 Lucas Imperfect-Information Model

The central idea in this model is the uncertainty about what price changes reflect from the point of view of the producer/firm. In other words, when a firm observes a price change, it is possible that the price is a change in relative prices, so that it will have implications on its production decision. On the other hand, if the price change reflects an aggregate price change in the market on the whole, there would not be any necessity for production decision alterations. In such a case, the rational response would be the optimize the expected value of the objective, and attribute a part of the observed price change to aggregate price change, and the other part to relative price change. This then
implies that aggregate supply would be upward sloping since even when there is just an aggregate price change, producers not being able to discern between the two types of price change, would alter output. Models of this genre typically include two types of shocks: real shocks in the form of preference shifts that alter relative demand for goods in the market; and monetary shocks that changes the aggregate price levels that have no real effects, if the changes are observed, but otherwise, it can have real effects as well.

1.1 Considering Perfect Information

To fully appreciate the impact of imperfect information, we will first examine a perfect information case, thereby understand the necessity of the model in generating incomplete adjustments. To that end, we will first adopt the following assumptions.

1. There are many differing goods produced in the economy.

2. Let the typical producer have the following production function:

\[ Q_i = L_i \tag{1} \]

where as usual, \( Q_i \) is the output produced by the individual producer \( i \), using \( L_i \) amount labour hours/weeks/months.

3. The individual consumer consumes \( C_i \), and based on the usual budget constraint requirement, is equal to the producer’s real income. In other words,

\[ C_i = \frac{P_i C_i}{P} \tag{2} \]

where \( P_i \) is the price of the good produced, and \( P \) is the price index is the price index of all goods consumed in the economy.

4. The individual’s utility is,

\[ U_i = C_i - \frac{1}{\gamma} L_i^\gamma \tag{3} \]

for \( \gamma > 1 \). This thus means that the marginal utility from consumption is constant, and marginal disutility from labour is increasing.

5. The demand is assumed to be dependent on three factors:

(a) real income,
(b) the good’s relative price, and
(c) random disturbance to preferences.

and is assumed to be log-linear,

\[ q_i = y + z_i - \eta(p_i - p) \]  

(4)

where \( y \) is log aggregate real income, \( z_i \) is demand shock to the demand of good \( i \), \( \eta > 1 \) is the elasticity of demand for each good, and \( q_i \) is the log of the demand per producer of good \( i \) (⇒ total demand is \( \ln N + y + z_i - \eta(p_i - p) \)). In addition, \( E(z_i) = 0 \) across all goods, implying that these are relative demand shocks. Also,

\[ y = \bar{q}_i \]  

(5)

\[ p = \bar{p}_i \]  

(6)

6. The aggregate demand in the economy is,

\[ y = m - p \]  

(7)

where \( m = \ln M \), and following your textbook, we will think of as a variable affecting aggregate demand as opposed to money. However, you can read your text regarding the various ways it can be interpreted.

Solving the individual’s choice first, when aggregate price level is known, then the individual’s problem can be written as,

\[ \max_{L_i} U_i = \frac{P_i L_i}{P} - L_i^{\gamma} \]  

(8)

\[ \Rightarrow \frac{P_i}{P} = L_i^{\gamma-1} \]  

(9)

\[ \Rightarrow L_i^* = \left( \frac{P_i}{P} \right)^{\frac{1}{\gamma-1}} \]  

(10)

\[ \Rightarrow \ln L_i = l_i = \frac{1}{\gamma - 1} (\ln P_i - \ln P) = \frac{1}{\gamma - 1} (p_i - p) \]  

(11)

The last equation implies that individual labour supply is increasing in the difference between the relative price of the individual’s output.
The equilibrium for good \(i\) in this model would as usual require demand for good \(i\) equating with supply. For output, since \(Q_i = L_i\), we have \(\ln Q_i = q_i = l_i = \ln L_i\). Therefore,

\[
\frac{p_i - p}{\gamma - 1} = y + z_i - \eta(p_i - p) \quad (12)
\]

\[
\Rightarrow p_i = \frac{(\gamma - 1)y}{1 + \eta\gamma - \eta} + p \quad (13)
\]

However, we know that \(p = \bar{p}_i\), so that we have,

\[
p = \frac{(\gamma - 1)y}{1 + \eta\gamma - \eta} + p \quad (14)
\]

since \(E(z_i) = \bar{z}_i = 0\). The above is true if and only if \(y = 0\) (which means that output in levels, as opposed to log’s is 1), which in turn implies that \(p = m\), and money is neutral in this economy, the standard argument of classical economics. That is money has no real effects.

1.2 Considering Imperfect Information

We are now ready to jazz things up by assuming that producers observe the price of their good, but not the aggregate price level. In other words,

\[
p_i = p + (p_i - p) \quad (15)
\]

\[
\equiv p + r_i \quad (16)
\]

where \(r_i\) is just relative price of good \(i\). This means that although the individual producer would like to rely on the relative prices for his/her decision, because this is not observed, he/she must estimate the relative price, given \(p_i\). To solve this, two simplifying assumptions were made.

1. Individual producer substitutes \(E(r_i|p_i)\) for \(p_i - p\), so that (log) labour supply, and consequently (log) output is

\[
q_i = l_i = \frac{E(r_i|p_i)}{\gamma - 1} \quad (17)
\]

This is known as certainty equivalence, and differs from the maximization of expected utility.
2. The second assumption is what is known today as rational expectations, and says that \( \mathbb{E}(r_i|p_i) \) is derived from the joint distribution of the two random variables. Today, the attack on this assumption has resumed given our current economic debacle. To make this tractable, the following assumptions are made,

(a) \( m \sim N(\mu_m, \sigma_m^2) \), and \( z_i \sim N(0, \sigma_z^2) \), which in turn implies that \( p \) and \( r_i \) are independent and normally distributed, and in turn implies that \( p_i \) is normally distributed.

To solve for the model, we need to derive the aggregate demand in terms of the underlying parameters noted in the above assumptions, \( m \), and \( z_i \), and the endogenous aggregate output in terms of the same random variables. To begin, we solve the individual’s problem of arriving at the expectation relative prices, note that since both \( p_i \) and \( r_i \) are normally distributed,

\[
(r_i|p_i) \sim N \left( \mu_r + \frac{\sigma_r}{\sigma_i} \rho_{r,i} \left( p_i - \mu_i \right), (1 - \rho_{r,i}^2) \sigma_r^2 \right)
\]

\[
\Rightarrow \mathbb{E}(r_i|p_i) = \mu_r + \frac{\sigma_r}{\sigma_i} \rho_{r,i} \left( p_i - \mu_i \right)
= \left( \mu_r - \frac{\sigma_r \rho_{r,i}}{\sigma_i} \mu_i \right) + \frac{\sigma_r \rho_{r,i}}{\sigma_i} p_i
= \alpha + \beta p_i
\]

The last equality simply highlights the general structure of the expectation. Next, note that

\[
\mu_i = \mathbb{E}(p + r_i) = \mu_p + \mu_r
\]

\[
\sigma_i^2 = \text{Var}(p + r_i) = \sigma_p^2 + \sigma_r^2
\]

\[
\text{Cov}(r_i, p_i) = \text{Cov}(r_i, p + r_i)
= \text{Cov}(r_i, p) + \text{Var}(r_i) = \sigma_r^2
\]

\[
\Rightarrow \rho_{r,i} = \frac{\sigma_r^2}{\sigma_r \sigma_i}
= \frac{\sigma_r}{\sqrt{\sigma_p^2 + \sigma_r^2}} = \frac{\sigma_r}{\sqrt{\sigma_p^2 + \sigma_r^2}}
\]
Therefore,

\[
E(r_i|p_i) = \left( \mu_r - \frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} (\mu_p + \mu_r) \right) + \frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} p_i
\]

\[
= \frac{\sigma_p^2 \mu_r + \sigma_r^2 \mu_r - \sigma_r^2 \mu_p - \sigma_r^2 \mu_r}{\sigma_p^2 + \sigma_r^2} + \frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} p_i
\]

\[
= \frac{\sigma_p^2 \mu_r - \sigma_r^2 \mu_p}{\sigma_p^2 + \sigma_r^2} + \frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} p_i
\]

\[
\Rightarrow E(r_i|p_i) = -\frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} \mu_p + \frac{\sigma_r^2}{\sigma_p^2 + \sigma_r^2} p_i
\]

\[
\Rightarrow E(r_i|p_i) = \frac{\sigma_r^2}{\sigma_r^2 + \sigma_p^2} (p_i - \mu_p)
\]

(25)

where the last two equalities follows since \( \mu_r = 0 \). This implies that,

1. Therefore, if \( p_i \geq E(p) \), then \( E(r_i|p_i) \) \( \geq 0 \).

2. The fraction of deviation is \( \frac{\sigma_r^2}{\sigma_r^2 + \sigma_p^2} \).

With this, the individual labour supply of equation (17), and the aggregate output are,

\[
q_i = l_i = \frac{1}{\gamma - \sigma_r^2 + \sigma_p^2} (p_i - E(p))
\]

(26)

\[
\Rightarrow y = q_i = \frac{1}{\gamma - 1 \sigma_r^2 + \sigma_p^2} (p_i - E(p))
\]

\[
\Rightarrow y = \frac{1}{\gamma - \sigma_r^2 + \sigma_p^2} (p - E(p))
\]

(27)

\[
: y = b(p - E(p))
\]

where equation (27) is the \textit{Lucas supply curve}, which says that any deviation of aggregate output from the mean (= 0) is a positive function of the deviation in the price level from its mean. Notice also that the equation is very similar to the \textit{expectations augmented Phillips curve}, and both highlights that deviations in output is positively dependent on deviations in inflation from expectation. The significance here however is that this was derived endogenously.
Combining the aggregate demand of equation \((7)\), and the supply \((27)\), we obtain

\[
y = m - p \\
b(p - E(p)) = m - p \\
p = \frac{m}{1 + b} + \frac{b}{1 + b} E(p) \\
\Rightarrow y = \frac{b}{1 + b} (m - E(p))
\]

We need to now solve for \(E(p)\). To that end, we need to realize first that \textit{ex post}, after \(m\) is determined, the aggregate demand \((28)\) must hold. This then implies that \textit{ex ante}, the expectations on both sides need to hold, that is

\[
E(p) = \frac{E(m)}{1 + b} + \frac{b}{1 + b} E(p) \\
\Rightarrow E(p) = E(m)
\]

Therefore, we can rewrite the aggregate demand, and output as,

\[
p = \frac{1}{1 + b} (E(m) + (m - E(m))) + \frac{b}{1 + b} E(m) \\
= E(m) + \frac{1}{1 + b} (m - E(m))
\]

and

\[
y = \frac{b}{1 + b} (m - E(m))
\]

Before we move on to the key insight, notice that \(p\) is a linear function of the random variable \(m\), which is normally distributed, and consequently, so too is \(p\). The key insight of the model is in the two equations. Recall that the individuals in the model “observe” \(E(m)\), since they form their own expectation. However, the individuals of the model do not observe the deviations, \((m - E(m))\). Notice then that \(E(m)\) affects only \(p\), that is the observed affects only prices. However, the unobserved deviation has real effects in the economy, since it affects aggregate output.

We can now complete the solution by expressing the coefficient \(b\) in terms of the underlying parameters. From before,

\[
b = \frac{\sigma_r^2}{(\gamma - 1)(\sigma_r^2 + \sigma_p^2)}
\]

However from the above,

\[
\sigma_p^2 = \text{Var}(p) = \frac{\sigma_m^2}{(1 + b)^2}
\]
We still need to solve for $\sigma_r^2$. We know that the demand curve is

$$q_i = y + z_i - \eta(p_i - p)$$
$$\Rightarrow q_i = b(p_i - E(p)) + z_i - \eta(p_i - p) \quad (36)$$

and the labour supply curve

$$q_i = l_i = b(p_i - E(p))$$
$$\Rightarrow l_i = b(p_i - p) + b(p - E(p)) \quad (37)$$

Before we move on, notice that $r_i$ is a linear function of $z_i$, the underlying normally distributed random variable. Consequently, $r_i$ is normally distributed. In addition, since $z_i$ and $m$ are independent, then so too are $p$ and $r_i$. This then implies that,

$$z_i - \eta(p_i - p) = b(p_i - p)$$
$$\Rightarrow r_i = p_i - p = \frac{z_i}{b + \eta} \quad (38)$$

$$\Rightarrow \sigma_r^2 = \frac{\sigma_z^2}{(b + \eta)^2} \quad (39)$$

This then implies that,

$$b = \frac{\sigma_z^2/(b + \eta)^2}{(\gamma - 1)[(\sigma_z^2/(b + \eta)^2) + (\sigma_m^2/(1 + b)^2)]}$$
$$= \frac{\sigma_z^2}{\gamma - 1}\frac{1}{\sigma_z^2 + \frac{(b + \eta)^2}{(1 + b)^2}\sigma_m^2} \quad (40)$$

and implicitly defined our solution for $b$ in terms of the underlying parameters. Proof to yourself that $b$ is increasing in $\sigma_z^2$ and decreasing in $\sigma_m^2$.

### 1.3 Implications

The key insight from Lucas's model is that an unexpected or unanticipated deviation in aggregate demand can have real effects on output, and prices, and consequently predicts a positive relationship between output and inflation. Consider the following example, where we assume that money follows a random walk with drift,

$$m_t = \alpha + m_{t-1} + \epsilon_t \quad (41)$$
$$E(m_t) = \alpha + m_{t-1} \quad (42)$$
where $\epsilon_t$ is the white noise term, in relation to Lucas’s model, corresponds to the unobserved component. Substituting this assumption into our aggregate demand, and output equations of the previous section,

\[ p_t = E(m_t) + \frac{1}{1 + b} \epsilon_t \]

\[ \Rightarrow p_t = \alpha + m_{t-1} + \frac{1}{1 + b} \epsilon_t \]  \hspace{1cm} (43)

\[ \Rightarrow p_{t-1} = \alpha + m_{t-2} + \frac{1}{1 + b} \epsilon_{t-1} \]  \hspace{1cm} (44)

and,

\[ y_t = \frac{b}{1 + b} \epsilon_t \]  \hspace{1cm} (45)

Combining these together, then inflation can be written as,

\[ p_t - p_{t-1} = \pi_t = \Delta m_{t-1} + \frac{1}{1 + b} \Delta \epsilon_t \]

\[ = \alpha + \frac{b}{1 + b} \epsilon_{t-1} + \frac{1}{1 + b} \epsilon_t \]  \hspace{1cm} (47)

where $\Delta = (1 - L)$, and $u_t \forall t$ are uncorrelated. Therefore, based on the last equation, output and inflation are positively correlated $\Rightarrow$ the Phillips curve. However, although the model generates this relationship, it cannot be exploited by the policy maker. The idea that even if the policy maker raises growth in money through $\alpha$ without any fanfare, so that it is not known to the public, thereby causing deviations in output, it will largely have only a short run effect, since once individuals realizes this, adjustments will be made, and every other period from then will yield no real change in output. Needless to say, if the change is made with full knowledge of the public, expectations changes instantaneously, and there is no real effects. And there lies the irony, the relationship exists as long as we do not meddle in it, but if we do, the relationship “breaks down”, and this is known as the Lucas critique.

Applying this idea to understanding whether there is any relevance in stabilization policies, consider the following line of reasoning. Monetary policy has an (real) effect, and thus can be considered a policy tool in stabilizing an economy if and only if the policy maker has in its possession information not publicly available. However, if we ponder the idea of interventionist policies, we would realize that it is commonly reactionary in nature, to events and consequent outcomes that are publicly acknowledged. Further, even
if the policy maker has superior information, it is far more efficient for the policy maker to share the information, rather than implementing an elaborate policy plan.

Although the model has its merits, it remains without doubt that publicly announced alterations to monetary policy does have real effects. You need not go to far to notice how private financial institutions hang all every policy hint, and thereby altering their interests, and consequently creatint real effects that permeate through the economy. In addition, the model is reliant on labour supply choices responding to changes in work incentives that are generated by the economy. However, as we had noted in RBC models, employment fluctuations are minimal in reality. Further, in current state of knowledge across the board, information is not difficult to come by, so that the case that the only the policy maker is privy to exclusive information is alittle strong to say the least. Of course, that is not to say that the model cannot be salvaged, and in did it can, but we will relegate this to your graduate school.